MTH 30 Midterm Exam Review

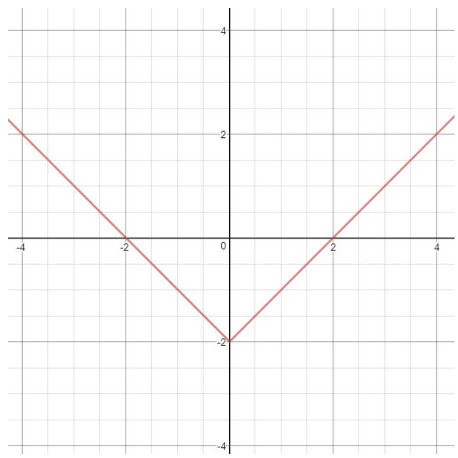
1. Find the *domain* and the *range* of the relation defined by { (a, 5), (c, 7), (d, 12), (f, 8), (g, 0), (b, 5)} ?

2. Is every *function* a *relation*?

3. Consider equation $y=2x^2-5x+7$. Is it a *function*?

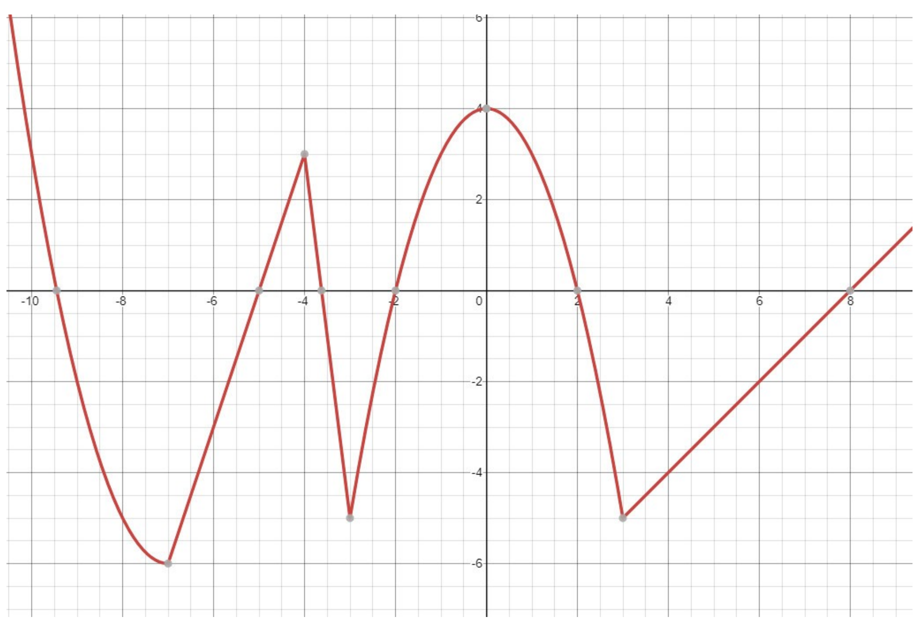
4. Is the *relation* defined by {(a,3), (b,4), (c,1), (d,2), (a,7), (f,5), (b,11)} a *function*?

5. Does the graph represent a function?



Recommendation: Review the graphs when the vertical line test fails.

6. Consider the graph of a function f(x):

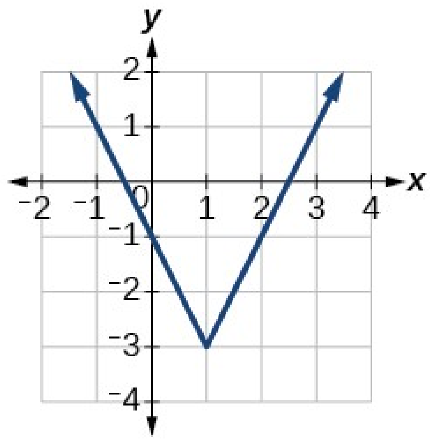


Give its *domain* and *range*, *relative/local* and *absolute maxima* and *minima* (if any), intervals of *increase* and *decrease*. At what values f(x) = -5?

7. Determine whether the given functions are odd/even/neither:

(a) $f(x)=x^2\sqrt{1-x^2}$ (b) $g(x)=2x^3-6x^5$ (c) $t(x)=x^2-x$

8. Determine whether the function represented by its graph is even/odd or neither?



9. Draw a graph of the function with the following properties:
domain: [-5,5]
range: [-4,4]
f(-2) = 3
f(-1) = 3
f(5) = 3
x-intercepts: 2, 4 (only *x*-coordinates are given)
y-intercept: -2

10. Find the domain of the given functions:

(a) $f(x)=\sqrt{2-6x}$ (b) $g(x)=\frac{1}{x}$ c) $h(x)=\frac{\sqrt{x-3}}{\sqrt{7-x}}$

11. For the function $f(x)=\frac{1}{2x}$. Find

(a) f(x+3)
(b) f(-7)

12. For the functions $f(x)=\sqrt{1-x}$ and $g(x)=\frac{1}{2+x}$. Find

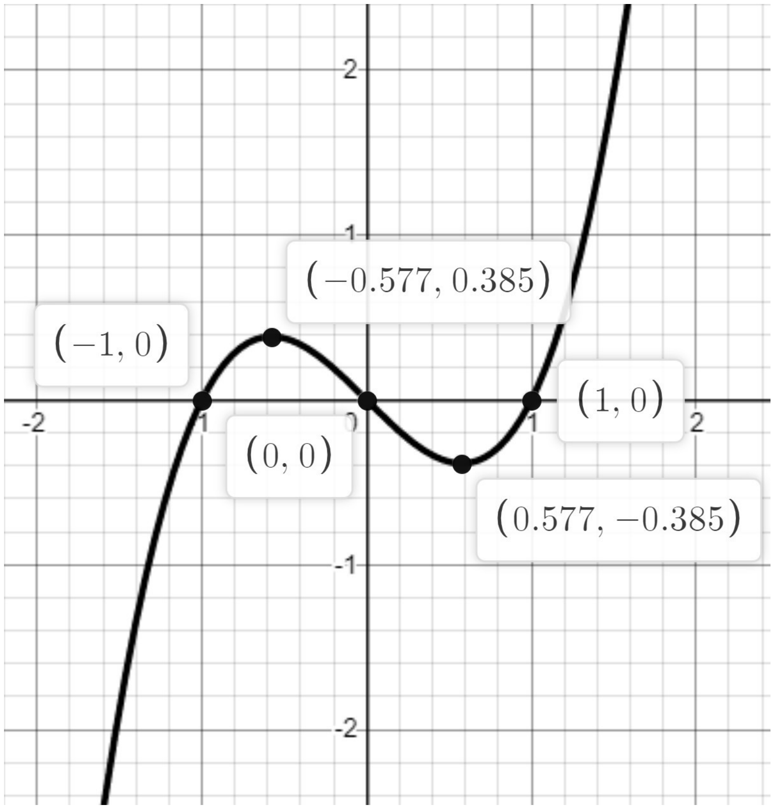
(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(fg)(x)$ (d) $\left(\frac{f}{g}\right)(x)$

(e) $(f\,o\,g)(x)$ (f) $(f\,o\,g)(4)$ (g) $(g\,o\,f)(x)$ (h) $(g\,o\,f)(-3)$

and their domain

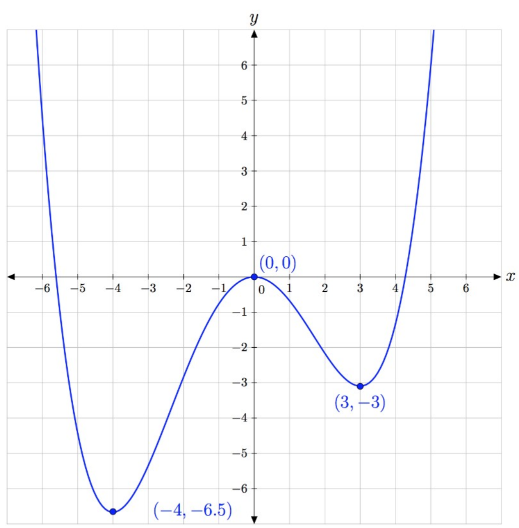
13. Let $f(x)=x^2$ and $g(x)=-2(x-3)^2+10$.
What transformation can be used to get the graph of g(x) from the graph of f(x)?

14. Use the graph of f(x) to graph g(x) = 3f(-2x)



15. Check if $f(x)=\frac{2}{x-5}$ and $g(x)=\frac{2}{x}+5$ are *inverse functions*.

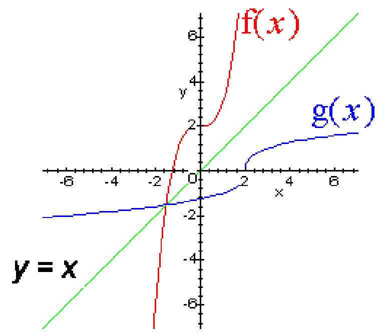
16. The function is given by its graph. Does it have an *inverse*?



17. Find the *inverse function* for the given functions

(a) $f(x)=\frac{4}{x}+9$ (b) $g(x)=\frac{2x-3}{x+1}$

18. Determine from the graphs of the functions whether they are inverse functions.



19. Is $f(x)=\dfrac{x^2-x-12}{x^2+2x-8}$ a polynomial function?

20. Find the *degree* and the *leading coefficient* of the polynomial function $f(x)=7x^{12}-\dfrac{\sqrt{5}}{2}x^{15}+\sqrt{9}x^{10}-9$.

21. What are the *vertical asymptotes* of the graph of the equation $f(x)=\dfrac{x^2-x-12}{x^2+2x-8}$?

22. What is the remainder of the division of $2x^3-5x^2+x+2$ by x+2?
Hint: use Remainder Theorem

23. Check if $2x^3-5x^2+x+2$ is divisible by (x-1)
Hint: use Factor theorem, or either long division or synthetic division

24. Check if $-\dfrac{1}{2}$ is a root of $f(x)=2x^3-5x^2+x+2$

25. What is the *horizontal asymptote* of the graph of $h(x)=\dfrac{3x-6x^2+10}{(2x-3)(x+5)}$?

26. Find all the zeros of the functions:

(a) $f(x)=\dfrac{(x+2)(x-4)(2x-7)}{(x-3)(x+9)(x-7)}$ (b) $g(x)=6x^2-11x-35$

(c) $h(x)=(x^2-1)(x-2)(3x+5)$

27. List all the *possible rational zeros* of the polynomial function $f(x)=-4x^2+x^3+8x-5$.

28. The polynomial function $f(x)=3x^7-12x^6+25x^9+3x^2-2x+8x^4$ has *at most* __________ *turning points*. Fill out the missing space.

29. Find the *y*-intercepts of the following functions:

(a) $f(x)=(x-3)^2-6$ (b) $g(x)=\dfrac{x-4}{x^2-x-6}$ (c) $h(x)=9x^2-10+6x^8-12x^5+8x$

30. Find all the *zeros* and their *multiplicity* for the function $f(x)=(x-2)^2(x-3)^2(2x+5)$?
What can you say about the graph behavior around these zeros?

31. What can you conclude if you are told that f(x) is a polynomial function,
in addition f(-2) = 7 and f(5) = -9?
Hint: use the *Intermediate Value Theorem*

32. Determine without graphing, whether the quadratic function $f(x)=2(x+1)^2-6$ has an *absolute/global minimum* or an *absolute/global maximum* value, find it.

33. Determine without graphing, whether the quadratic function $f(x)=-4x^2+2x+4$ has an *absolute minimum* or an *absolute maximum* value, find it.

34. For the given *piecewise function*:

$$f(x)=\begin{cases} 2x^2-7 & ,x<2 \\ x+2 & ,x\geq 2 \end{cases}$$

Find **(a)** f(0)
(b) f(2)
(c) f(3)

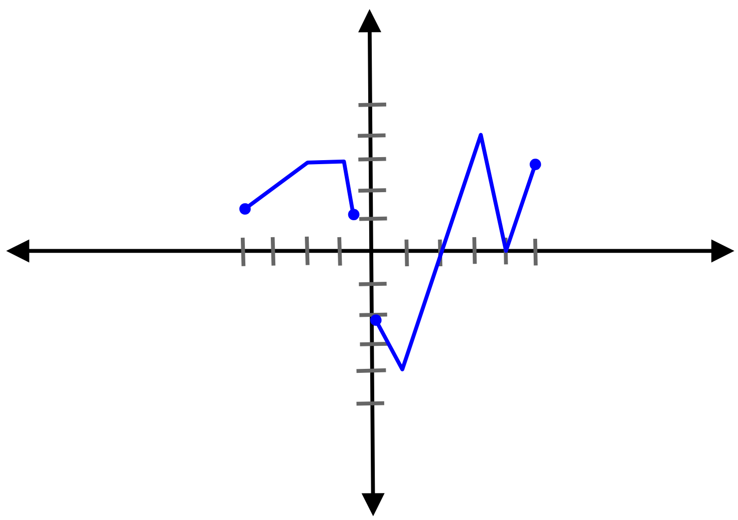
35. Solve *absolute value equation* $18=3|2x-7|$.

Comment: Note that you might be asked to find all zeros of absolute value function $f(x)=3|2x-7|-18$, which will yield the same solution and the same answer.

36. Solve the absolute value inequalities
(a) $18>3|2x-7|$ **(b)** $18\leq 3|2x-7|$

Answers:

1. domain: {a, b, c, d, f, g}, and range: {0, 5, 7, 8, 12}
2. Yes
3. Yes
4. No, because of (a,3) and (a,7) or (b,4) and (b,11). Every value from the domain corresponds to at most one value from the range.
5. yes, it does (using the vertical line test).
6. domain: (- ∞, ∞), range: [-6, ∞),
   relative/local maxima: 3, 4 relative/local minima: -6, -5
   no absolute maximum absolute minimum: -6
   increasing: (-7,-4) ∪ (-3,0) ∪ (3,∞) decreasing: (-∞, 7) ∪ (-4, -3) ∪ (0,3)
   f(x) = -5 at -8, -6.75, -3, 3
7. (a) even (b) odd (c) neither
8. neither
9. a possible graph (there are some variations in possible graphs):



10. (a) (- ∞, 1/3] (b) (- ∞, 0) ∪ (0, ∞) (c) [3,7)
11. (a) $f(x+3)=\frac{1}{2x+6}$ (b) f(-7) = -1/14
12. (a) $(f+g)(x)=\sqrt{1-x}+\frac{1}{2+x}$ (b) $(f-g)(x)=\sqrt{1-x}-\frac{1}{2+x}$

    (c) $(fg)(x)=\frac{\sqrt{1-x}}{2+x}$ (d) $\left(\frac{f}{g}\right)(x)=\sqrt{1-x}(2+x)$

    domain for (a) – (d) : (-∞,-2) ∪ (2, 1]

    (e) $(f\,o\,g)(x)=\sqrt{\frac{1+x}{2+x}}$ domain: (- ∞, -2) [-1,∞)

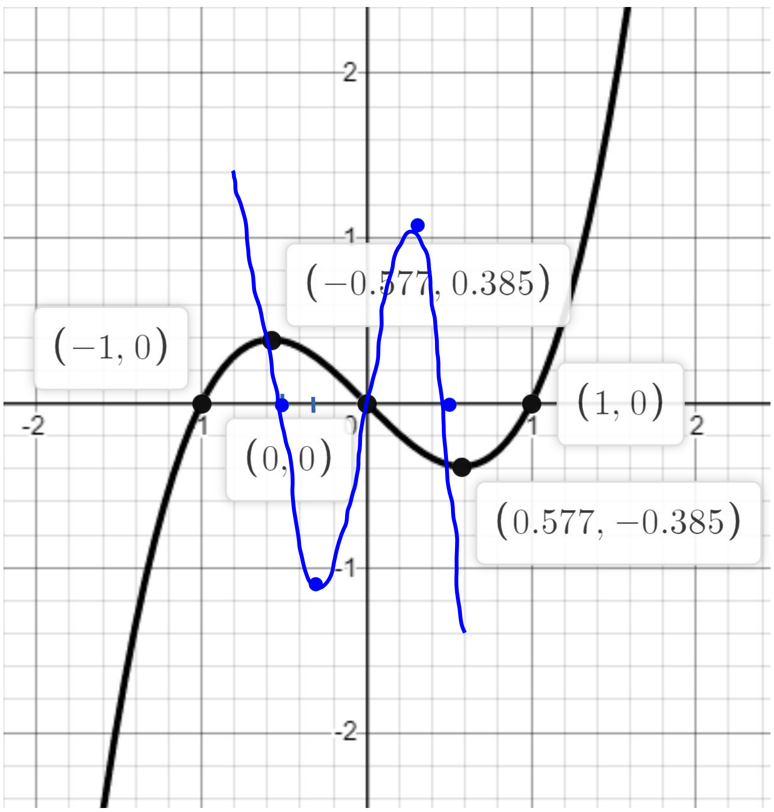
    (f) $(f\,o\,g)(4)=\sqrt{\frac{5}{6}}$

(g) $(g\circ f)(x)=\frac{1}{2+\sqrt{1-x}}$ domain: (-∞,1]

(h) $(g\circ f)(-3)=\frac{1}{4}$

13. horizontal shift 3 units to the right
    reflection about the x-axis
    vertical stretching (every y-coordinate is multiplied by 2)
    vertical shift 10 units up

14. the original graph together with the answer:



15. f and g are inverse functions
16. No, the horizontal line test fails.
17. (a) $f^{-1}(x)=\frac{4}{x-9}$ (b) $g^{-1}=\frac{x+3}{2-x}$
18. yes, they are inverse functions
19. No, it is not.
20. degree = 15 leading coefficient = $\frac{\sqrt{5}}{2}$
21. x = 4 and x = -2
22. -36
23. yes, it is. $\frac{2x^3-5x^2+x+2}{x-1}=2x^2-3x-2$
24. yes, it is, f(-1/2) = 0
25. y = -3 Note: there are three cases for horizontal asymptotes …. review all of them

26. (a) x = -2, 4, 3.5 (b) x = 7/2, -5/3 (c) x = -5/3, -1, 1, 2
27. 1, -1, 5, -5
28. 8
Explanation: degree of the polynomial – 1 = 9 – 1 = 8
29. (a) (0, 3) or simply 3 (if only y-coordinate is asked for)
(b) (0, 2/3) or simply 2/3 (if only y-coordinate is asked for)
(c) (0, -10) or simply -10 (if only y-coordinate is asked for)
30. 2 is a zero of f(x) of multiplicity 2, the graph touches the x-axis at this zero and turns around, it also flattens out near this zero;
3 is a zero of f(x) with multiplicity 3, the graph intersects the x-axis at this zero and at the same time flattens out near this zero;
5/2 is a zero of multiplicity 1, the graph of f(x) intersects the x-axis at it.
31. f(x) has a zero on the interval (-2,5), i.e. there exists a value c from the interval (-2,5) such that f (c) = 0.
32. absolute minimum of -6
33. absolute maximum of 4.25
34. f(0) = -7 (the first expression is used), f(2) = 4 (the second expression is used), f(3) = 5 (the second expression is used)
35. $\left\{\frac{1}{2}, \frac{13}{2}\right\}$ or $x = \left\{\frac{1}{2}, \frac{13}{2}\right\}$
36. (a) $\frac{1}{2} < x < \frac{13}{2}$, in interval notation: $\left(\frac{1}{2}, \frac{13}{2}\right)$
(b) $x \geq \frac{13}{2}$ or $x \leq \frac{1}{2}$, in interval notation: $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{13}{2}, \infty\right)$