

N2 $f(t) = 0.04t^4 - 0.08t^3$, $t \geq 0$ t -seconds
 $s = f(t)$ in feet.

(a) $v(t) = f'(t) = \underline{0.04t^3 - 0.12t^2}$

(b) $v(3) = 0.04 \cdot 3^3 - 0.12 \cdot 3^2 = 1.08 - 1.08 = \underline{0} \text{ ft/t}$

(c) the particle is at rest when $v(t) = 0$

$$0.04t^3 - 0.12t^2 = 0 \quad t^2(0.04t - 0.12) = 0$$

$$t = 0, \quad t = \frac{0.12}{0.04} = \frac{12}{4} = 3 \quad \underline{t = 0, 3 \text{ sec.}}$$

(d) the particle is moving in a positive direction when

$$v(t) > 0 \quad 0.04t^3 - 0.12t^2 > 0$$

$$t^2(0.04t - 0.12) > 0$$

$$0.04t - 0.12 > 0$$

$$\underline{t > 3 \text{ sec.}}$$

(e) Find the total distance traveled during the first 8 sec.

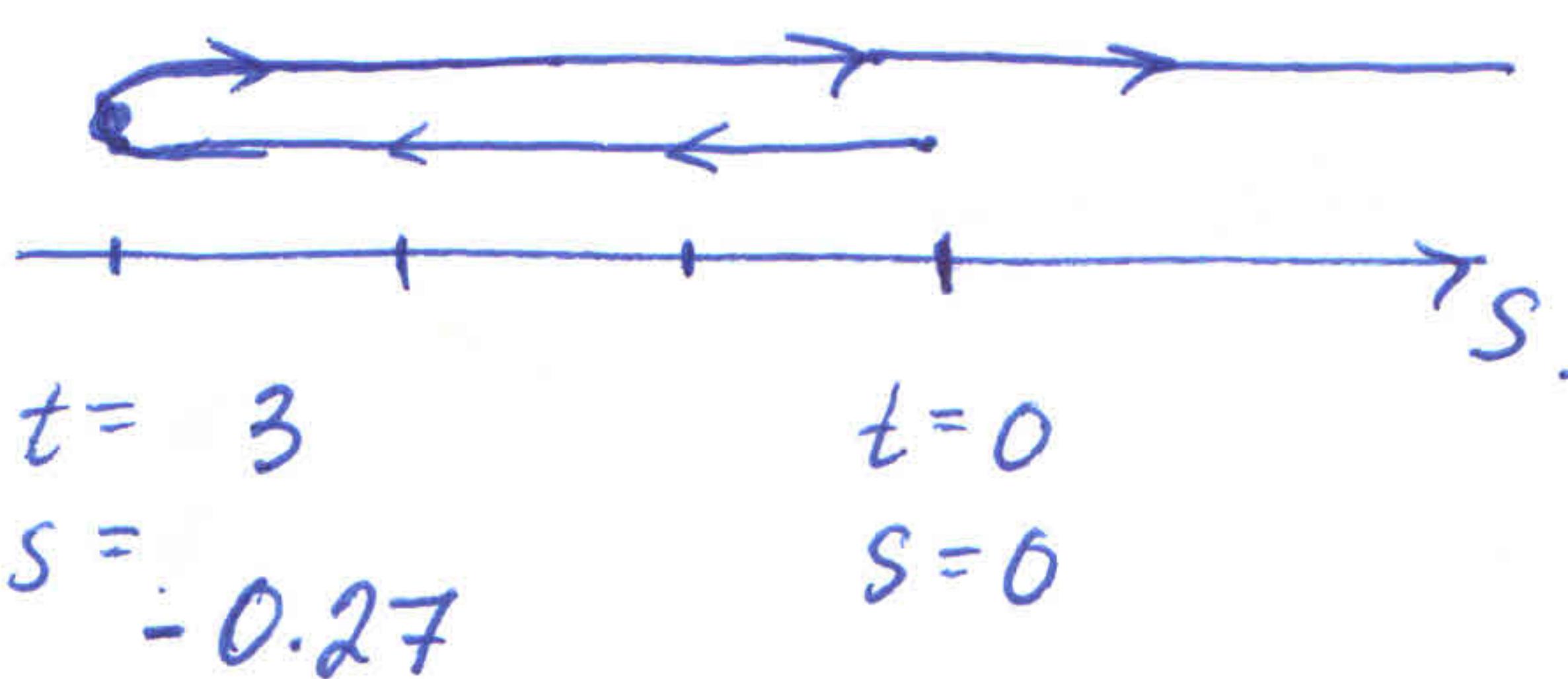
$$D = D_{\text{sec}}(0, 3) + D_{\text{sec}}(3, 8).$$

$$D_{\text{sec}}(0, 3) = \left| f(3) - f(0) \right| = |-0.27 - 0| = 0.27$$

$$D_{\text{sec}}(3, 8) = \left| f(8) - f(3) \right| = |20.48 - (-0.27)| = 20.75$$

$$\underline{D = 20.75 \text{ feet} + 0.27 \text{ feet} = 21.02 \text{ ft}}$$

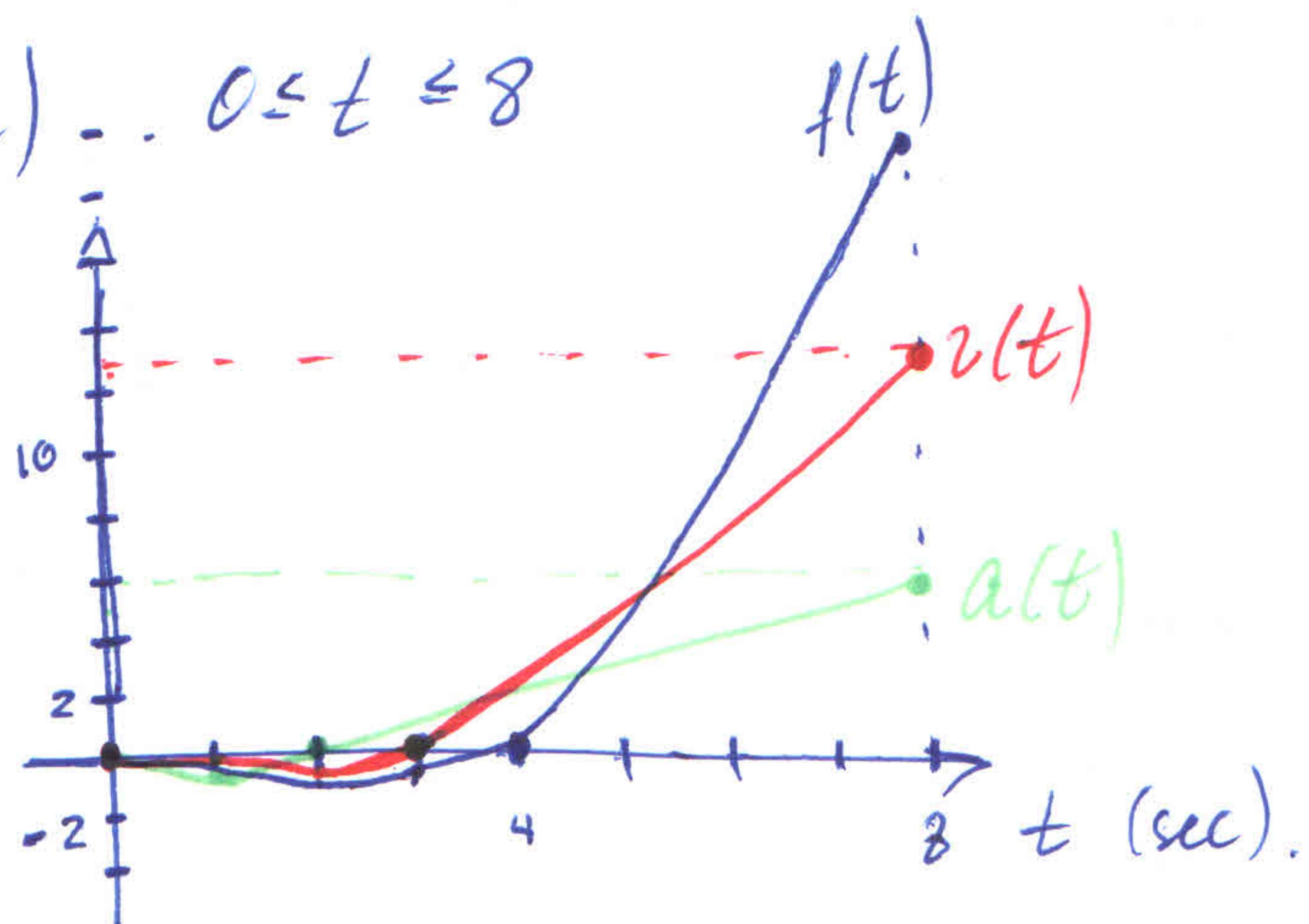
(f) draw a diagram to illustrate the motion of the particle.



$$(g) a(t) = v'(t) = \underline{0.12t^2 - 0.24t}$$

$$a(3) = 0.12 \times 9 - 0.24 \times 3 = \underline{0.36 \text{ ft/s}^2}$$

$$(h) \therefore 0 \leq t \leq 8$$



$$f(t) = 0$$

$$0.01t^4 - 0.04t^3 = 0$$

$$t^3(0.01t - 0.04) = 0$$

$$t = 0 \text{ or } t = 4$$

$$f(1) = 0.01 - 0.04 = -0.03$$

$$f(2) = -0.32$$

$$v(t) = 0.04t^3 - 0.12t^2$$

$$v(t) = 0 \text{ if } t = 0, 3$$

$$v(1) = 0.04 - 0.12 = -0.08$$

$$v(2) = -0.16$$

$$v(8) = 12.8$$

$$a(t) = 0.12t^2 - 0.24t$$

$$a(t) = 0 \text{ if } t = 0, 2$$

$$a(1) = -0.12$$

$$a(8) = 5.76$$

(i) the particle is speeding up when the signs of acceleration and velocity are the same.

- $v(t) > 0$ if $t > 3$, $v(t) < 0$, $t < 3$

- $a(t) = 0.12t^2 - 0.24t > 0$ $0.12t(t-2) > 0$
 $t > 0$ and $t > 2$ or $t < 0$ and $t-2 < 0$
 $t > 2$

hence $a(t) > 0$ if $t > 2$ and $a(t) < 0$ when $t < 2$.

Therefore, the particle is speeding up after 3 seconds,
and on the time interval from 0 sec to 28 seconds.

The particle is slowing down after 2 seconds and before 3 sec.

N8

$$v_{\text{initial}} = 80 \text{ ft/s}$$

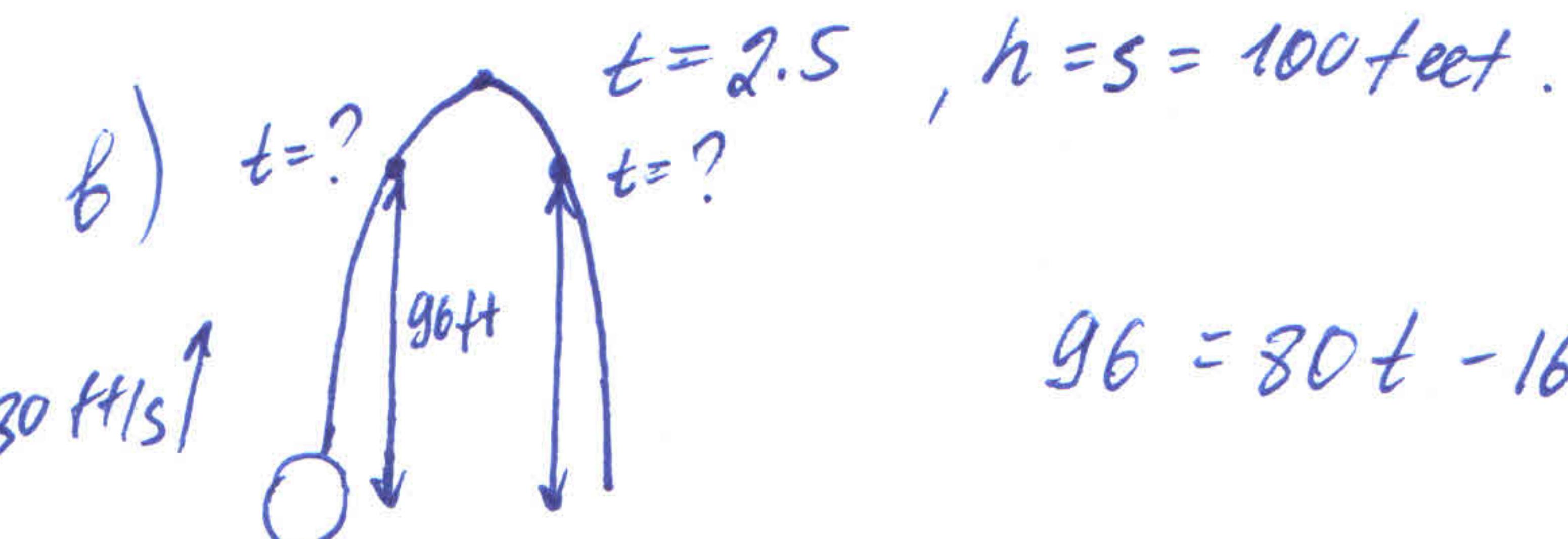
$$s = 80t - 16t^2 - \text{height (ft) after } t \text{ (sec).}$$

a) maximum height is reached when $v(t) = 0$

$$v(t) = s'(t) = 80 - 32t$$

$$v(t) = 0 \quad \text{i.e. } 80 - 32t = 0 \quad t = \frac{80}{32} = \frac{10}{4} = \frac{5}{2} = 2.5 \text{ sec.}$$

$$s(2.5) = 80 \times 2.5 - 16 \cdot (2.5)^2 = \underline{\underline{100 \text{ ft}}}.$$



$$96 = 80t - 16t^2$$

- let's solve it to find
the time.

$$16t^2 - 80t + 96 = 0$$

$$16(t^2 - 5t + 6) = 0$$

$$t^2 - 5t + 6 = 0 \quad \text{or} \quad (t-2)(t-3) = 0 \quad t = \underline{\underline{2 \text{ sec}, 3 \text{ sec}}}$$

$$\text{Hence, } v(2) = 80 - 32 \times 2 = 80 - 64 = 16 \text{ ft/s}$$

$$v(3) = 80 - 32 \times 3 = 80 - 96 = -16 \text{ ft/s.}$$

On the way up, velocity is 16 ft/s, and
on the way down, velocity is -16 ft/s

IN30

$$C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$$

a) Find and interpret $C'(100)$

$$C'(x) = 25 - 0.18x + 0.0012x^2 \quad - \text{marginal cost}$$

$$C'(100) = 25 - 18 + 12 = 19 \approx \text{the cost of producing}$$

101st item

b) $C(101) = 339 + 25 \times 101 - 0.09 \times 101^2 + 0.0004 \times 101^3 = 2358.0304$

$$C(100) = 339 + 2500 - 900 + 400 = 2339$$

Cost of producing the 101st item is 19.0304

the approximation and the actual price are very close
(19 and 19.0304).