

N6 $g(t) = 4 \sec t + \tan t$ differentiate

Solution: $g'(t) = 4(\sec t)' + (\tan t)' =$

$$= \boxed{4 \sec t \tan t + \sec^2 t} = 4 \cdot \frac{1}{\cos t} \cdot \frac{\sin t}{\cos t} + \frac{1}{\cos^2 t} =$$

or

$$= \boxed{\frac{4 \sin t + 1}{\cos^2 t}}$$

N14 $y = \frac{1 - \sec x}{\tan x}$

Solution: $y' = \frac{-\sec x \cdot \tan x \cdot \tan x - (1 - \sec x) \cdot \sec^2 x}{\tan^2 x} =$

$$= \frac{-\sec x \tan^2 x - \sec^2 x + \sec^3 x}{\tan^2 x} =$$

$$= \frac{-\frac{1}{\cos x} \cdot \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} + \frac{1}{\cos^3 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{-\sin^2 x - \cos x + 1}{\cos^3 x} \quad \begin{array}{l} 1 - \sin^2 x = \\ = \cos^2 x \end{array}$$

$$= \frac{\cos^2 x - \cos x}{\cos^3 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{\cos^2 x - \cos x}{\cos x \sin^2 x} = \frac{\cos x (\cos x - 1)}{\cos x \sin^2 x} =$$

$$= \frac{\cos x - 1}{\sin^2 x} = \frac{\cos x - 1}{1 - \cos^2 x} = \frac{\cancel{\cos x - 1}^{-1}}{(\cancel{1 - \cos x})(1 + \cos x)} = \boxed{-\frac{1}{1 + \cos x}}$$

$$\sin^2 x = 1 - \cos^2 x$$

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N22

N22 Find an equation of the tangent line to the curve $y = (1+x)\cos x$ at $(0,1)$

Solution: 1) $y' = m = 1 \cdot \cos x + (1+x)(-\sin x) =$
 $= \cos x - (1+x)\sin x$

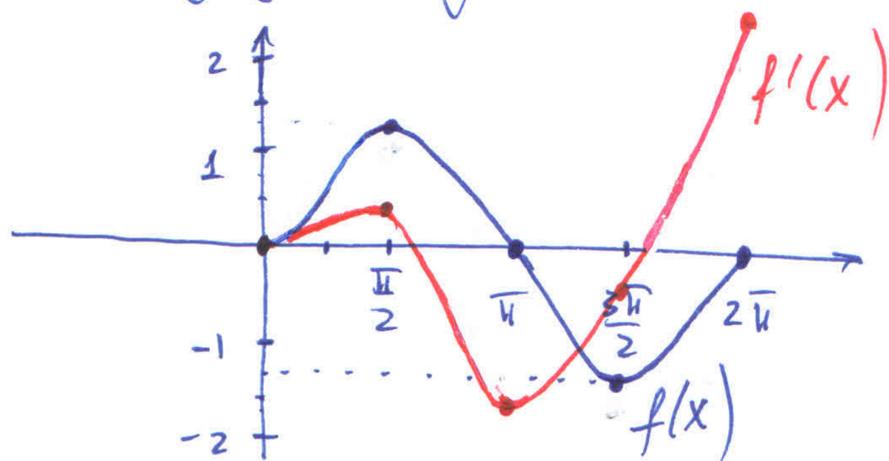
2) $m_{(0,1)} = \cos 0 - (1+0)\sin 0 = 1 - 0 = 1$

3) $y - 1 = 1(x - 0)$ $y = x + 1$

N28 (a) If $f(x) = \sqrt{x} \sin x$, find $f'(x)$

Solution: $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}\sin x + \sqrt{x}\cos x = \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x =$
 $\frac{\sqrt{x}\sin x}{2x} + \sqrt{x}\cos x$

(b) Check to see that your answer to part (a) is reasonable by graphing both f and f' for $0 \leq x \leq 2\pi$



x	f'
0	undefined.
$\pi/2$	≈ 0.4
π	$-\sqrt{\pi}$
2π	≈ 2.5
$3\pi/2$	≈ -0.4

x	f
0	0
$\pi/2$	$\sqrt{\pi/2}$
π	0
$3\pi/2$	$-\sqrt{\pi/2}$
2π	0
$\pi/4$	≈ 0.6

$\approx 2x$

N32 suppose $f\left(\frac{\pi}{3}\right) = 4$, $f'\left(\frac{\pi}{3}\right) = -2$, and let
 $g(x) = f(x) \sin x$ and $h(x) = \frac{\cos x}{f(x)}$.

Find

(a) $g'\left(\frac{\pi}{3}\right)$ (b) $h'\left(\frac{\pi}{3}\right)$.

Solution: (a) $g'(x) = f'(x) \sin x + f(x) \cos x$, hence

$$g'\left(\frac{\pi}{3}\right) = (-2) \sin \frac{\pi}{3} + 4 \cdot \cos \frac{\pi}{3} = -2 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} =$$

$$= -\sqrt{3} + 2 \quad \boxed{g'\left(\frac{\pi}{3}\right) = 2 - \sqrt{3}}$$

(b) $h'\left(\frac{\pi}{3}\right)$ $h'(x) = \frac{-\sin x f(x) - \cos x f'(x)}{(f(x))^2}$ & hence

$$h'\left(\frac{\pi}{3}\right) = \frac{-\sin \frac{\pi}{3} \cdot 4 - \cos \frac{\pi}{3} \cdot (-2)}{4^2} = \frac{-\frac{\sqrt{3}}{2} \cdot 4 - \frac{1}{2} \cdot (-2)}{16} =$$

$$= \frac{-2\sqrt{3} + 1}{16} = \frac{1}{16} - \frac{\sqrt{3}}{8} \quad \boxed{h'\left(\frac{\pi}{3}\right) = \frac{1}{16} - \frac{\sqrt{3}}{8}}$$

?
x =
2x

N44 $\lim_{x \rightarrow 0} \frac{\sin 3x \sin 5x}{x^2}$

Solution: we know that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$; so let's re-write the given limit:

$$\lim_{x \rightarrow 0} \frac{\sin 3x \sin 5x}{x^2} \stackrel{*15}{=} \lim_{x \rightarrow 0} \frac{15 \sin 3x \sin 5x}{3x \cdot 5x} =$$

$$= 15 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 15 \cdot 1 \cdot 1 = \boxed{15}$$