

N4

$y = x - x^3$  at  $(1,0)$  (find slope of the tangent line)

a) using Definition 1:

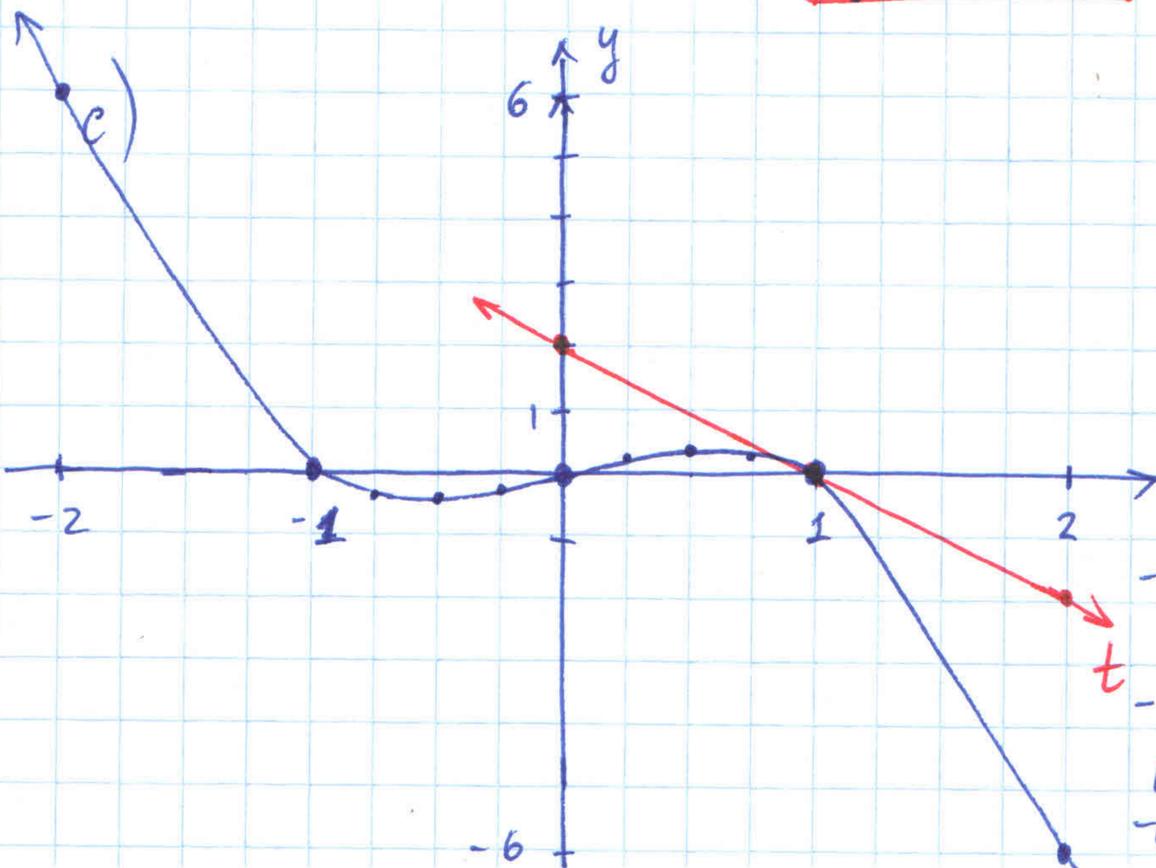
$$\begin{aligned}
 m &= \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - x^3) - (1 - 1^3)}{x - 1} = \\
 &= \lim_{x \rightarrow 1} \frac{x - x^3}{x - 1} = \lim_{x \rightarrow 1} \frac{x(1 - x^2)}{x - 1} = \lim_{x \rightarrow 1} \frac{x(1-x)(1+x)}{x-1} = \\
 &= \lim_{x \rightarrow 1} -x(1+x) = \boxed{-2}
 \end{aligned}$$

using formula 2:  $m = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} =$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{((1+h) - (1+h)^3) - (1 - 1^3)}{h} = \lim_{h \rightarrow 0} \frac{1+h - (1+3h+3h^2+h^3)}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{1+h - 1 - 3h - 3h^2 - h^3}{h} = \lim_{h \rightarrow 0} (-2 - 3h - h^2) = \boxed{-2}
 \end{aligned}$$

b)  $m = -2$ , point  $(1,0)$  let's use  $y - y_1 = m(x - x_1)$

$$y - 0 = -2(x - 1) \quad \boxed{y = -2x + 2} \text{ - equation of the tangent line at point } (1,0)$$



x	y = x - x <sup>3</sup>
0	0
1	0
2	2 - 8 = -6
-1	-1 + 1 = 0
-2	-2 + 8 = 6
0.5	0.375
-0.5	-0.375
0.25	≈ 0.2
-0.25	≈ -0.2
0.75	≈ 0.3
-0.75	≈ -0.3

x	y = -2x + 2
0	2
1	0
2	-2

**N<sup>o</sup> 8**

$y = \frac{2x+1}{x+2}$ , (1,1) find an equation of the tangent line

Solution:

$$1) m = \lim_{x \rightarrow 1} \frac{\frac{2x+1}{x+2} - \frac{2 \cdot 1 + 1}{1+2}}{x-1} = \lim_{x \rightarrow 1} \frac{\frac{2x+1}{x+2} - \frac{3}{3}}{x-1} =$$

$$= \lim_{x \rightarrow 1} \frac{(2x+1) - (x+2)}{(x+2)(x-1)} = \lim_{x \rightarrow 1} \frac{x-1}{(x+2)(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x+2} = \underline{\underline{\frac{1}{3}}}$$

2) use  $y - y_1 = m(x - x_1)$  :  $y - 1 = \frac{1}{3}(x - 1)$ , so

$$y = \frac{1}{3}x + \frac{2}{3}$$

**N<sup>o</sup> 10**

$$y = \frac{1}{\sqrt{x}}$$

a)  $m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h} =$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{a} - \sqrt{a+h}}{\sqrt{a}\sqrt{a+h}h} \cdot (\sqrt{a} + \sqrt{a+h}) =$$

$$= \lim_{h \rightarrow 0} \frac{a - (a+h)}{\sqrt{a}\sqrt{a+h}(\sqrt{a} + \sqrt{a+h})} = \lim_{h \rightarrow 0} \frac{-h}{\sqrt{a}\sqrt{a+h}h(\sqrt{a} + \sqrt{a+h})}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{a}\sqrt{a+h}(\sqrt{a} + \sqrt{a+h})} = \frac{-1}{\sqrt{a} \cdot \sqrt{a} (\sqrt{a} + \sqrt{a})} =$$

$$= \frac{-1}{a \cdot 2\sqrt{a}} = \frac{-1}{2a\sqrt{a}} = \frac{-\sqrt{a}}{2a^2}$$

$$m = \frac{-\sqrt{a}}{2a^2}$$

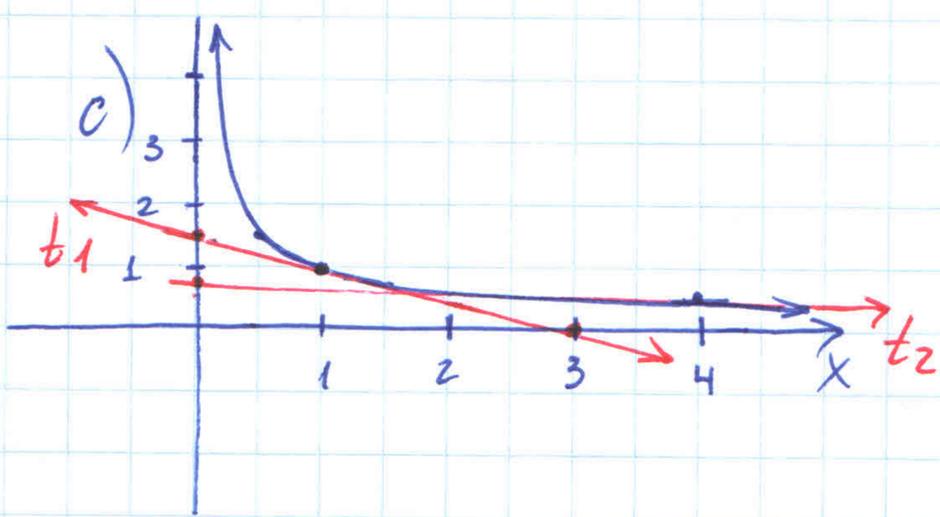
b) at (1,1):  $m = -\frac{1}{2}$   $y - 1 = -\frac{1}{2}(x - 1)$

at  $(4, \frac{1}{2})$   $m = \frac{-2}{2 \cdot 16} = -\frac{1}{16}$   $y - \frac{1}{2} = -\frac{1}{16}(x - 4)$

$$y = -\frac{1}{2}x + \frac{3}{2} \quad t_1$$

$$y = -\frac{1}{16}x + \frac{3}{4} \quad t_2$$

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**Nº14**  $H = 10t - 1.86t^2$

a) - will do after (b)

b)  $v(a) = \lim_{t \rightarrow a} \frac{H(t) - H(a)}{t - a} = \lim_{t \rightarrow a} \frac{10t - 1.86t^2 - (10a - 1.86a^2)}{t - a} =$

$= \lim_{t \rightarrow a} \frac{10t - 1.86t^2 - 10a + 1.86a^2}{t - a} = \lim_{t \rightarrow a} \frac{10t - 10a - 1.86t^2 + 1.86a^2}{t - a} =$

$= \lim_{t \rightarrow a} \frac{10(t - a) - 1.86(t^2 - a^2)}{t - a} = \lim_{t \rightarrow a} \frac{(t - a)(10 - 1.86(t + a))}{t - a} =$

$= \lim_{t \rightarrow a} 10 - 1.86(t + a) = 10 - 1.86 \cdot 2a = 10 - 3.72a$

**$v(a) = 10 - 3.72a$**

a) at  $a = 1$   $v(a) = 10 - 3.72 =$   **$6.28 \text{ m/s.}$**

c)  $H = 0$   $10t - 1.86t^2 = 0$   $t(10 - 1.86t) = 0$

$t = 0$  (not interested, since it is when we throw it up)

$10 - 1.86t = 0$

$t = \frac{10}{1.86} \approx$   **$5.4 \text{ seconds}$**

in about 5.4 seconds, the rock will hit the surface

d)  $v(5.4) = 10 - 3.72 \times 5.4 = -10.088$  (sign for direction)

**$10.088 \text{ m/s.}$**

Nº 20

$y = f(x)$ , tangent to it at  $(4, 3)$ , it also passes  $(0, 2)$ . Find  $f(4)$  and  $f'(4)$ .

Solution:  $f(4) = 3$  (from point  $(4, 3)$ )

$f'(4)$  - slope of the tangent line.

since  $(4, 3)$  and  $(0, 2)$  are points on the tangent line, by using the slope formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  we can find its slope.

$$m = \frac{2 - 3}{0 - 4} = \frac{-1}{-4} = \frac{1}{4}$$

$$f'(4) = \frac{1}{4}$$

Nº 22

Sketch the graph of the function, for which

(1)  $g(0) = g(2) = g(4) = 0$

(2)  $g'(1) = g'(3) = 0$ , ←

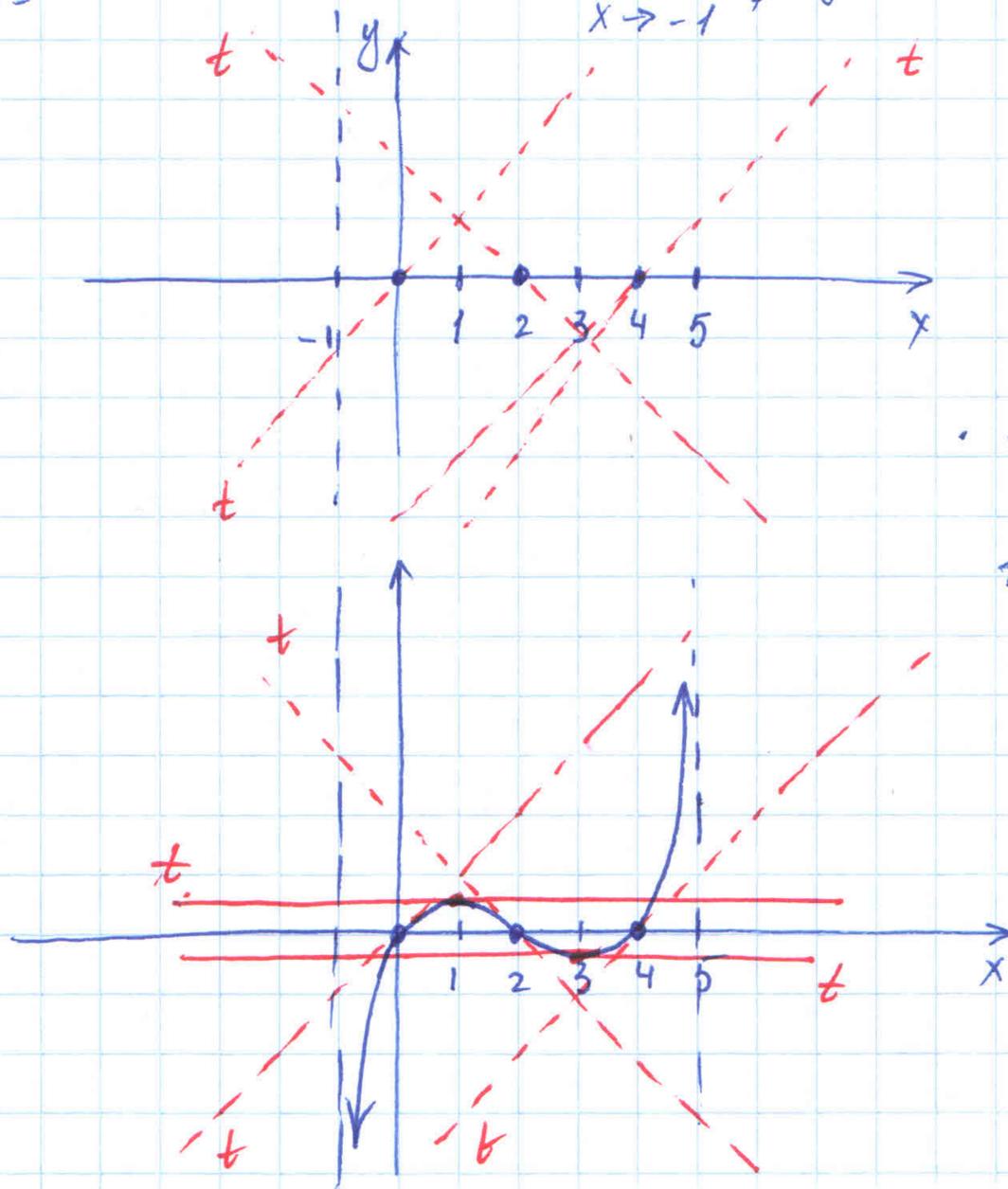
horizontal tangent line

(3)  $g'(0) = g'(4) = 1$  ← like  $y = x$

(4)  $g'(2) = -1$ , ← like  $y = -x$

$\lim_{x \rightarrow 5^-} g(x) = \infty$

(6)  $\lim_{x \rightarrow -1^+} g(x) = -\infty$



- First, we sketched zeros (1),
- then we sketched tangent lines at them (4), (3);
- then we begin to draw the curve from the left  $(-\infty)$  and we satisfy the requirements from (2) and (6) along the way

BT

**N28**  $f(t) = 2t^3 + t$

$$f'(a) = \lim_{t \rightarrow a} \frac{f(t) - f(a)}{t - a} = \lim_{t \rightarrow a} \frac{2t^3 + t - (2a^3 + a)}{t - a} =$$

by formula

$$= \lim_{t \rightarrow a} \frac{2t^3 + t - 2a^3 - a}{t - a} = \lim_{t \rightarrow a} \frac{2(t^3 - a^3) + 1(t - a)}{t - a} =$$

$$= \lim_{t \rightarrow a} \frac{2(t - a)(t^2 + at + a^2) + 1(t - a)}{t - a} =$$

$$= \lim_{t \rightarrow a} \frac{(t - a)(2(t^2 + at + a^2) + 1)}{t - a} =$$

$$= \lim_{t \rightarrow a} 2t^2 + 2at + 2a^2 + 1 = 2a^2 + 2a^2 + 2a^2 + 1 = 6a^2 + 1$$

$f'(a) = 6a^2 + 1$  or  $f'(t) = 6t^2 + 1$

**N32**  $f(x) = \frac{4}{\sqrt{1-x}}$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4}{\sqrt{1-(a+h)}} - \frac{4}{\sqrt{1-a}}}{h}$$

by definition

$$= \lim_{h \rightarrow 0} \frac{4\sqrt{1-a} - 4\sqrt{1-(a+h)}}{h\sqrt{1-(a+h)}\sqrt{1-a}} = \lim_{h \rightarrow 0} \frac{4(\sqrt{1-a} - \sqrt{1-(a+h)})}{h\sqrt{1-(a+h)}\sqrt{1-a}}$$

$$= \lim_{h \rightarrow 0} \frac{4((1-a) - (1-(a+h)))}{h\sqrt{1-(a+h)}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-(a+h)})} = \lim_{h \rightarrow 0} \frac{4 \cdot h}{h\sqrt{1-(a+h)}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-(a+h)})}$$

$$= \lim_{h \rightarrow 0} \frac{4}{\sqrt{1-(a+h)}\sqrt{1-a}(\sqrt{1-a} + \sqrt{1-(a+h)})} = \frac{4}{2(1-a)\sqrt{1-a}}$$

Hence,  $f'(x) = \frac{2}{(1-x)\sqrt{1-x}} = \frac{2\sqrt{1-x}}{(1-x)^2}$

$$\boxed{N34} \quad \lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

Solution: recall definition:  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ ,

compare it with the limit above:

$$f(a) = 2, \quad f(\underline{a+h}) = \sqrt[4]{\underline{16+h}}$$

We can conclude, that  $\boxed{a = 16}$ , and  $\boxed{f(x) = \sqrt[4]{x}}$

(by checking  $f(16) = \sqrt[4]{16} = 2$  we conclude that we got them right)

$$\boxed{N40} \quad s = f(t) \quad s \text{ in m, } t \text{ is seconds.}$$

$t = 5$ , find the velocity and the speed.

$$f(t) = t^{-1} - t = \frac{1}{t} - t$$

Solution:  $s = f(t)$  - distance

• velocity: (the speed) + (direction)

$$\bullet \quad s' = f'(t) = \lim_{x \rightarrow t} \frac{f(x) - f(t)}{x - t} = \lim_{x \rightarrow t} \frac{\left(\frac{1}{x} - x\right) - \left(\frac{1}{t} - t\right)}{x - t} =$$

$$= \lim_{x \rightarrow t} \frac{\frac{1-x^2}{x} - \frac{1-t^2}{t}}{x-t} = \lim_{x \rightarrow t} \frac{(1-x^2)t - x(1-t^2)}{xt(x-t)} =$$

$$= \lim_{x \rightarrow t} \frac{t - x^2t - x + xt^2}{xt(x-t)} = \lim_{x \rightarrow t} \frac{t-x - x^2t + xt^2}{xt(x-t)} =$$

$$= \lim_{x \rightarrow t} \frac{-1(-t+x) - xt(x-t)}{xt(x-t)} = \lim_{x \rightarrow t} \frac{(-1-xt)(x-t)}{xt(x-t)} = \frac{-1-t^2}{t^2}$$

$$\text{So, } \boxed{f'(t) = -\frac{1+t^2}{t^2}}$$

$$\bullet \quad \text{speed} = |f'(5)| = \frac{1+25}{25} = \boxed{\frac{26}{25} \text{ m/s}}, \quad \text{velocity} = f'(5) = \boxed{-\frac{26}{25} \text{ m/s}} \quad 16$$