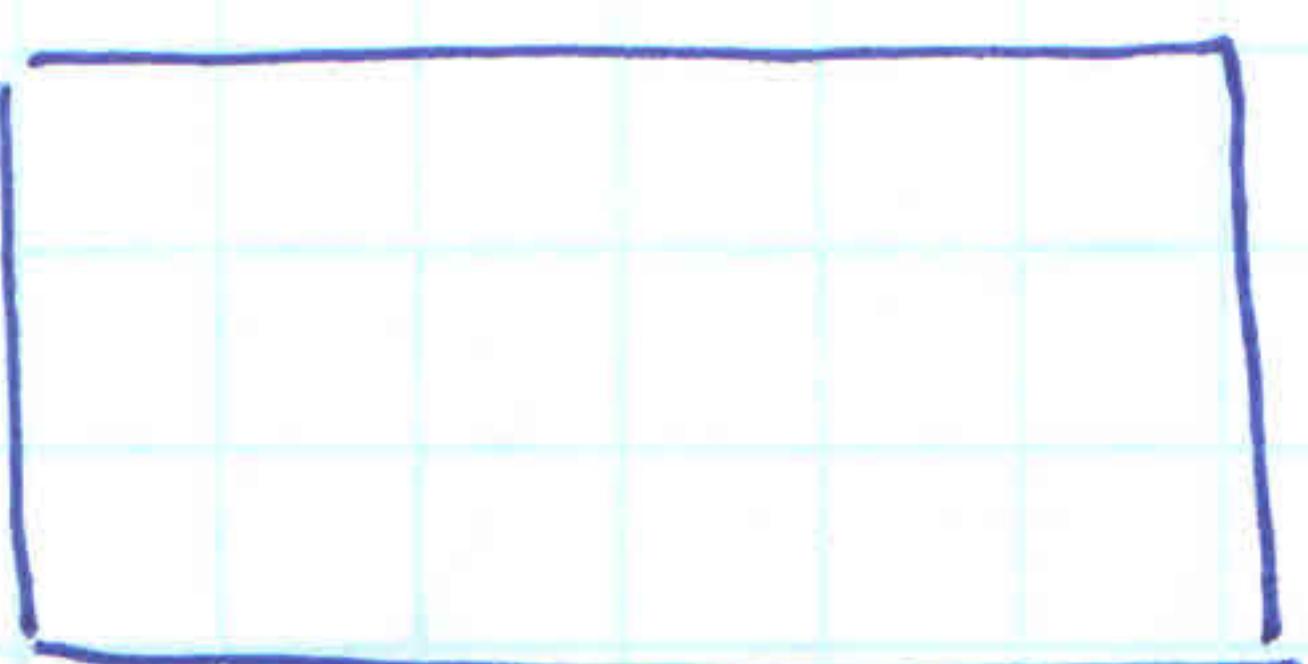


N4



length is increasing at a rate of 8 cm/s
width is increasing at a rate of 3 cm/s

When the length is 20 cm, and the width is 10 cm, how fast is the area of rectangle is increasing?

Solution: $A = L \times W$

$$\frac{dA}{dt} = \frac{d}{dt}(L \times W)$$

$$\frac{dA}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt}$$

8 cm/s 10 cm 20 cm 3 cm/s

$$\frac{dA}{dt} = 80 + 60 = 140 \text{ cm}^2/\text{s}$$

Answer:

$$\boxed{\frac{dA}{dt} = 140 \text{ cm}^2/\text{s}}$$

N12



snowball melts

its surface area decreases at the rate of 1 cm²/min
what is the rate of diameter decrease, when d = 10 cm.

Solution: $A = 4\pi r^2$ - surface area formula of a sphere.

$$\frac{dA}{dt} = \frac{d}{dt}(4\pi r^2)$$

$$\frac{dA}{dt} = 4\pi \frac{d}{dt} r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

1 cm²/min

$$d = 10, \text{ hence } r = 5$$

$$1 = 8\pi \times 5 \frac{dr}{dt}$$

$$1 = 4 \times \pi \times 5 \frac{d(2r)}{dt}$$

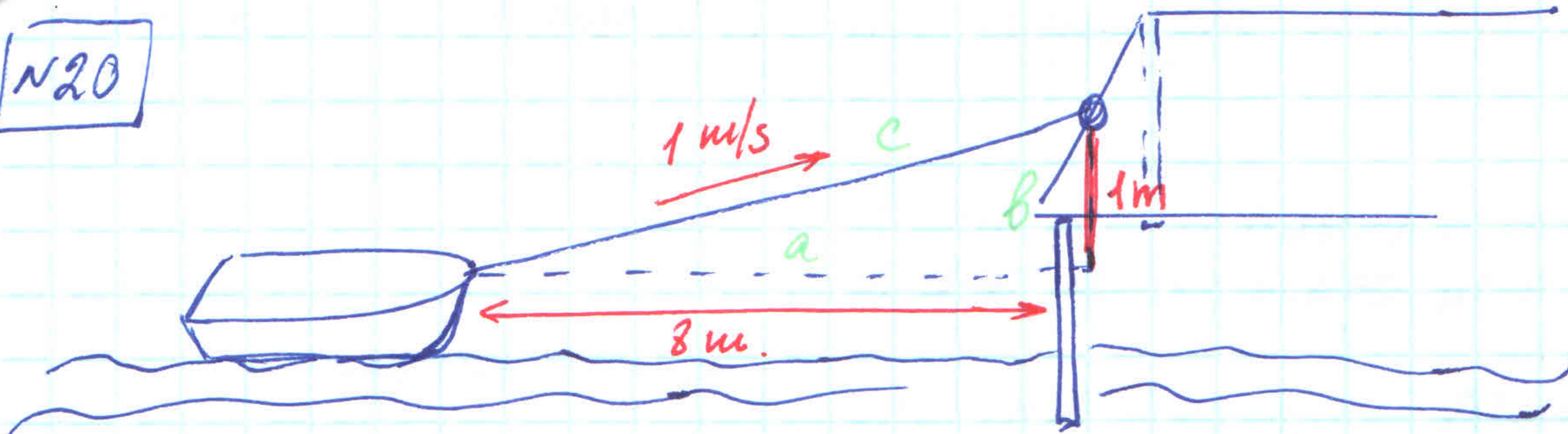
d(2r) = diameter

$$1 = 20\pi \frac{d2r}{dt}$$

$$\frac{d2r}{dt} = \boxed{\frac{1}{20\pi}} \approx 0.016 \text{ cm/min} \quad \leftarrow \text{Answer}$$

Comment: we could also incorporate diameter d in the sphere area formula, and work with it: $A = \pi d^2$ - we'll get the same result.

N20



Solution: let's recall Pythagorean theorem:

$$a^2 + b^2 = c^2$$

$$\frac{d}{dt} (a^2 + b^2) = \frac{d}{dt} c^2 \quad 2a \frac{da}{dt} + 0 = 2c \frac{dc}{dt}$$

$b=1$ (constant), hence

$$\frac{db}{dt} = 0$$

$$2a \frac{da}{dt} = 2c \frac{dc}{dt}$$

$$a \frac{da}{dt} = c \frac{dc}{dt}$$

$$8 \text{ m} \quad \cancel{1 \text{ m/s}}$$

$$8 \frac{da}{dt} = \sqrt{65}$$

$$c^2 = 1^2 + 8^2$$

$$c^2 = 65$$

$$c = \sqrt{65} \text{ (m)}$$

$$\boxed{\frac{da}{dt} = \frac{\sqrt{65}}{8} \approx 1.008 \text{ m/s.}}$$