

Diagnostic Test C : Functions

①

(a) $f(-1) = \boxed{-2}$

(b) $f(2) \approx 2.75$
2.8 in the answers

(c) $f(x) = 2$ when $\boxed{x = -3, 1}$

(d) $f(x) = 0$ when $\boxed{x \approx -2.5, 0.4}$

(e) domain : on what interval f is defined

$$[-3, 3]$$

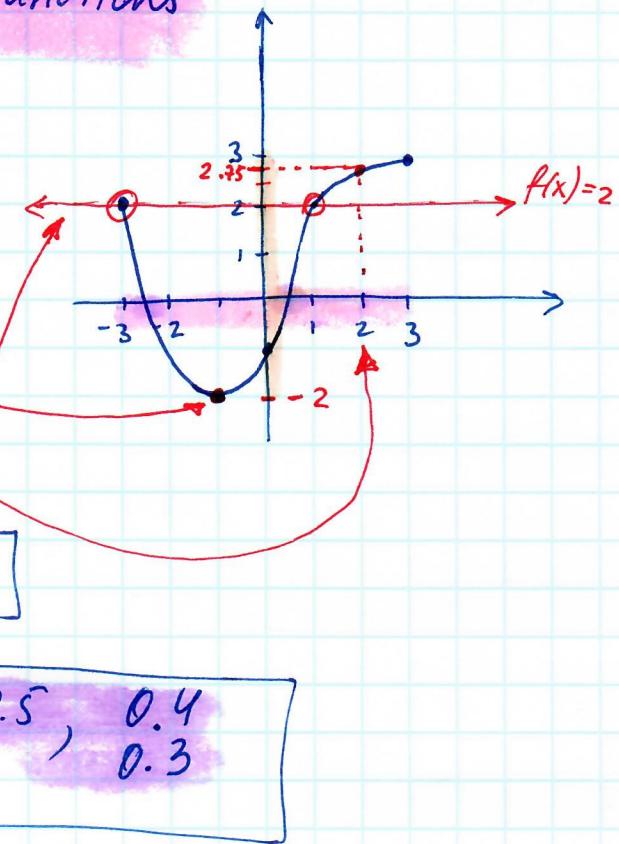
range : values of f

$$[-2, 3]$$

② $f(x) = x^3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$\cancel{+h^3 - x^3} = \frac{12xh + 6x^2h + h^3}{h} = \boxed{12x + 6x^2 + h^2}$$



$$(3) \quad (a) \quad f(x) = \frac{2x+1}{x^2+x-2}$$

zeros of denominator:
 (to be excluded from
 the domain)

$$x^2+x-2=0$$

using trial method...

$$(x+2)(x-1)=0$$

$$x = 1, -2$$

domain: $\mathbb{R} - \{1, -2\}$ ← sets

or

$$\boxed{(-\infty, -2) \cup (-2, 1) \cup (1, \infty)} \quad \text{← intervals}$$

$$(b) \quad g(x) = \frac{\sqrt[3]{x}}{x^2+1}$$

- cube root preserves the sign, so everything is well there
- x^2+1 is never 0

thus, the domain is \mathbb{R} or $\boxed{(-\infty, \infty)}$

$$(c) \quad h(x) = \sqrt{4-x} + \sqrt{x^2-1}$$

- we need to exclude those values of x when

$$4-x < 0 \quad \text{or} \quad x^2-1 < 0 :$$

- $4-x < 0$

$x > 4$
 exclude

- $x^2-1 < 0$

$x^2 < 1$
 $-1 < x < 1$
 exclude!

Therefore, the

domain is:

$$\boxed{(-\infty, -1] \cup [1, 4]}$$

4 (a) $y = -f(x)$
 all ~~not~~ points (x, y) because $(x, -y)$
 so it is reflection about the x-axis
 of $f(x)$

(b) $y = 2f(x) - 1$
 stretching vertically (multiply by 2)
 move 1 mark down (unit)

(c) $y = f(x-3) + 2$
 horizontal shift 3 units to the right
 vertical shift 2 units up

5 comments only, see the answers in the book

$$(a) y = x^3$$

note that if (a, b) belongs to the graph, then $(-a, -b)$ also.

x	1	-1	2	-2
y	+1	+1	8	-8

...

(b) $y = (x+1)^3$ grab the graph from a),
 shift it 1 unit to the left

(c) $y = (x-2)^3 + 3$ grab the graph from a),
 shift it 2 units right and
 3 units up.

(d) $y = 4 - x^2$

grab the graph of $y = -x^2$ ~~↓~~ and shift it 4 units up.

(e) $y = \sqrt{x}$

note that $x \geq 0$

(f) $y = 2\sqrt{x}$

stretch the graph of $y = \sqrt{x}$ from e by multiplying every point's y-coordinate by 2.

(g) $y = -2^x$

note that $y < 0$

(h) $y = 1 + x^{-1}$

graph $y = \frac{1}{x}$, shift is 1 unit up.

6 (a) $f(-2) = 1 - (-2)^2 = 1 - 4 = \boxed{-3}$
using $\frac{1-x^2}{-x^2}$
because $-2 < 0$

$f(1) = 2 \cdot 1 + 1 = 2 + 1 = \boxed{3}$
using $2x+1$
because $1 > 0$

(b) see the answer in the book

! try using a graphing calculator

(7) $f(x) = x^2 + 2x - 1$
 $g(x) = 2x - 3$

(a) $f \circ g (x) = f(g(x)) = f(2x-3) =$
 $= (2x-3)^2 + 2(2x-3) - 1 =$
 $= 4x^2 - 12x + 9 + 4x - 6 - 1 = \boxed{4x^2 - 8x + 2}$

(b) $g \circ f (x) = g(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 =$
 $= 2x^2 + 4x - 2 - 3 = \boxed{2x^2 + 4x - 5}$

(c) $g \circ g \circ g (x) = g(g(g(x))) = g(g(2x-3)) =$
 $= g(2(2x-3) - 3) = g(4x - 6 - 3) = g(4x-9) =$
 $= 2(4x-9) - 3 = 8x - 18 - 3 = \boxed{8x-21}$