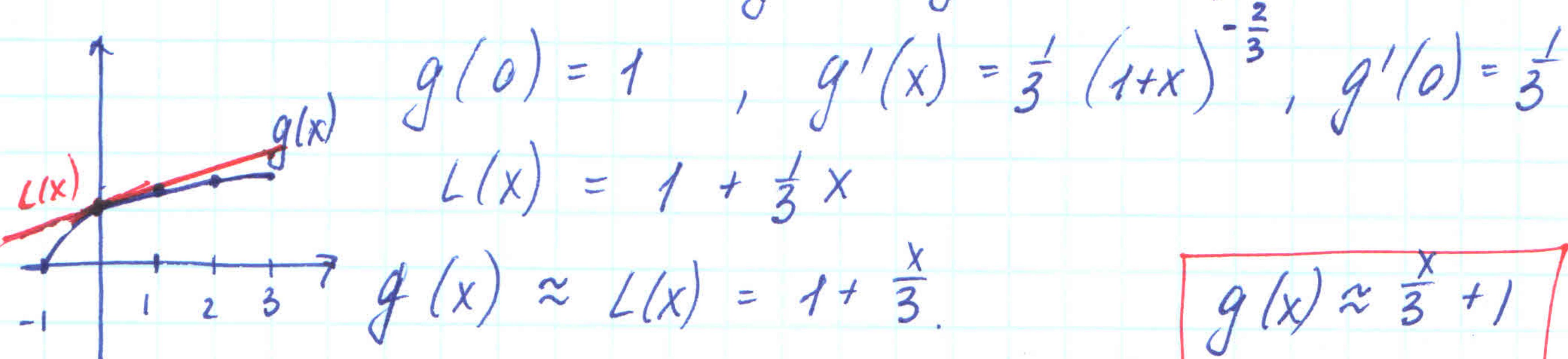


[N6]  $g(x) = \sqrt[3]{1+x}$  at  $a=0$

approximate  $\sqrt[3]{0.95}$  and  $\sqrt[3]{1.1}$ ; Graph.

Solution:  $L(x) = g(a) + g'(a)(x-a)$



$$g(x) \approx \frac{x}{3} + 1$$

$$\sqrt[3]{0.95} \approx \sqrt[3]{1+(-0.05)} \approx 1 + \frac{-0.05}{3} \approx 0.98$$

$$\sqrt[3]{0.95} \approx 0.98$$

$$\sqrt[3]{1.1} = \sqrt[3]{1+0.1} \approx 1 + \frac{0.1}{3} \approx 1.03$$

$$\sqrt[3]{1.1} \approx 1.03$$

[N10]  $\tan x \approx x$  at  $a=0$

1) let  $f(x) = \tan x$

$$f'(x) = \sec^2 x \quad f(0) = 0, \quad f'(0) = 1 \quad L(x) = 0 + 1(x-0)$$

$$f(x) \approx x$$

2)  $|\tan x - x| < 0.1$

$$-0.1 < \tan x - x < 0.1$$

$$-\tan x - \tan x - \tan x$$

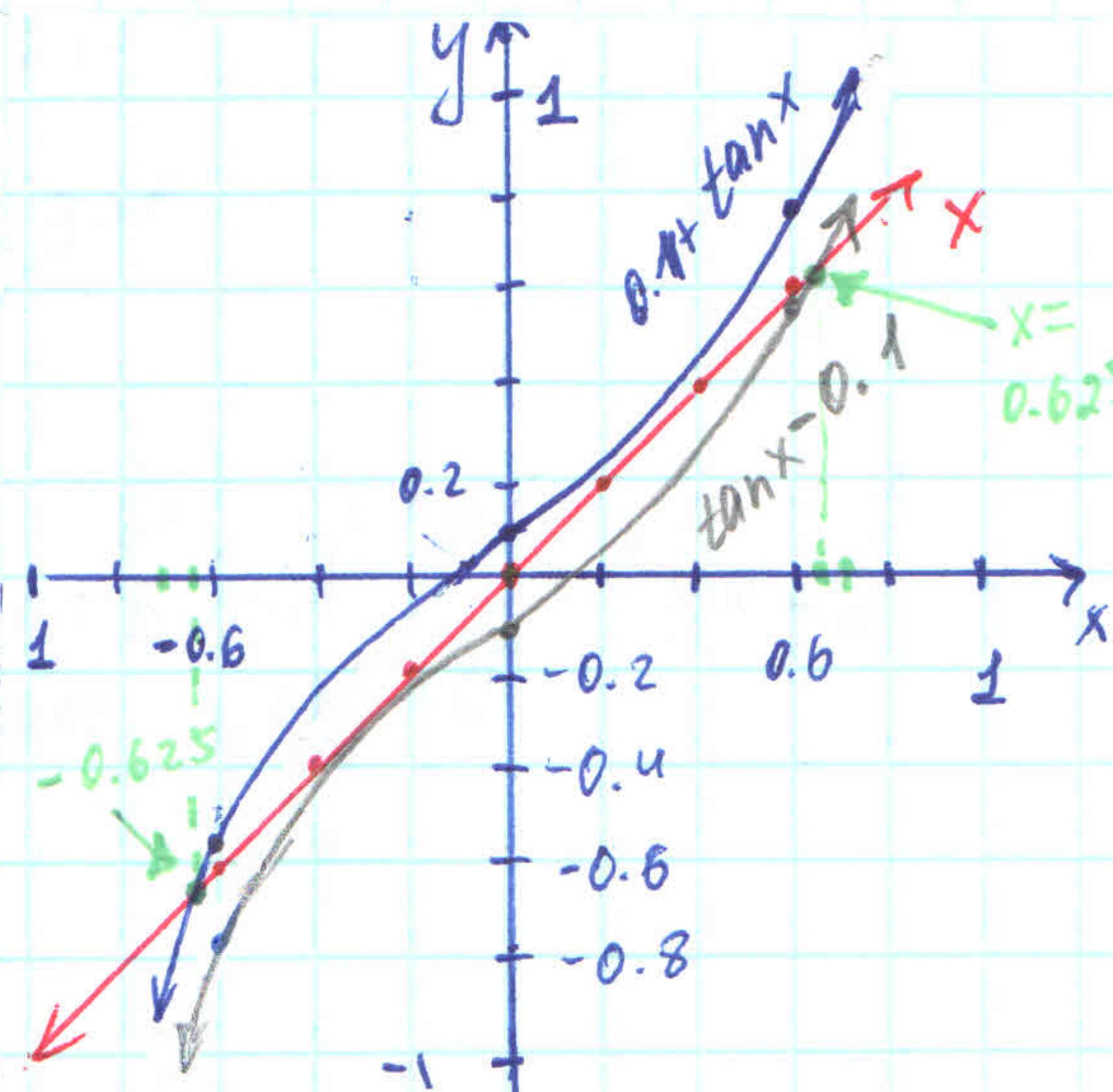
$$-0.1 - \tan x < -x < 0.1 - \tan x$$

$$\star (-1) \quad \star (-1) \quad \star (-1)$$

$$0.1 + \tan x > x > \tan x - 0.1$$

$$-0.625 < x < 0.625$$

(according to graphs)



N18

$$y = \frac{x+1}{x-1}, \quad x=2, \quad dx=0.05$$

Find  $dy$  and evaluate for the given values.

Solution:  $\frac{dy}{dx} = \frac{(x-1) - (x+1)}{(x-1)^2}$

$$\frac{dy}{dx} = \frac{-2}{(x-1)^2}$$

$$dy = \frac{-2 \, dx}{(x-1)^2}$$

$$dy = \frac{-2 \cdot 0.05}{(2-1)^2} = \underline{\underline{-0.1}}$$

N22

$$y = x^3, \quad x = 1, \quad \Delta x = 0.5$$

compute  $\Delta y$  and  $dy$ ; sketch a diagram showing  $dx$ ,  $dy$ ,  $\Delta y$ .

Solution:

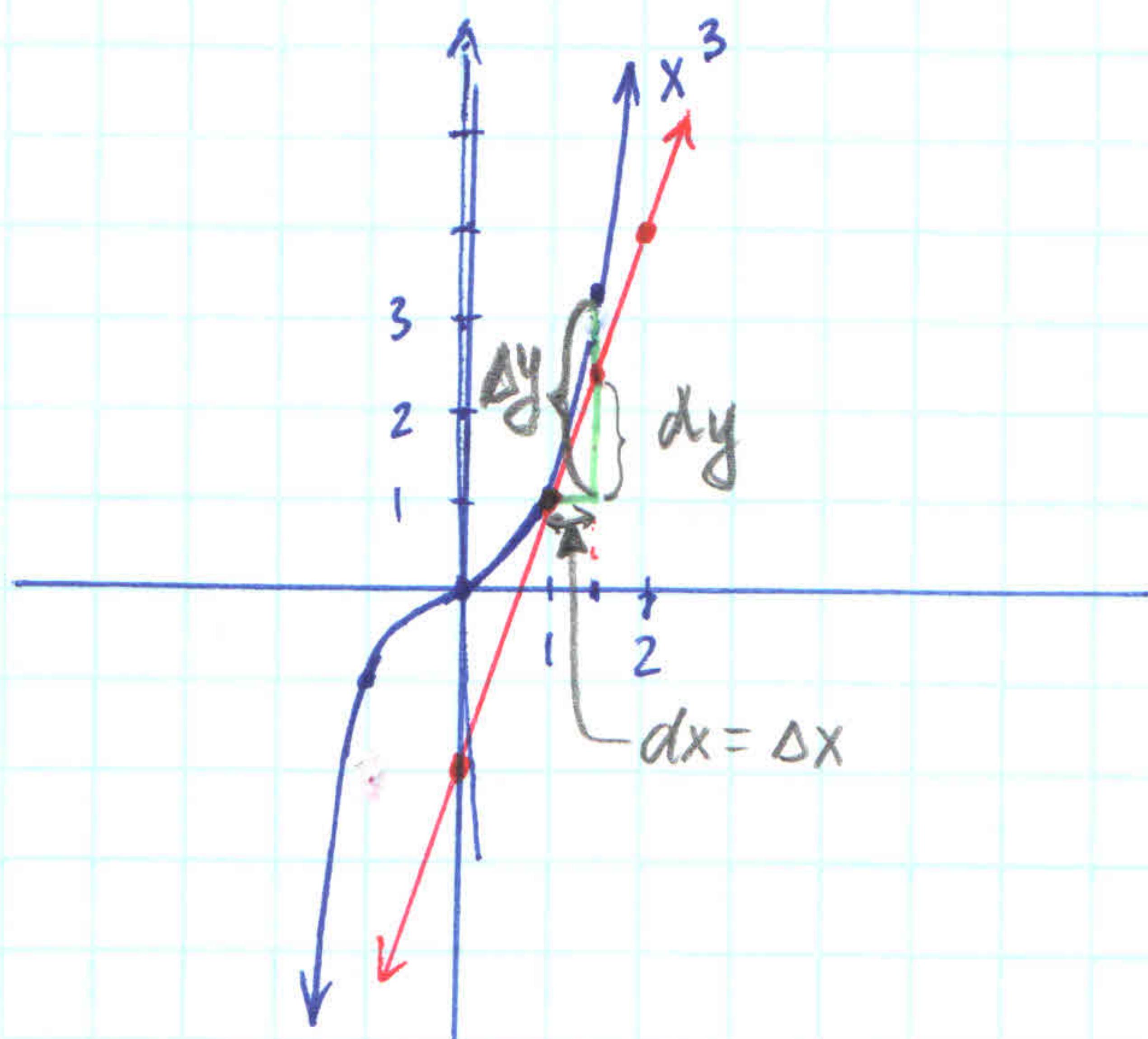
$$\frac{dy}{dx} = 3x^2 \quad dy = 3x^2 dx.$$

$$\text{at } x=1, \quad dx = \Delta x = 0.5, \quad \text{hence } \underline{\underline{dy = 3 \cdot 1^2 \cdot 0.5 = 1.5}}$$

$x$  changes from 1 to 1.5 (because  $\Delta x = 0.5$ ).

$$f(1) = 1^3 = 1, \quad f(1.5) = 1.5^3 = 3.375$$

$$\underline{\Delta y = f(1.5) - f(1) = 3.375 - 1 = 2.375.}$$



$$m = y' = 3x^2 \quad \text{at } x=1, m=3 \\ y - 1 = 3(x-1) \quad (point (1,1)) \quad y = 3x - 2$$

N24  $\sin 1^\circ$  use linear approximation or differentials.

Solution:

1) using linear approximation of  $y = \sin x$  at  $0^\circ$ :

$$y' = \cos x \quad L(x) = f(0) + f'(0)(x-0)$$

$$f(0) = \sin 0^\circ = 0, \quad f'(0^\circ) = \cos 0^\circ = 1$$

hence  $y \approx x$  i.e.  $f(x) = \sin x^\circ \approx x$ .

$$\text{Therefore, } \underline{\sin 1^\circ \approx 1^\circ = \frac{\pi}{180} \approx 0.017}$$

2) using differentials:  $x=0^\circ, \Delta x = 1^\circ$

$$\frac{dy}{dx} = \cos x \quad dy = \cos x dx \quad dy = \cos 0^\circ \cdot 1^\circ = 1^\circ$$

$\Delta y \approx dy = 1^\circ$  (change in function's value is approximately equal to change in tangent's line value)

at  $x=0$ , the  $\sin 0^\circ = 0$ , so  $\underline{\sin 1^\circ \approx 0 + dy = 1^\circ = \frac{\pi}{180} \approx 0.017}$

$$\left\{ \begin{array}{l} 1^\circ - x \\ 180^\circ - \pi \end{array} \right. \quad \begin{array}{l} x \cdot 180^\circ = 1^\circ \cdot \pi \\ x = \frac{1^\circ \cdot \pi}{180^\circ} \approx 0.017 \end{array}$$