Answers to problems 1-8 for the Chapter 3 Final Exam Review

1.
(a) False, the counterexample: the function $f(x)=-|x|$ has a local maximum at $x = 0$, but $f'(0)$ doesn't exist.

(b) True, see Section 3.3, Second Derivative Test

(c) True, one the the derivative's rules: The Sum Rule

(d) False, the counterexample: $f(x)=|x|$ is continuous at $x = 0$, but is not differentiable

(e) False, $f'(x) = 0$

(f) True, see Theorem 4 from Section 2.2'

(g) True, see the Increasing/Decreasing Test in Section 3.3

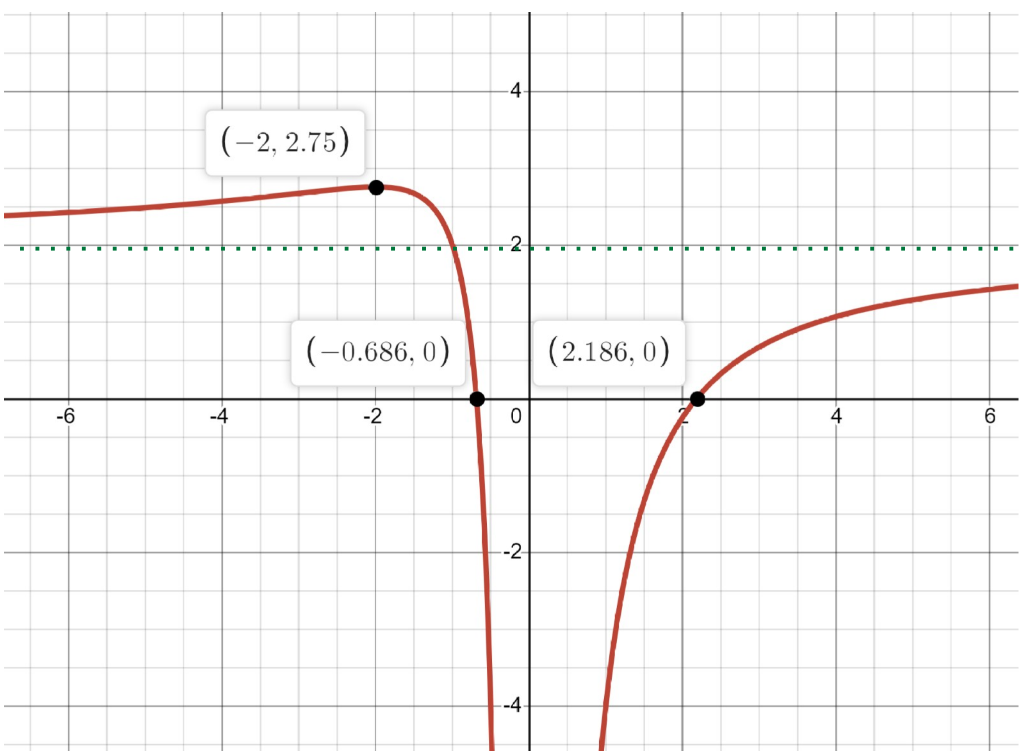
2. $f(x)$ has a local minimum only at $x = -\frac{1}{2}$

3. key moments: local maximum: $(-2, 2\frac{3}{4})$ roots of $f(x)$: $x=\frac{3\pm\sqrt{33}}{4}\approx 2.2, -0.7$

vertical asymptote: $x = 0$, from the left of 0 and from the right of 0, as $x \to 0$, $f(x) \to \infty$
horizontal asymptote: $y = 2$

$f(x)$ is increasing on $(-\infty, -2)$ and $(0,\infty)$
$f(x)$ is decreasing on $(-2,0)$
$f(x)$ is concave up on $(-\infty, -3)$
$f(x)$ is concave down on $(-3,0)$ and $(0,\infty)$
$f(x)$ has inflection point at $(-3, 2\frac{2}{3})$

Additional points we found for plotting: $f(1) = \ldots = -4$, $f(4) = \ldots = 1.0625$



4. x = 8, y = 6

5. f(x) has two horizontal asymptotes: y = $-\frac{1}{15}$ (on the right) and y = $\frac{1}{15}$ (on the left)

$$\lim_{x\to\infty} f(x) = -\frac{1}{15} \quad \text{and} \quad \lim_{x\to-\infty} f(x) = \frac{1}{15}$$

6. on the interval [-1,5],

f(x) has absolute maximum at x=5, it is 509, and

f(x) has absolute minimum at x = -1, it is -25

7. The volume of the box is maximized for the cut-outs of size $5\sqrt{2}$ cm wide and long.
The max. volume in this case is $4000\sqrt{2}$ cubic centimiters

8. $x_2 = -1.5$, $x_3 = -1.5 - \frac{-0.875}{7.75} \approx -1.387097$

9. f(x) = $-x^3 + 2x^2 + 2.5x + 2$

10. f(x) = $x^3 - 30$, and initial approximation $x_1 = 3$, since $\sqrt[3]{27} = 3$ - closest to $\sqrt[3]{30}$ and “extractable”.

$x_2 = 3\frac{1}{9} \approx 3.11111111$, $x_3 \approx 3.10723734$, and finally $x_4 \approx 3.107232506$

We stop with x_4 because the first four digits after the decimal point are the same in x_3 and x_4.
Answer: $\sqrt[3]{30} \approx 3.107232506$

11. f(4) ≤ 9