

#21 $f(x)$ is a rational function,
its v. asymptotes are zeros of denominator, i.e.

$$x^2 + 2x - 8 = 0$$

$$(x-2)(x+4) = 0$$

$$x-2 = 0 \quad \text{or} \quad x+4 = 0$$

$$x = 2 \quad \text{or} \quad x = -4$$

V. Asymptotes : $x = 2$ and $x = -4$

#22 you could do the long division or synthetic division, however, the ~~Factor~~ Theorem can do it faster!
Remainder

Remainder theorem: if the polyn. $f(x)$ is divided by $x-c$, then the remainder is $f(c)$

$x+2$ $x-c$ what is c in $x+2$?

$c = -2$ because $x+2 = x - (-2)$

hence if $f(x) = 2x^3 - 5x^2 + x + 2$, then the remainder from the division of $f(x)$ by $x+2$ is $f(-2)$.

$$\begin{aligned} f(-2) &= 2(-2)^3 - 5(-2)^2 + (-2) + 2 = \\ &= -16 - 20 - 2 + 2 = \boxed{-36} \end{aligned}$$

#23 we can quickly check divisibility by checking the remainder from division of $2x^3 - 5x^2 + x + 2$ by $x-1$. $f(x)$

Again, we will use the remainder theorem (with $c=1$)

$$f(1) = 2 \cdot 1^3 - 5 \cdot 1^2 + 1 + 2 = 2 - 5 + 1 + 2 = 0$$

Then, by the Factor Theorem, since $f(1) = 0$ then $x-1$ is a factor of $2x^3 - 5x^2 + x + 2$, hence it is divisible by $x-1$.