MTH 30 Trigonometry Review
Additional Review Problems

(1) If $\cos(\alpha) = -\frac{4}{5}$, and angle α is in the quadrant II, i.e. 90 ≤ α ≤ 180, find the remaining trigonometric functions without using a calculator. Simplify your answer, rationalize the denominator if needed (i.e. there should be no radicals in the denominator).

(2) Find all solutions, if possible, of each equation:

(a) $\sin\left(\frac{2\theta}{3}\right) = -1$ (b) $2\sin^2 x + \sin x = 0$ (c) $\tan^2 x \cos x = \tan^2 x$

(3) verify the identity $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$

(4) Find the exact value of $\cos(\alpha + \beta)$, if $\tan(\alpha) = -\frac{3}{4}$, $\cos(\beta) = \frac{1}{3}$, and α lies in quadrant II and β lies in quadrant I.

Answers:

(1) $\sin(\alpha)=\frac{3}{5}$, $\tan(\alpha)=-\frac{3}{4}$, $\cot(\alpha)=-\frac{4}{3}$, $csc(\alpha)=\frac{5}{3}$, $sec(\alpha)=-\frac{5}{4}$

(2) (a) $\theta=\frac{9\pi}{4}+3\pi n, n\in Z$ (b) $x=\pi n, \frac{7\pi}{6}+2\pi n, \frac{11\pi}{6}+2\pi n, n\in Z$ (c) $x=\pi n, n\in Z$

(3) let's assume that the identity is true, and apply the cross-product property:
$\cos x\cdot\cos x=(1-\sin x)\cdot(1+\sin x)$
then re-write the left side and open the parentheses on the right side (difference of squares):
$\cos^2 x=1-\sin^2 x$ (*)
recall the identity: $\cos^2 x+\sin^2 x=1$, solve it for $\cos^2$x: $\cos^2 x=1-\sin^2 x$ - this is exactly what we got in line (*).
Therefore, our assumption that the identity is true was correct.

(4) $\cos(\alpha+\beta)=-\frac{4+6\sqrt{2}}{15}$