

N2

$$y = (2x^3 + 5)^4$$

Solution: $f(g(x))$ $u = g(x)$ - inner function
 $y = f(u)$ - outer function

$$u = 2x^3 + 5 \text{ - inner function}$$

$$y = u^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ where } \frac{dy}{du} = 4u^3 \quad \frac{du}{dx} = 6x^2; \text{ thus}$$

$$y' = \frac{dy}{dx} = 4(2x^3 + 5)^3 \cdot 6x^2 = \boxed{24x^2(2x^3 + 5)^3}$$

N10

$$f(x) = \frac{1}{(1 + \sec x)^2} = (1 + \sec x)^{-2}$$

Solution: $u = 1 + \sec x$

$$y = \frac{1}{u^2} = u^{-2}$$

$$y' = -2(1 + \sec x)^{-3} (1 + \sec x)' = -2(1 + \sec x)^{-3} \cdot \sec x \cdot \tan x$$

$$y' = \boxed{\frac{-2 \sec x \tan x}{(1 + \sec x)^3}} = \frac{-2 \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\left(1 + \frac{1}{\cos x}\right)^3} = \frac{-2 \sin x}{\cos^2 x} \cdot \frac{\cos^3 x}{(\cos x + 1)}$$

OR

$$= \boxed{\frac{-2 \sin x \cos x}{(\cos x + 1)^3}} = \boxed{\frac{-\sin 2x}{(\cos x + 1)^3}}$$

OR

$$\boxed{N18} \quad g(x) = (x^2+1)^3 (x^2+2)^6$$

Solution:

$$\begin{aligned} g(x) &= f_1(x) \cdot f_2(x) & g'(x) &= f_1'(x)f_2(x) + f_1(x)f_2'(x) \\ g'(x) &= \left((x^2+1)^3 \right)' (x^2+2)^6 + (x^2+1)^3 \left((x^2+2)^6 \right)' = \\ &= \underline{3(x^2+1)^2 \cdot 2x} (x^2+2)^6 + (x^2+1)^3 \underline{6(x^2+2)^5 \cdot 2x} = \\ &= 6x(x^2+1)^2(x^2+2)^6 + 12x(x^2+1)^3(x^2+2)^5 = \\ &= 6x(x^2+1)^2(x^2+2)^5 \left(x^2+2 + 2(x^2+1) \right) = \\ &= \underline{6x(x^2+1)^2(x^2+2)^5(3x^2+4)} \end{aligned}$$

$$\boxed{N28} \quad y = \frac{\cos \pi x}{\sin \pi x + \cos \pi x}$$

Solution:

$$\begin{aligned} y' &= \frac{-\sin \pi x \cdot \pi (\sin \pi x + \cos \pi x) - \cos \pi x (\cos \pi x \cdot \pi - \sin \pi x \cdot \pi)}{(\sin \pi x + \cos \pi x)^2} \\ &= \frac{-\pi \sin \pi x (\sin \pi x + \cos \pi x) - \pi \cos \pi x (\cos \pi x - \sin \pi x)}{(\sin \pi x + \cos \pi x)^2} = \\ &= \frac{-\pi \sin^2 \pi x - \cancel{\pi \sin \pi x \cos \pi x} - \pi \cos^2 \pi x + \cancel{\pi \cos \pi x \sin \pi x}}{(\sin \pi x + \cos \pi x)^2} = \frac{-\pi (\sin^2 \pi x + \cos^2 \pi x)}{(\sin \pi x + \cos \pi x)^2} \\ &= \frac{-\pi}{(\sin \pi x + \cos \pi x)^2} = \frac{-\pi}{\sin^2 \pi x + 2\sin \pi x \cos \pi x + \cos^2 \pi x} = \frac{-\pi}{1 + \sin 2\pi x} \end{aligned}$$