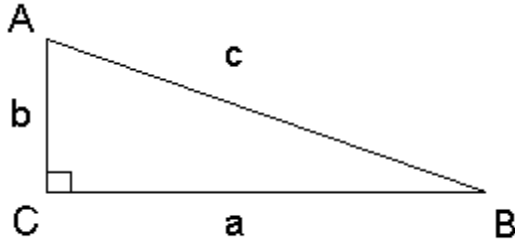


Right triangles and the Pythagorean Theorem

A right triangle is a triangle with one right angle, that is, one angle that measures 90° . By convention we usually call a right triangle $\triangle ABC$, and give the angles and sides the labels



shown in this picture. The vertex of the right angle is labeled with the capital letter C; the other vertices are labeled A and B. The right angle is often marked with a small square.

The lowercase letters c, a, and b are used to represent the lengths of the sides of the right triangle. The label of a vertex and the label of the opposite side correspond. So the side opposite the right angle C is labeled c; this special side is called the hypotenuse of the right triangle. The side opposite angle A is labeled a; the side opposite angle B is labeled b. These other two sides (not the hypotenuse) are called the legs of the right triangle.

The Pythagorean Theorem gives an important relationship among the sides of a right triangle. This Theorem can be used to find the third side of a right triangle when two sides are known.

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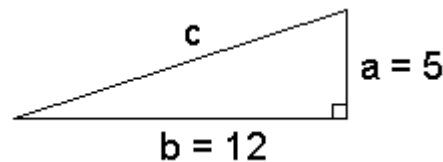
Pythagorean Theorem: Suppose $\triangle ABC$ is a right triangle with right angle C. Suppose c represents the length of the hypotenuse, and a and b are the lengths of the legs. Then

$$c^2 = a^2 + b^2.$$

Also, if $c^2 = a^2 + b^2$ for any triangle $\triangle ABC$, then the triangle is a right triangle with right angle C.

Example 1: Find the hypotenuse of a right triangle whose legs have lengths 5 inches and 12 inches.

If a picture isn't given, draw one and label what you know.



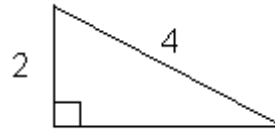
You want to find the hypotenuse, which is labeled c. Use the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 5^2 + 12^2 \\ c^2 &= 25 + 144 = 169 \\ \sqrt{c^2} &= \sqrt{169} = 13 \\ c &= 13 \text{ feet} \end{aligned}$$

Substitute the values you know and solve for c. In the square root step, we take only the positive value since c is a length.

Answers to odd exercises on reverse: 1. 10 feet; 3. $5\sqrt{5}$ inches; 5. $10\sqrt{3}$ feet; 7. $4\sqrt{3}$; 9. $8\sqrt{3}$; 11. $\sqrt{2}$ x ; 13. no, since $c^2 \neq a^2 + b^2$; $36 \neq 16 + 25 = 41$

Example 2: Find the missing side of this triangle.



The unknown side is one of the legs, say a .

Use the Pythagorean Theorem and substitute.

Solve for a . Again, since we know a is a length, we take only the positive square root. Be sure to simplify the radical.

$$c^2 = a^2 + b^2, \quad c = 4 \text{ and } b = 2$$

$$4^2 = a^2 + 2^2$$

$$16 = a^2 + 4$$

$$16 - 4 = 12 = a^2$$

$$\sqrt{12} = \sqrt{a^2} = a$$

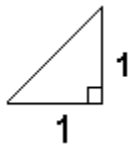
$$a = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

Exercises:

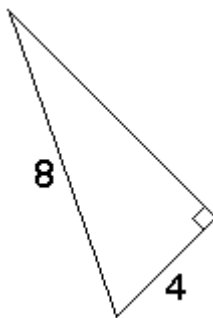
In exercises 1-10, find the missing side or sides of the right triangle.

1. $a = 6$ feet, $b = 8$ feet
2. $b = 8$ meters, $c = 17$ meters
3. $a = 10$ inches, $c = 15$ inches
4. $a = 10$ feet, $b = 20$ feet
5. $b = 10$ feet, $c = 20$ feet

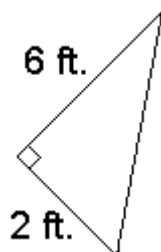
6.



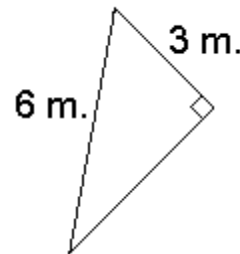
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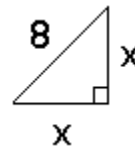
8.



9.



10.



11. If the legs of a right triangle have the same length, what is the length of the hypotenuse? (Hint: Choose a letter to represent the length of a leg.)

12. Suppose a triangle has sides $a = 3$ inches, $b = 4$ inches and $c = 5$ inches. Is angle C a right angle?

13. Suppose a triangle has sides $a = 4$ inches, $b = 5$ inches and $c = 6$ inches. Is angle C a right angle?