

**Nº 2**

$$(a) \lim_{x \rightarrow 2} (f(x) + g(x)) = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$$

**not from HW** (b)  $\lim_{x \rightarrow 1} (f(x) + g(x)) = \lim_{x \rightarrow 1} f(x) + \lim_{x \rightarrow 1} g(x) = 1 + \text{doesn't exist} = \text{doesn't exist.}$

$$(c) \lim_{x \rightarrow 0} (f(x)g(x)) = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.5 = 0$$

**not from HW assignment** (d)  $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow -1} f(x)}{\lim_{x \rightarrow -1} g(x)} = \frac{-1}{0} = \text{doesn't exist}$

$$(e) \lim_{x \rightarrow 2} (x^3 f(x)) = \lim_{x \rightarrow 2} x^3 \cdot \lim_{x \rightarrow 2} f(x) = 8 \cdot 2 = 16$$

**not from HW** (f)  $\lim_{x \rightarrow 1} \sqrt[7]{3+f(x)} = \sqrt[7]{\lim_{x \rightarrow 1} (3+f(x))} = \sqrt[7]{3+\lim_{x \rightarrow 1} f(x)} = \sqrt[7]{3+1} = 2$

**N8**

$$\lim_{t \rightarrow 2} \left( \frac{t^2-2}{t^3-3t+5} \right)^2 = \left( \frac{4-2}{8-6+5} \right)^2 = \left( \frac{2}{7} \right)^2 = \frac{4}{49}$$

DD prop-ty

**N14**

$$\lim_{x \rightarrow -1} \frac{x^2-4x}{x^2-3x-4} = \lim_{x \rightarrow -1} \frac{x(x-4)}{(x-4)(x+1)} = \lim_{x \rightarrow -1} \frac{x}{x+1} = \text{doesn't exist}$$

R it didn't help.

**N30**

$$\lim_{x \rightarrow -4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x \rightarrow -4} \frac{\sqrt{x^2+9}-5}{x+4} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} =$$

rationalizing numerator

$$= \lim_{x \rightarrow -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x \rightarrow -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x \rightarrow -4} \frac{(x-4)(x+4)}{(x+4)(\dots)}$$

$$= \lim_{x \rightarrow -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-4-4}{\sqrt{16+9}+5} = \frac{-8}{5+5} = -\frac{8}{10} = -\frac{4}{5}$$

N36 use the squeeze theorem to show that

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0$$

Illustrate by graphing functions  $f, g, h$ .

Solution:

we know that  $-1 \leq \sin \frac{\pi}{x} \leq 1$ , hence

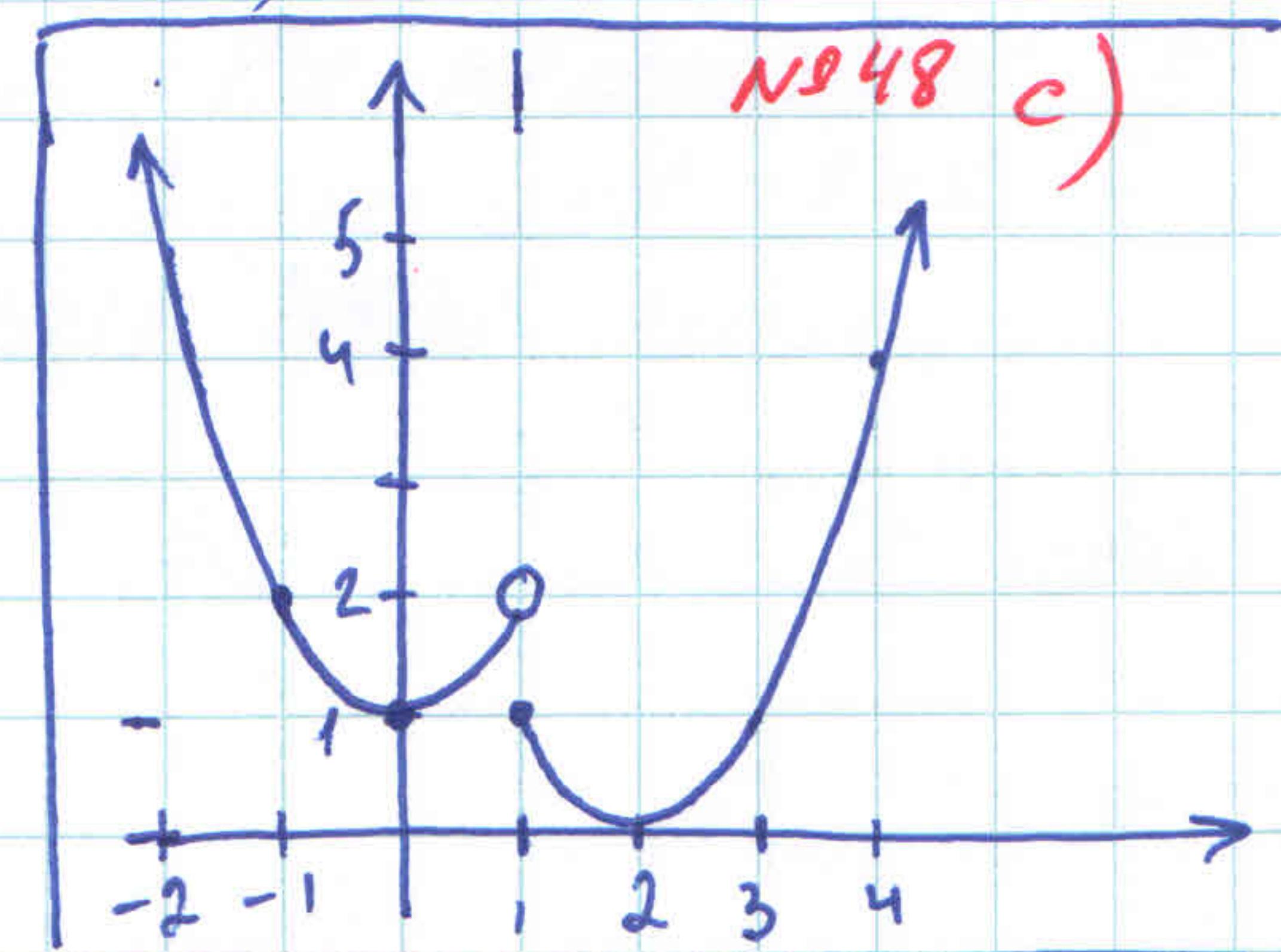
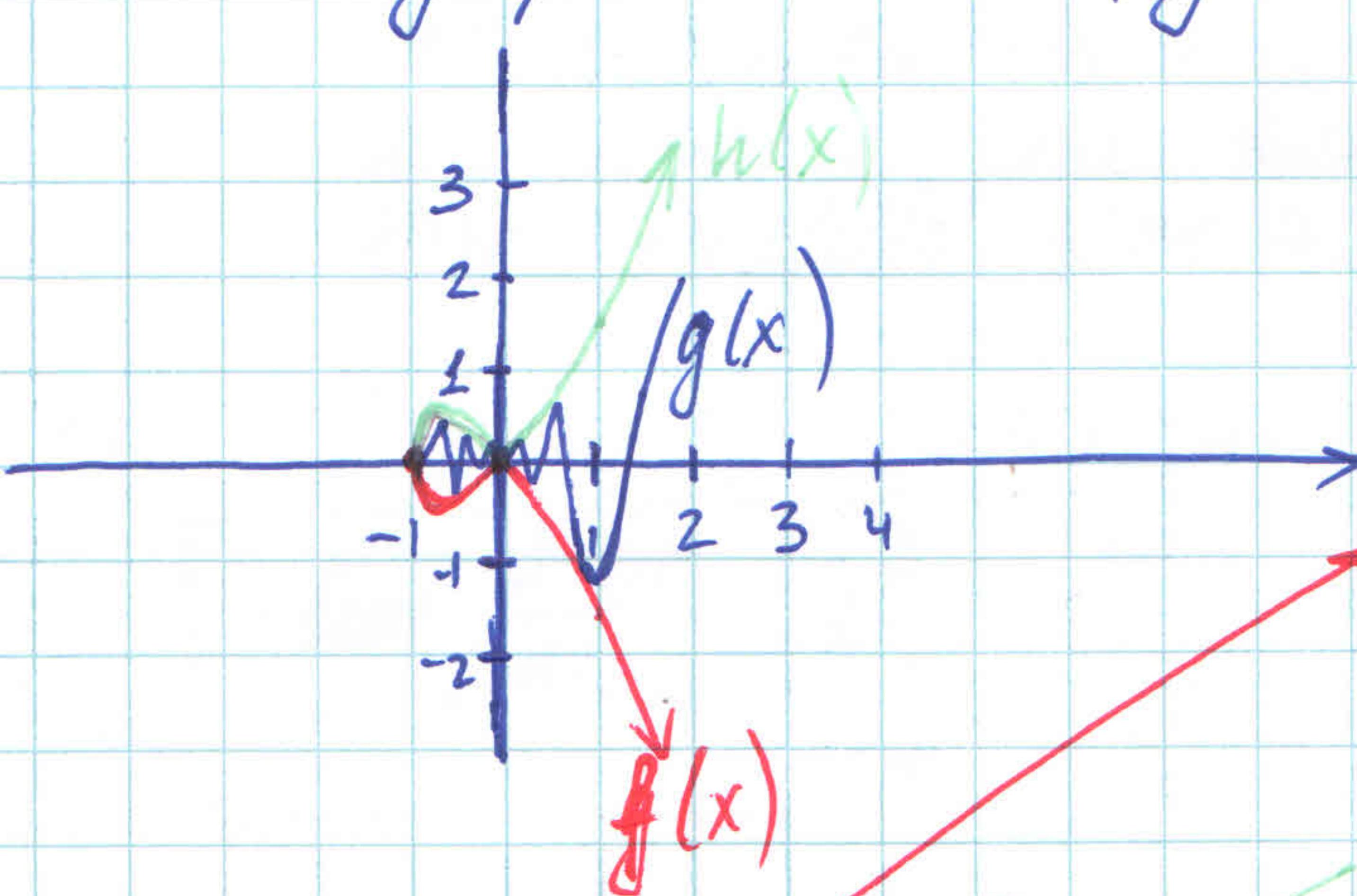
$$\underbrace{-\sqrt{x^3+x^2}}_{f(x)} \leq \underbrace{\sqrt{x^3+x^2} \sin \frac{\pi}{x}}_{g(x)} \leq \underbrace{\sqrt{x^3+x^2}}_{h(x)}$$

let  $f(x)$        $g(x)$        $h(x)$

$$\lim_{x \rightarrow 0} -\sqrt{x^3+x^2} = 0, \quad \lim_{x \rightarrow 0} \sqrt{x^3+x^2} = 0, \text{ hence}$$

by the squeeze theorem  $\lim \sqrt{x^3+x^2} \sin \frac{\pi}{x} = 0.$  ||

Let's see the graphs of  $f(x), g(x), h(x)$



as  $x \rightarrow 0.5$  from the left,  
top is negative, bottom is positive

N43  $\lim_{x \rightarrow 0.5^-} \frac{2x-1}{|2x^3-x^2|} = \lim_{x \rightarrow 0.5^-} \frac{2x-1}{x^2 |2x-1|} = \lim_{x \rightarrow 0.5^-} \frac{(-1) \cdot \frac{1}{x^2}}{2x-1} = \frac{-1}{(0.5)^2} = -4$

N48 a)  $\lim_{\substack{x \rightarrow 1^- \\ (\text{wscd } x^2+1)}} f(x) = 2$ ,  $\lim_{\substack{x \rightarrow 1^+ \\ (\text{wscd } (x-2)^2)}} f(x) = 1$  b)  $\lim_{x \rightarrow 1} f(x)$  doesn't exist

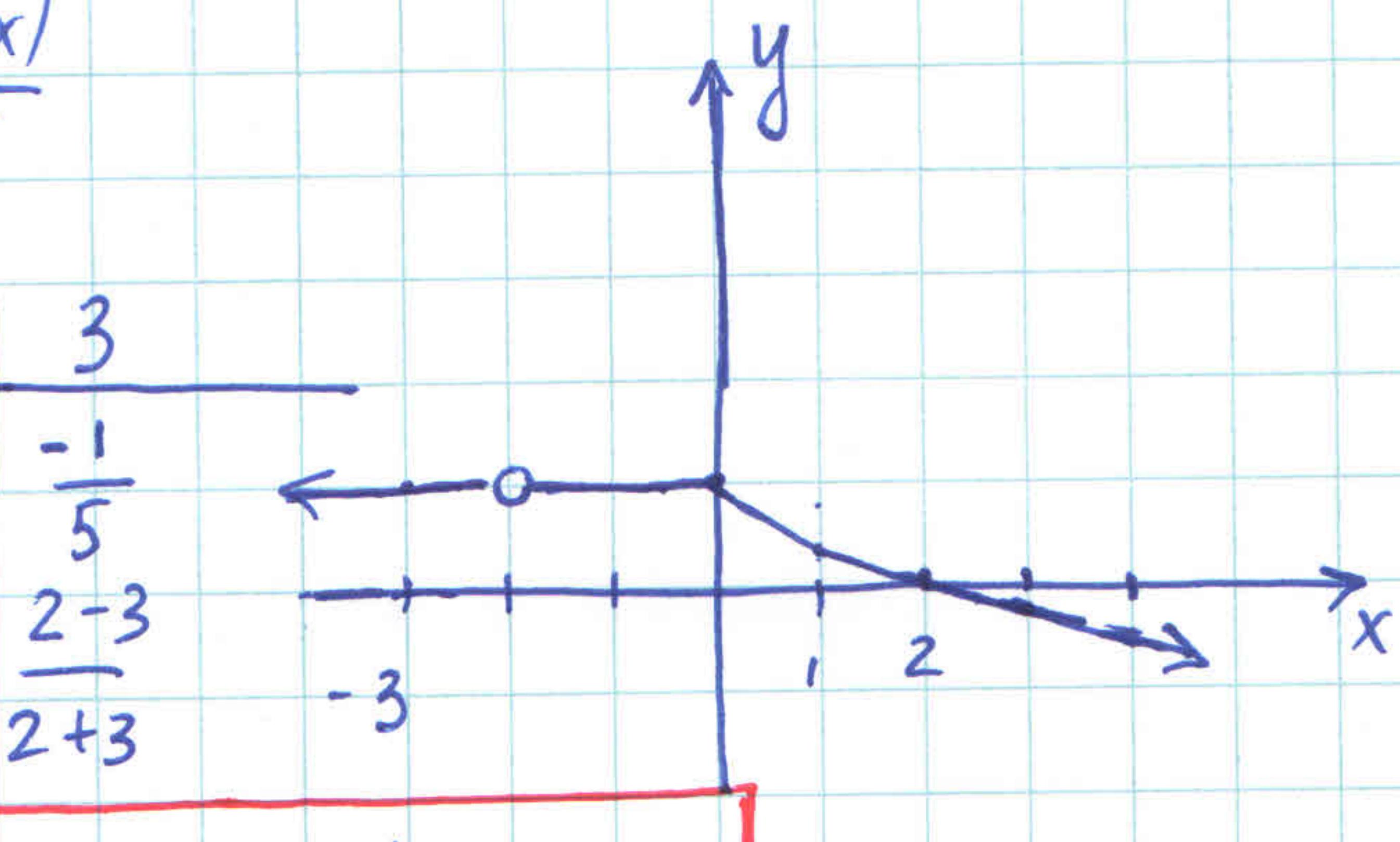
N44

$$\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x}$$

Solution:

1<sup>st</sup> way: let's graph  $f(x) = \frac{2 - |x|}{2 + x}$

x	0	1	-1	-1.5	-2.5	-3	2	3
$f(x)$	1	$\frac{1}{3}$	1	1	1	1	0	$-\frac{1}{5}$
	$\frac{2-1}{2+1}$	$\frac{2-1}{2+1}$	$\frac{2-1.5}{2+1.5}$	$\frac{2-2.5}{2+2.5}$	$\frac{2-3}{2+3}$			
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2-1.5}{2+1.5}$	$\frac{2-2.5}{2+2.5}$	$\frac{2-3}{2+3}$			



by looking at the graph,

$$\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} = 1$$

2<sup>nd</sup> way: let's discuss what happens as  $x \rightarrow -2^+$  and as  $x \rightarrow -2^-$

as  $x \rightarrow -2^+$ , the numerator is positive, the denominator is also positive, and both  $2 - |x|$  and  $2 + x$  have values close to each other, so it looks like we have behaviour of  $\frac{x}{x} = 1$ .

as  $x \rightarrow -2^-$ , the numerator is negative, the denominator is also negative, and both  $2 - |x|$  and  $2 + x$  have values close to each other again.

(At the same time we do understand, that  $f(-2)$  is undef.)

Therefore

$$\lim_{x \rightarrow -2} \frac{2 - |x|}{2 + x} = 1$$