MTH30 Final Exam Review, Part 2

problem 8: Given $f(x)=2x^3-7x^2+9x-3$, find the remainder from the division by (x+11).
Hint: use the *Remainder Theorem*.

Solution:
Indeed, the *Remainder Theorem* says:

If the polynomial *f(x)* is divided by *x-c*, then the remainder is *f(c)*.

We just need to detect the *c* in the divider (x+11). We can re-write (x+11) = (x - (-11)), hence c = -11.
So let's find f(-11):

$$f(x)=2(-11)^3-7(-11)^2+9(-11)-3=-2662-847-99-3=-3611$$

Answer: -3611

problem 9: For the rational function $f(x)=\frac{x^2+4x+3}{(x+2)^2}$, find

- Its *vertical asymptotes*, if any
- Its *horizontal asymptote*, if any
- *x-intercepts*, if they exist
- *y-intercept*, if it exists

vertical asymptotes: where f(x) is undefined, i.e. when x+2 = 0.
Hence x = -2 is the vertical asymptote of f(x)

horizontal asymptote: $f(x)=\frac{x^2+4x+3}{(x+2)^2}=\frac{1x^2+4x+3}{1x^2+4x+4}$

the degree of the numerator polynomial = *the degree of the denominator polynomial*, therefore f(x) has a horizontal asymptote with the equation

$$y=\frac{\text{leading coefficient of the numerator polynomial}}{\text{leading coefficient of the denominator polynomial}}=\frac{1}{1}=1$$

Hence, y = 1 is the horizontal asymptote of f(x)

x-intercepts: we can find them by setting f(x) to 0, an solving the equation: $\frac{x^2+4x+3}{(x+2)^2}=0$

$\frac{x^2+4x+3}{(x+2)^2}=0$ only when $x^2+4x+3=0$, which we can factor: $(x+3)(x+1)=0$, and solve for each factor separately: x+3 = 0 or x+1 = 0, which gives us: x = -3 or x = -1.
Therefore, (-3,0) and (-1,0) are the x-intercepts

y-intercept: when x = 0: $f(0)=\frac{0^2+4\cdot 0+3}{(0+2)^2}=\frac{3}{4}$, hence (0,3/4) is the y-intercept

problem 10: Find the domain of the rational function $f(x)=\frac{x+1}{x^2+2x-3}$

Solution: the function f(x) is undefined when $x^2+2x-3=0$
If we solve it: (x+3)(x-1) = 0, then x+3 = 0 or x-1 = 0, hence x = -3 or x = 1
We will get that at x = -3 and at x = 1 the function f is undefined, therefore we need to exclude them.

The domain of f(x) is $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$