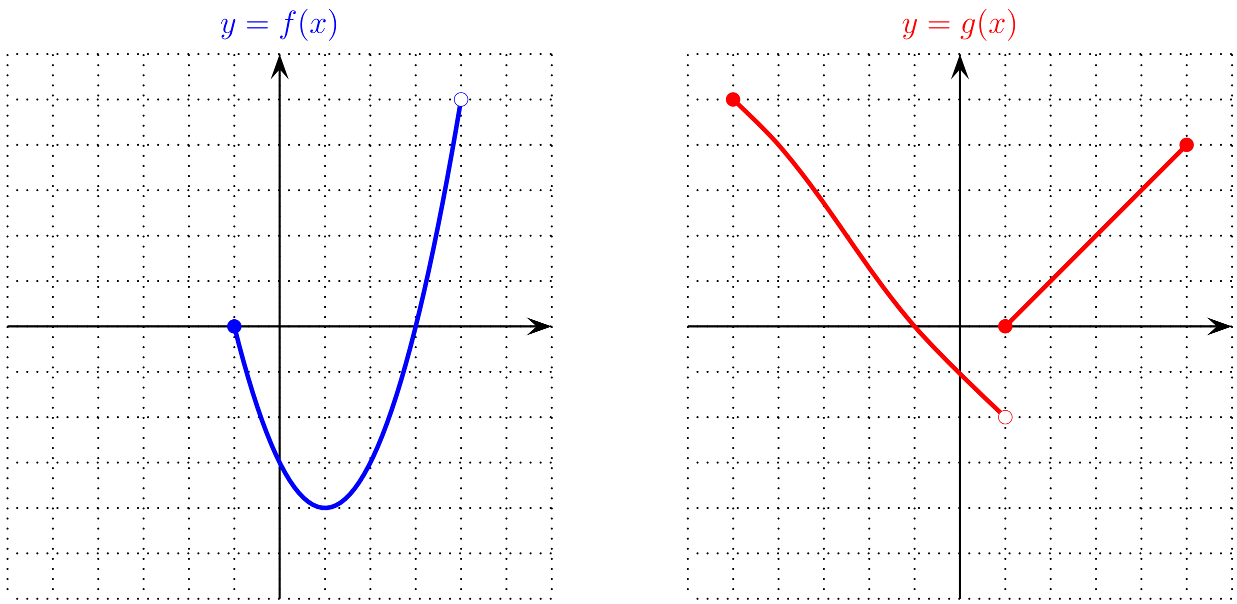
BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

MTH30 Review Sheet

1. Given the functions f and g described by the graphs below:



   (a) Find:
      i. The domain of f
      ii. The range of f
      iii. An interval on which f is increasing
      iv. An interval on which f is decreasing
      v. The domain of g
      vi. The range of g
      vii. An interval on which g is one-to-one

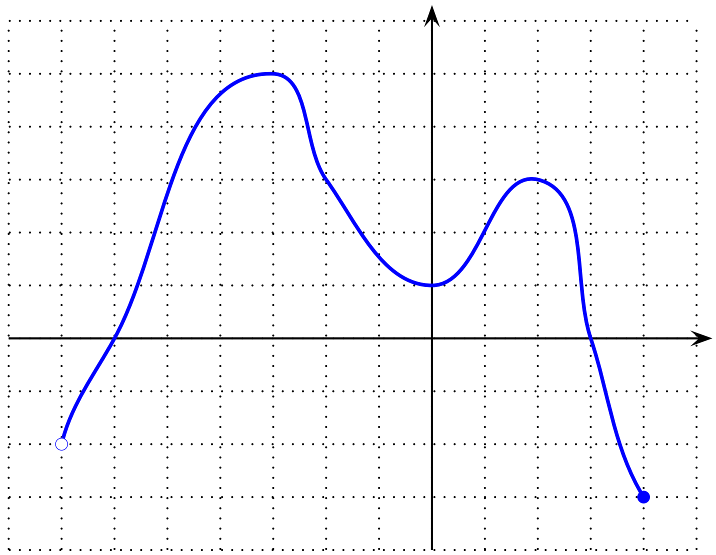
   (b) Evaluate the following, if they exist:
      i. $g(1)$
      ii. $(f+g)(1)$
      iii. $(f-g)(4)$
      iv. $\left(\frac{g}{f}\right)(-1)$
      v. $(f \circ g)(1)$
      vi. $(g \circ g)(-5)$
      vii. $(f \circ f)(3)$

2. Let $f(x) = \sqrt{x^2+4x+4}$ and $g(x) = \dfrac{x^2-1}{\sqrt{1-x}}$.

   (a) Find the domains of f and g. Give your answer using interval notation.

(b) Evaluate, if defined: $f(g(0))$; $g(f(0))$; $(f \cdot g)(0)$

3. Given the graph of $y = f(x)$, answer the following questions.



(a) Find the domain of f

(b) Find the range of f

(c) Over which intervals is f increasing?

(d) Over which intervals is f decreasing?

(e) Find $f(-3)$ and $f(4)$

(f) Find all solutions to the equation $f(x) = 3$

(g) Find the zeros of the function.

(h) Does f have an inverse function? Explain.

4. For each of the functions f given below:

A. $f(x) = \dfrac{x}{x+1}$ B. $f(x) = e^{2x-1}$ C. $f(x) = \log_2(3-x)$

(a) Find the inverse function f^{-1}.

(b) Verify that $f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$

(c) Sketch a graph of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of coordinates.

5. Consider the functions: $f(x) = e^{x^2}$ and $g(x) = \sqrt{\ln x}$. Are f and g a pair of inverse functions? Justify your answer.

6. For each pair of functions f and g given below find $f \circ g$ and $g \circ f$.

(a) $f(x) = 2x^2 - 3x + 5$; $g(x) = 5 - 2x$.

(b) $f(x) = \dfrac{2x}{x-5}$; $g(x) = \dfrac{5x}{x-2}$

(c) $f(x) = x^2 - 4$; $g(x) = \sqrt{x+5}$

7. The graph of a parabola $y = f(x)$ has axis of symmetry $x = -1$, vertex $(-1, 5)$, and $f(0) = 3$.

(a) Write the equation of the parabola in standard form.

(b) State the domain and the range of f.

(c) Sketch a graph of $y = f(x)$.

8. For each of the the following polynomials $p(x)$:

   A. $p(x) = x^3 - 3x^2 + 4$ B. $p(x) = -x^3 + 4x^2 - x - 6$ C. $p(x) = 2x^4 + 7x^3 + 6x^2 - x - 2$

   (a) List all possible rational roots of $p(x)$, according to the Rational Zeros Theorem.
   (b) Factor $p(x)$ completely.
   (c) Find all roots of the equation $p(x) = 0$.
   (d) Determine the end behavior of the graph of $y = p(x)$.
   (e) Determine the y–intercept of the graph of $y = p(x)$
   (f) Determine the x–intercepts of the graph $y = p(x)$
   (g) Determine the local behavior of $y = p(x)$ near the x-intercepts.
   (h) Use the above information to sketch a graph of $y = p(x)$.

9. Find the remainder of the division of $x^{122} - 20x^{51} + 60x^{34} + 1$ when divided by $x - 1$.

10. For each of the following rational functions f

    A. $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 2}$ B. $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 2x - 3}$ C. $f(x) = \dfrac{x^2 - 9}{x^2 - x - 2}$ D. $f(x) = \dfrac{2 - x}{x^2 + x - 2}$

    E. $f(x) = \dfrac{x^2}{x^2 + 1}$

    (a) Factor numerator and denominator and simplify if possible.
    (b) Find the x and y intercepts of the graph of $y = f(x)$ if they exist.
    (c) Find any vertical or horizontal asymptotes.
    (d) Determine how the sign of $f(x)$ changes.
    (e) Use the above information to sketch a graph of $y = f(x)$.

11. Solve the following inequalities. Express your answer using interval notation.

    A. $x^4 + x^3 - 7x^2 - x + 6 \geq 0$ B. $\dfrac{x + 4}{2x - 1} > 3$ C. $\dfrac{x^2 - 3x + 2}{x^3 - 6x^2 + 9x} \leq 0$

12. Evaluate the following expressions. Give exact values whenever possible:

    (a) $\log_2 \dfrac{1}{64}$
    (b) $\log_9 \dfrac{\sqrt{3}}{3}$
    (c) $\log_b x^3 y$, given that $\log_b x = 2$ and $\log_b y = 36$
    (d) e^{x-y} given that $e^x = 3$ and $e^y = 4$
    (e) $\log_a \left(\dfrac{x}{y}\right)$ given that $\log_a(x) = 12$ and $\log_a(y) = 4$
    (f) $\ln e^{\sqrt{2}}$
    (g) $\log 1000$
    (h) $\log_7 31$, rounded to the nearest hundredth
    (i) $\sin^{-1}\left(\sin \dfrac{\pi}{6}\right)$
    (j) $\cos^{-1}\left(\cos \dfrac{4\pi}{3}\right)$
    (k) $\cos\left(\sin^{-1}(-1)\right)$

(l) $\sin(a+b)$, if $\sin a = \frac{1}{3}$, $\cos b = \frac{3}{5}$ and $0 < a, b < \frac{\pi}{2}$

13. Find θ if
    (a) $\cos\theta = \frac{\sqrt{3}}{2}$, and $\frac{3\pi}{2} < \theta < 2\pi$.
    (b) $\sin\theta = -\frac{1}{2}$, and $\pi < \theta < \frac{3\pi}{2}$.
    (c) $\sin\theta = \frac{\sqrt{2}}{2}$, and $\frac{\pi}{2} < \theta < \pi$

    —Q14

14. Solve the following equations:
    (a) $\log_2 x - \log_2(x-1) = 1$
    (b) $7^{x+2} = 49$
    (c) $\sin^2 x = \frac{3}{4}$, where x is in the interval $[0, 2\pi)$
    (d) $2\cos^2 x + 3\cos x + 1 = 0$, where x is in the interval $[0, 2\pi)$

15. Verify the following identities:
    (a) $\tan^2 x + 1 = \sec^2 x$
    (b) $\csc x - \sin x = \cot x \cos x$
    (c) $\csc x - \cos x \cot x = \sin x$
    (d) $\cos^2 x - \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

16. For each of the following functions

    A. $f(x) = -\sin(4x - \pi)$ B. $f(x) = -3\cos(2x + \pi)$
    C. $f(x) = 2\sin(3x - \frac{\pi}{2})$ D. $f(x) = \frac{1}{2}\cos\left(\frac{x}{2} - \frac{\pi}{2}\right)$

    (a) Find the period of this function.
    (b) Find the amplitude of the graph of $y = f(x)$
    (c) Find the phase shift of the graph of $y = f(x)$
    (d) Sketch two complete cycles of the graph of $y = f(x)$

The answers

1. (a) **i**. $[-1, 4)$, **ii**. $[-4, 5)$, **iii**. $[1, 4)$, **iv**. $[-1, 1]$, **v**. $[-5, 5]$, **vi**. $(-2, 5]$, **vii**. $[-5, 1)$ or $[1, 5]$
   (b) **i**. 0, **ii**. -4, **iii**. undefined, **iv**. indeterminate, **v**. -3, **vi** 4, **vii** -3

2. (a) Domain of f is $(-\infty, \infty)$, domain of g is $(-\infty, 1)$
   (b) $f(g(0)) = 1$; $g(f(0))$ is undefined; $(f \cdot g)(0) = -2$

3. A. $(-7, 4]$ B. $[-3, 5]$ C. $(-7, -3]$ and $[0, 2]$ D. $[-3, 0]$ and $[2, 4]$ E. $f(-3) = 5$; $f(4) = -3$
   F. $\{-5, -2, 2\}$ G. -6, 3 H. No. It is not one-to-one.

4. For the graphs see Figure 1. A. $f^{-1}(x) = \frac{x}{1-x}$ B. $f^{1}(x) = \frac{\ln x + 1}{2}$ C. $f^{-1}(x) = 3 - 2^x$

5. They are not a pair of inverse functions: f is not one-to-one and thus it doesn't have an inverse function.

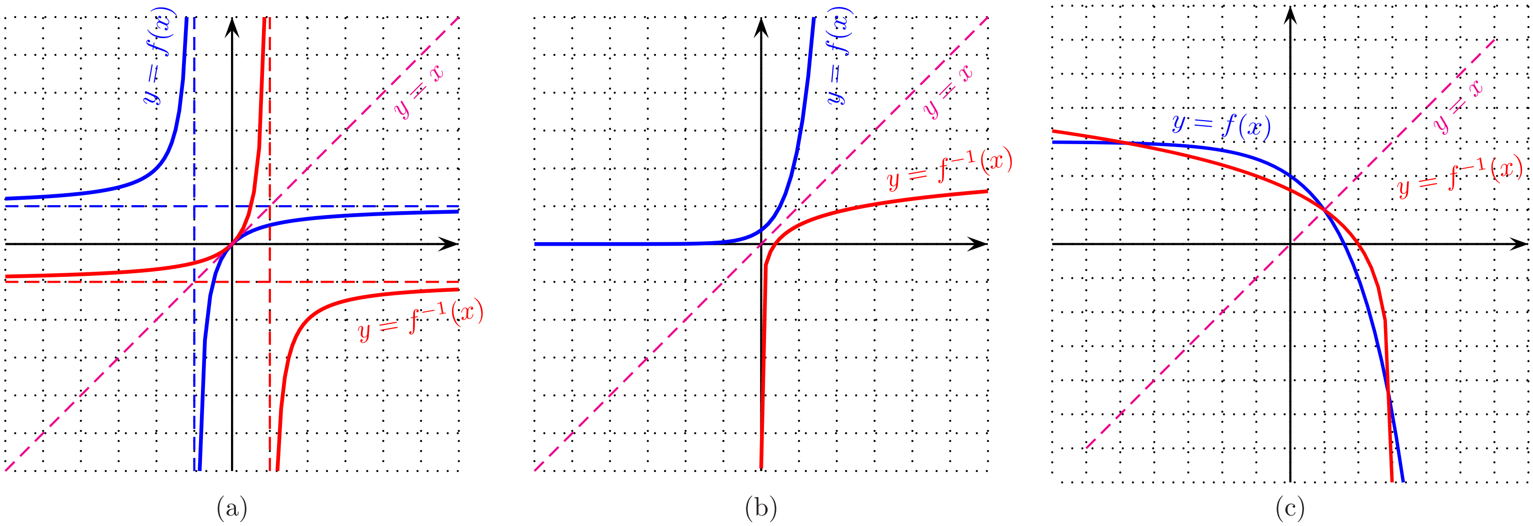


Figure 1: The graphs of Question 4

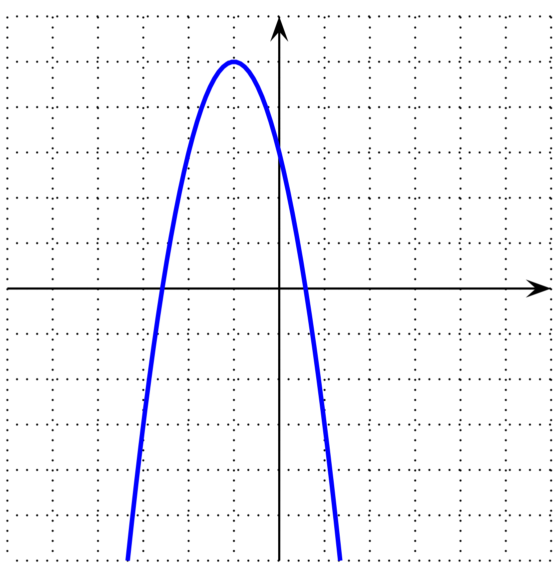


Figure 2: The parabola of Question 7

6. A. $(f \circ g)(x) = 8x^2 - 34x + 40$; $(g \circ f)(x) = -4x^2 + 6x - 5$ B. $(f \circ g)(x) = x$; $(g \circ f)(x) = x$
C. $(f \circ g)(x) = x + 1$; $(g \circ f)(x) = \sqrt{x^2 + 1}$

7. A. $y = -2(x+1)^2 + 5$ B. Domain is $(-\infty, \infty)$, Range is $(-\infty, 5]$ C. See Figure 2

8. (a) A. $\{\pm 1, \pm 2, \pm 4\}$ B. $\{\pm 1, \pm 2, \pm 3, \pm 6\}$ C. $\{\pm 1, \pm 2, \pm \frac{1}{2}\}$

   (b) A. $(x+1)(x-2)^2$ B. $(x+1)(2-x)(x-3)$ C. $(x+1)^2(x+2)(2x-1)$

   (c) A. $x = -1$, $x = 2$ B. $x = -1$, $x = 2$, $x = 3$ C. $x = -1$, $x = -2$, $x = \frac{1}{2}$

   For the remaining questions see Figure 3

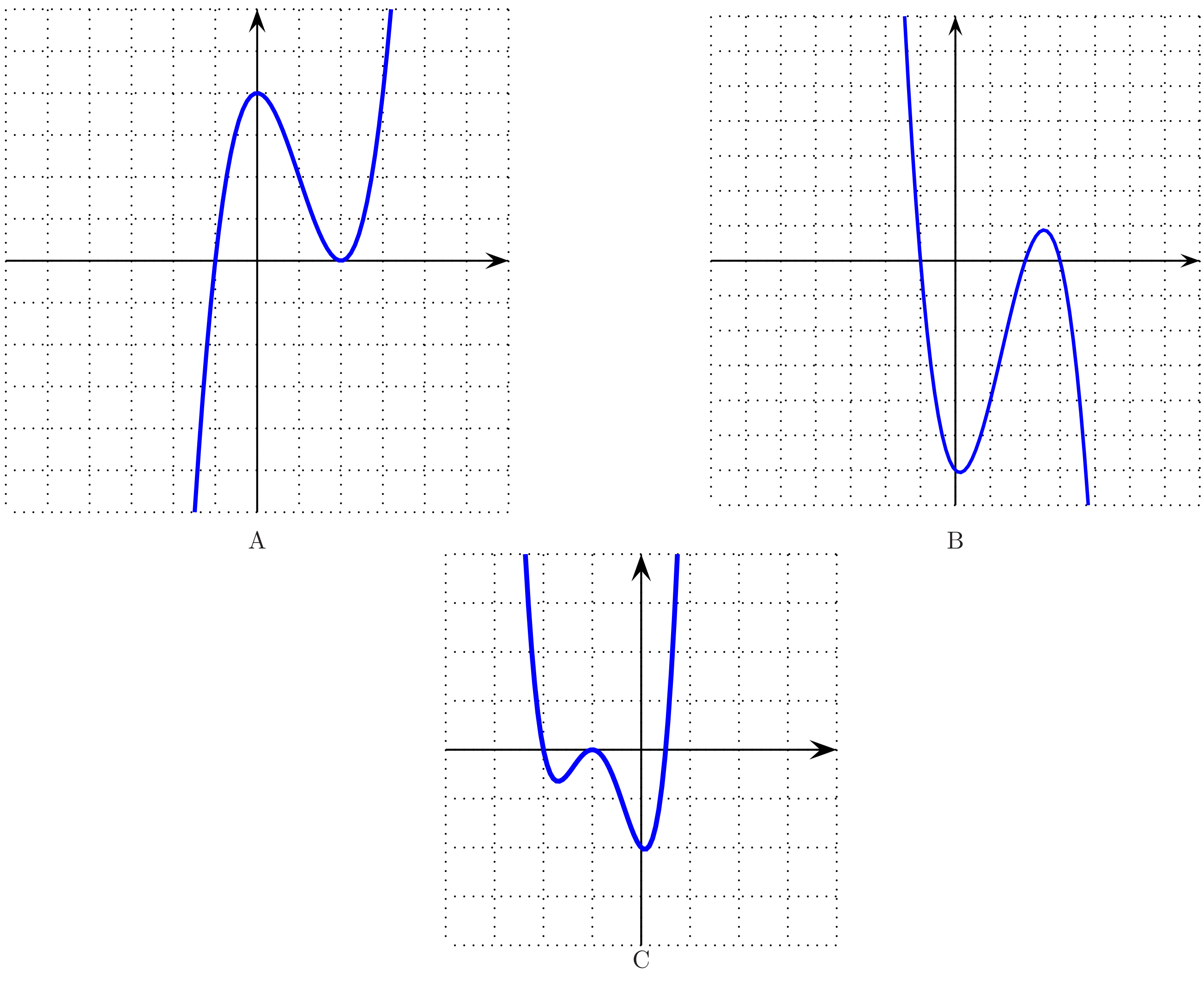


Figure 3: The graphs in Question 8

9. By the Remainder Theorem the answer is 42.

10. The first four graphs are shown in Figure 4. The fifth is shown in Figure 5.

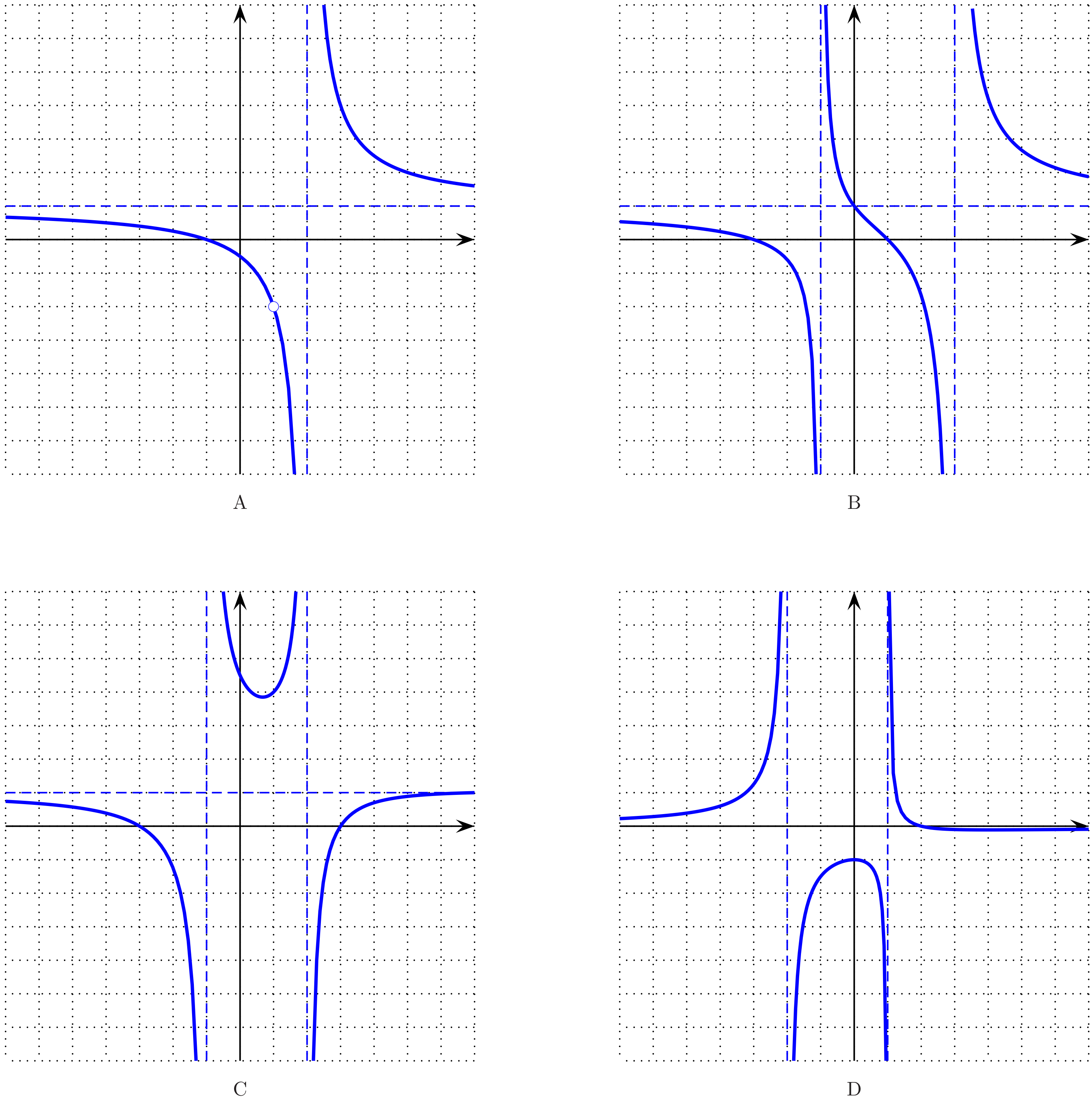


Figure 4: The first four graphs of Question 10

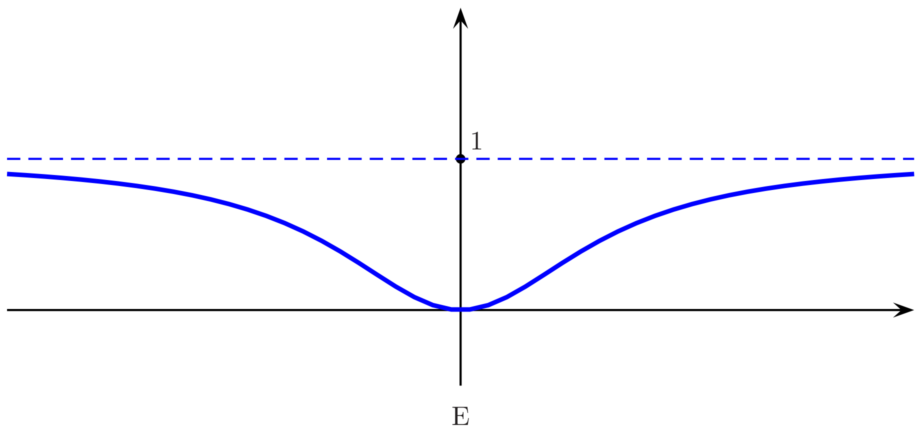


Figure 5: The fifth graph of Question 10

11. A. $(-\infty,-3]\cup[-1,1]\cup[2,\infty)$ B. $\left(\frac{1}{2},\frac{7}{5}\right]$ C. $(-\infty,0)\cup[1,2]$

12. A. -6 B. $-\frac{1}{4}$ C. 42 D. $\frac{3}{4}$ E. 8 F. $\sqrt{2}$ G. 3 H. 1.76 I. $\frac{\pi}{6}$ J. $\frac{2\pi}{3}$ K. 0 L. $\frac{3+8\sqrt{2}}{15}$

13. A. $\frac{11\pi}{6}$ B. $\frac{7\pi}{6}$ C. $\frac{3\pi}{4}$

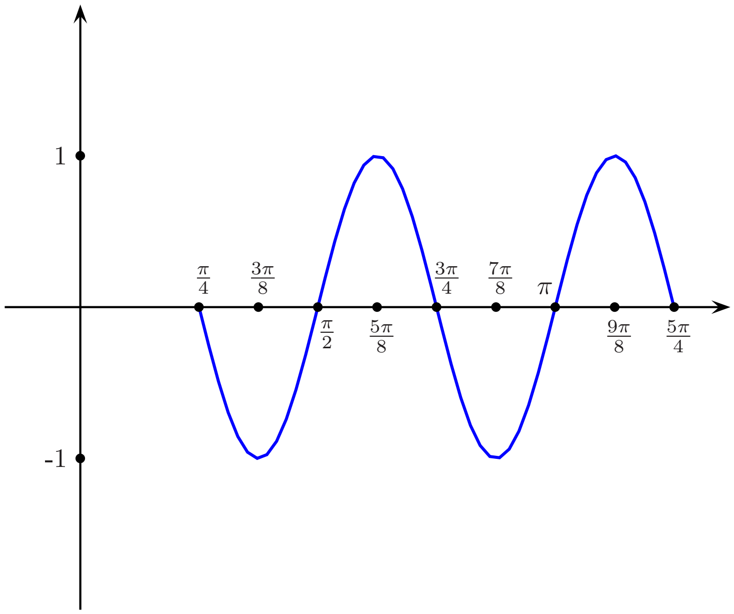
14. A. $x=2$ B. $x=0$ C. $x=\frac{\pi}{3}, x=\frac{2\pi}{3}, x=\frac{4\pi}{3}, x=\frac{5\pi}{3}$ D. $x=\pi, x=\frac{2\pi}{3}, x=\frac{4\pi}{3}$

15. To prove these identities, use algebra and the basic identities

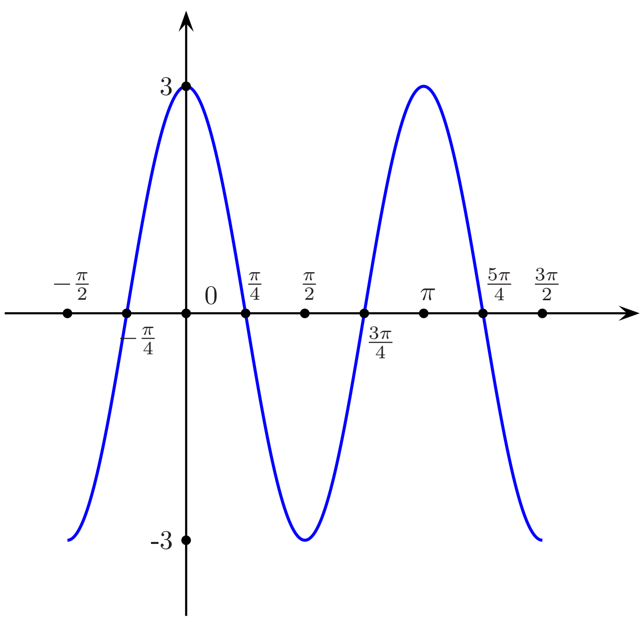
$$\begin{aligned}
\csc\theta &= \frac{1}{\sin\theta}\\
\sec\theta &= \frac{1}{\cos\theta}\\
\cot\theta &= \frac{1}{\tan\theta}\\
\tan\theta &= \frac{\sin\theta}{\cos\theta}\\
\cos^2\theta+\sin^2\theta &= 1
\end{aligned}$$

16. (a) A. $\frac{\pi}{2}$ B. π C. $\frac{2\pi}{3}$ D. 4π
    (b) A. 1 B. 3 C. 2 D. $\frac{1}{2}$
    (c) A. $\frac{\pi}{4}$ B. $-\frac{\pi}{2}$ C. $\frac{\pi}{6}$ D. π

The graphs are as follows:

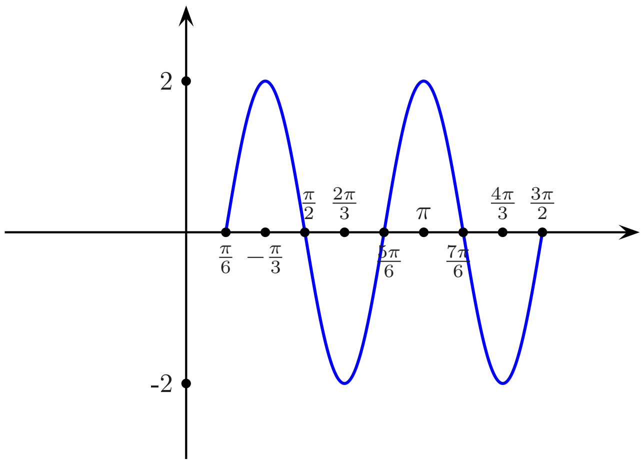


(a)



(b)

(c)


2
-2
$\frac{\pi}{6}$
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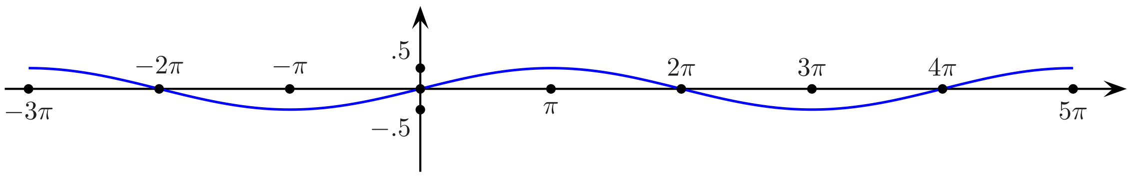
(d)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