Review problems

Chapter 3

Exercise 1: State whether each statement is *True* or *False*, provide brief explanation.

1. If a function has a local maximum at c, then $f'(c)$ exists and is equal to 0.

2. If $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ has a local minimum at c.

3. $(f(x) + g(x))' = f'(x) + g'(x)$.

4. Continuous functions are always differentiable.

Exercise 1: State whether each statement is *True* or *False*, provide brief explanation.

5. If $f(x) = e^2$, then $f'(x) = 2e$.

6. Differentiable functions are always continuous.

7. If $f'(x) < 0$ for all x in $(0,1)$, then $f(x)$ is decreasing on $(0,1)$.

Exercise 2: The derivative of the function $f(x)$ is $f'(x) = 2(x-1)^2(2x+1)$.

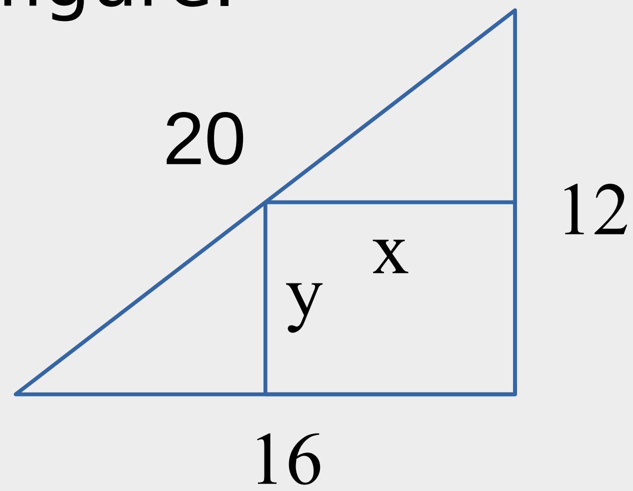
Find all the *critical numbers* of $f(x)$ and determine whether a *relative maximum*, *relative minimum*, or neither occurs at each critical number.

Exercise 3: Sketch the graph of

$$f(x) = 2 - \frac{3}{x} - \frac{3}{x^2}$$

Plot any stationary points and any points of inflection. Show vertical and horizontal asymptotes.

Exercise 4: Find the dimensions of the rectangles of maximum area which may be embedded in a right triangle with sides of length 12, 16, and 20 feet as shown in the figure.



Exercise 5: Does the function $f(x)=\frac{\sqrt{x^2+14x}}{5-15x}$ have a horizontal asymptote?

Exercise 6: Find the *absolute maximum* and *absolute minimum* of the function

$$f(x)=x^3-9x^2+24x+9$$

on the interval $[-1,5]$.

Exercise 7: If 1800 sq. cm. of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Exercise 8: Use Newton’s method to find the second and third approximation of a root of $x^3+x+4=0$, starting with $x_1 = -1$ as initial approximation.

Exercise 9:

Find f, if $f''(x) = 4\text{-}6x$, $f(0) = 2$, and $f(2) = 7$.

Exercise 10:

Use *Newton's Method* to find $\sqrt[3]{30}$ correct to four decimal points.

Exercise 11:

Suppose that $f(0) = 1$ and $f'(x) \leq 2$ for all values of x. Use the *Mean Value Theorem* to determine how large $f(4)$ can possibly be.