

Diagnostic test B : Analytic Geometry

- ① (a) point $(2, -5)$, slope $m = -3$

use point-slope form of equation of straight line :

$$y - y_0 = m(x - x_0)$$

\uparrow \uparrow \uparrow
 -5 -3 2

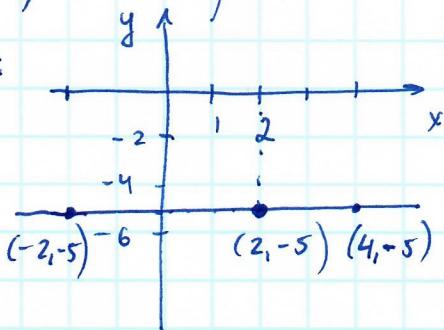
$$y - (-5) = (-3)(x - 2), \text{ then simplify/re-write:}$$

$$y + 5 = -3x + 6$$

$y = -3x + 1$

- (b) point $(2, -5)$, and is parallel to the x -axis

let's draw it :



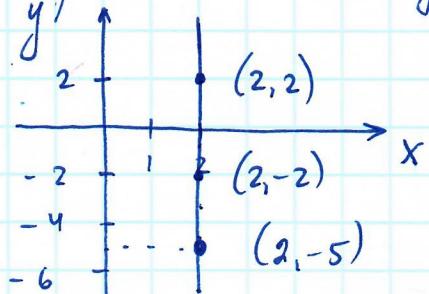
we can see that y -coordinate is fixed, hence

$y = -5$

- (c) point $(2, -5)$, and is parallel to the y -axis.

Similarly to case (b),
 x -coordinate is fixed

$x = 2$



(d) point $(2, -5)$ and is parallel to the line with equation $2x - 4y = 3$.

We know: parallel lines have the same slopes

STEP 1: find the slope of the line w/ eq. $2x - 4y = 3$

- solve for y :

$$2x - 4y = 3$$

$$\frac{2x - 3}{4} = \frac{4y}{4}$$

$$\frac{1}{2}x - \frac{3}{4} = y$$

$$\text{slope} \quad m = \frac{1}{2}$$

slope y-intercept
↓ ↓ $(0, b)$

recall $y = mx + b$

STEP2: Point $(2, -5)$ and slope $m = \frac{1}{2}$

This time, instead of using $y - y_0 = m(x - x_0)$,

let's use $y = mx + b$ twice.

- this is just another method of finding equation of a straight line using a point and a slope

1st time: $y = mx + b \rightarrow -5 = \frac{1}{2} \cdot 2 + b$

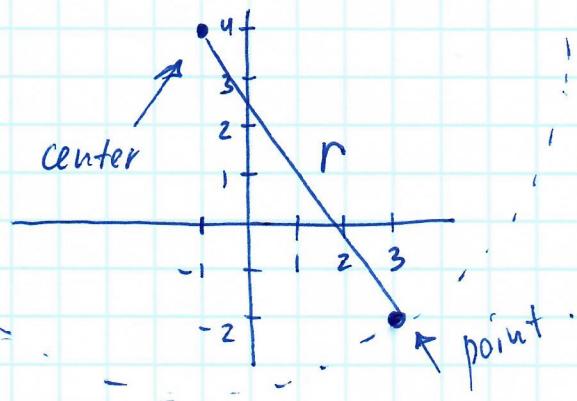
$$b = -6$$

$$\underline{\text{2nd time}}: y = mx + b$$

$$\rightarrow \boxed{y = \frac{1}{2}x - 6}$$

(2)

circle

center: $(-1, 4)$ point: $(3, -2)$ 

equation of circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is center of the circle, and r is radius.- we know h and k :

$$(x - (-1))^2 + (y - 4)^2 = r^2$$

$$(x + 1)^2 + (y - 4)^2 = r^2$$

let's find radius r by finding the distance between the center point $(-1, 4)$ and the point $(3, -2)$

$$r = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 3)^2 + (4 - (-2))^2} = \\ = \sqrt{16 + 36} = \sqrt{52}, \text{ hence } r^2 = 52$$

Equation of the circle: $(x + 1)^2 + (y - 4)^2 = 52$

(3)

Find the center and the radius of the circle w/ equation $\underline{x^2 + y^2 - 6x + 10y + 9} = 0$

- we need to complete the squares for x and for y , i.e. $(x \pm h)^2 + (y \pm k)^2 \pm \text{const} = 0$

consider $x^2 - 6x$: $(x-3)^2 = \underline{x^2 - 6x} + 9$

consider $y^2 + 10y$: $(y+5)^2 = \underline{y^2 + 10y} + 25$

so extra!

$$x^2 + y^2 - 6x + 10y + 9 = (x-3)^2 + (y+5)^2 - 25$$

$$(x-3)^2 + (y+5)^2 - 25 = 0$$

$$(x-3)^2 + (y+5)^2 = 25$$

compare
with

$$(x-h)^2 + (y-k)^2 = r^2$$

(h,k) is center
 r is radius

Mence, radius is 5
and the center is $(3, -5)$

(4) A (-7, 4) B (5, -12)

$$(a) m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 4}{5 - (-7)} = \frac{-16}{12} = -\frac{4}{3}$$

$\uparrow \quad \uparrow$
5 -7

slope

(b) using point $(-7, 4)$ and slope $m = -\frac{4}{3}$ and equation $y - y_0 = m(x - x_0)$

$$y - 4 = -\frac{4}{3}(x - (-7))$$

$$y - 4 = -\frac{4}{3}x - \frac{4}{3} \cdot 7 \quad \mid \quad 4 - \frac{4}{3} \cdot 7 = 4 - \frac{28}{3} = \frac{12-28}{3} =$$

$$= -\frac{16}{3}$$

$$y = -\frac{4}{3}x - \frac{16}{3}$$

* y-intercept: $(0, -\frac{16}{3})$ *

To find x-intercept, make $y = 0$

$$0 = -\frac{4}{3}x - \frac{16}{3} \quad \text{find } x:$$

$$\left(-\frac{3}{4}\right) \times \frac{16}{3} = -\frac{4}{3}x \times \left(-\frac{3}{4}\right)$$

$$-\frac{16}{4} = x$$

$$x = -4 \quad *$$

Equation: $y = -\frac{4}{3}x - \frac{16}{3}$

y-intercept: $(0, -\frac{16}{3})$

x-intercept: $(-4, 0)$

(c) midpoint of AB :

$$A(-7, 4) \quad B(5, -12)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$x = \frac{x_1 + x_2}{2}$$

$$y = \frac{y_1 + y_2}{2}$$

$$x = \frac{-7+5}{2} = \frac{-2}{2} = -1$$

$$y = \frac{-12+4}{2} = \frac{-8}{2} = -4$$

midpoint : $\boxed{(-1, -4)}$

(d) length of AB : use distance formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-7-5)^2 + (4-(-12))^2} =$$
$$= \sqrt{144 + 256} = \sqrt{400} = \boxed{20}$$

(e) equation perpendicular to AB , passes through its midpoint. $(-1, -4)$.

Recall: slopes of perpendicular lines are negative reciprocals of each other, i.e. $m_1 \cdot m_2 = -1$ or

$$m_2 = -\frac{1}{m_1} \quad \leftarrow \text{flip and change the sign}$$

Equation of line \overline{AB} : $y = -\frac{4}{3}x - \frac{16}{3}$ see (b)

hence $m_{AB} = -\frac{4}{3} \quad \downarrow \text{flip and change the sign}$

Therefore, $m = \frac{3}{4}$ slope of the line \perp to \overline{AB} .

We have: slope $m = \frac{3}{4}$ and point $(-1, -4)$
 Let's use $y - y_0 = m(x - x_0)$

$$y - (-4) = \frac{3}{4}(x - (-1))$$

$$y + 4 = \frac{3}{4}(x + 1)$$

$$y + 4 = \frac{3}{4}x + \frac{3}{4}$$

$$\begin{aligned} y &= \frac{3}{4}x + \frac{3}{4} - 4 \\ y &= \frac{3}{4}x - \frac{13}{4} \end{aligned}$$

$$\frac{3}{4} - 4 = \frac{3}{4} - \frac{16}{4} = -\frac{13}{4}$$

if we want to bring it to the standard form $Ax + By = C$,
 then multiply both sides of by 4, and collect terms
 with x and y on one side:

$$4x + y = \left(\frac{3}{4}x - \frac{13}{4}\right) \times 4$$

$$4y = 3x - 13$$

$$13 = 3x - 4y$$

$$3x - 4y = 13$$

(f) we have : 1) $(-1, -4)$ midpoint of AB and center point of the circle !
 2) from d) length of AB is 20, hence diameter of the circle is 20 and radius is 10

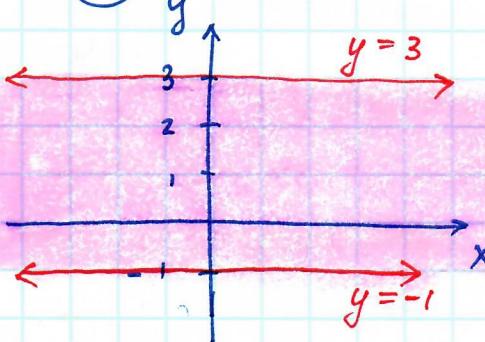
Using
$$(x-h)^2 + (y-k)^2 = r^2$$

$\uparrow \quad \uparrow \quad \uparrow$
 $-1 \quad -4 \quad 10$

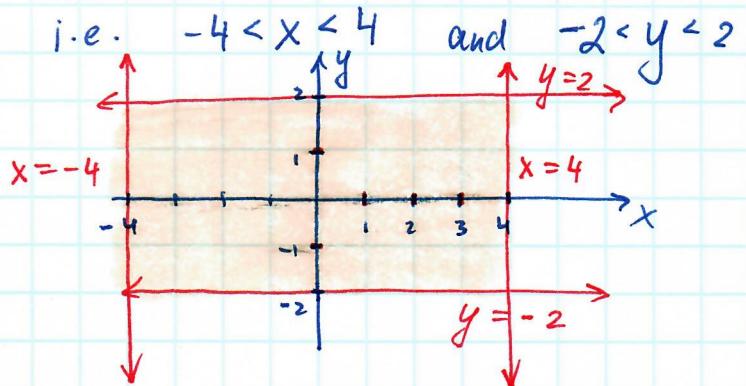
$$\boxed{(x - (-1))^2 + (y - (-4))^2 = 10^2}$$

$$\boxed{(x+1)^2 + (y+4)^2 = 100}$$

5 (a) $-1 \leq y \leq 3$

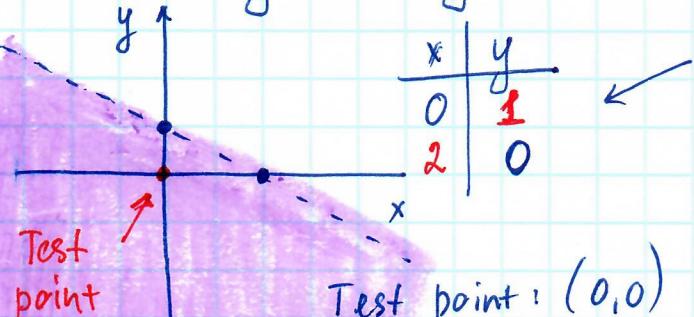


(b) $|x| < 4$ and $|y| < 2$



(c) $y < 1 - \frac{1}{2}x$

boundary line: $y = 1 - \frac{1}{2}x$

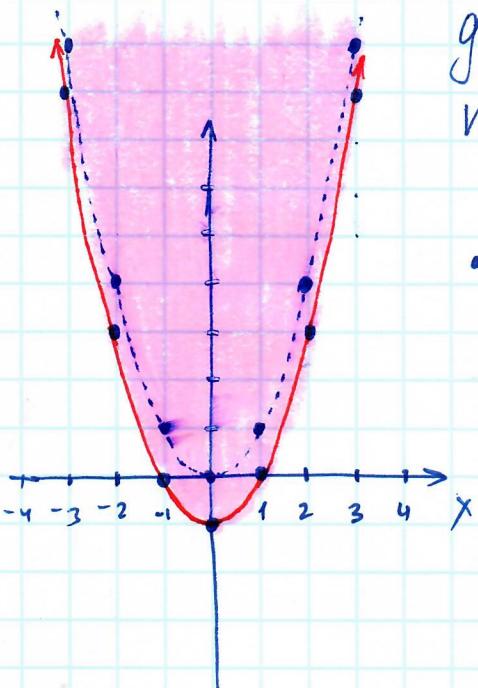


dashed, i.e. points on the boundary line are excluded.
 plotting the points of the boundary line
 • pick $x=0$, find y
 • pick $y=0$, find x

$0 < 1 - \frac{1}{2} \cdot 0$ True! ← shade the half-plane the test point belongs to.

$$(d) \quad y \geq x^2 - 1$$

- boundary line : $y = x^2 - 1$ it is a parabola!



grab graph of $y = x^2$ and shift it one mark down

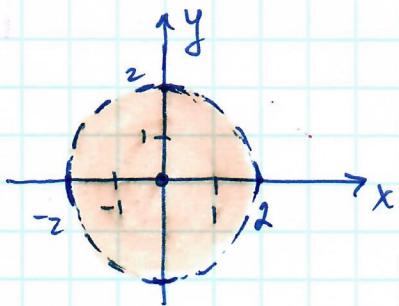
- shade "inside" the parabola

$$(e) \quad x^2 + y^2 < 4$$

boundary line : $x^2 + y^2 = 4$

dashed, i.e. points on it are not included.

$x^2 + y^2 = 4$ is equation of the circle of radius 2, with center (0,0)



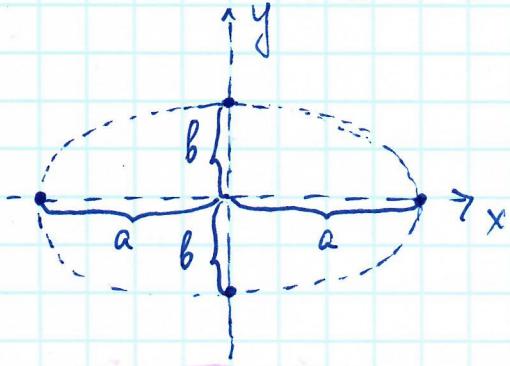
shade inside!

$$(f) \quad 9x^2 + 16y^2 = 144$$

recall ovals, to be more precise, ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

with center at $(0,0)$

a is distance to vertices, b is distance to co-vertices



$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

so