#12

$f(x) = \sqrt{1-x}$ domain: $1 - x \geq 0$ $1 \geq x$ or $x \leq 1$

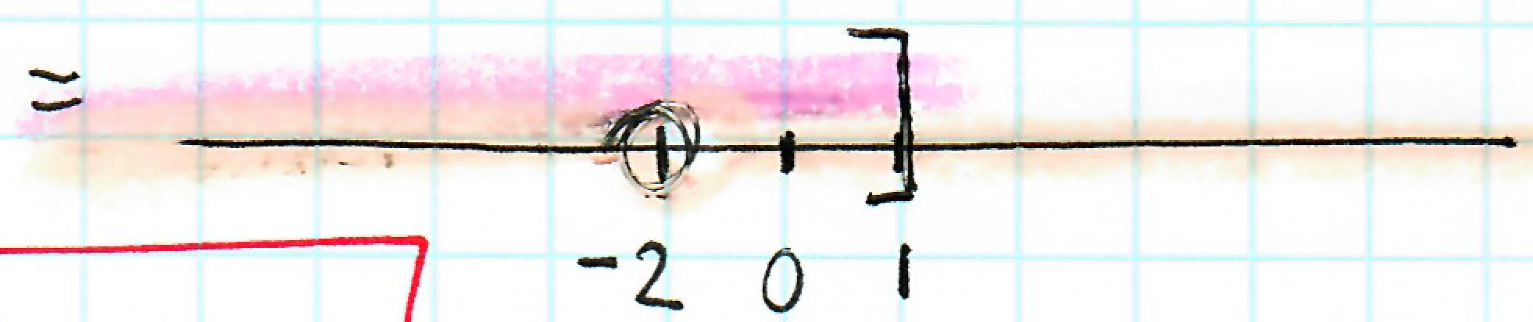
domain: D_f $(-\infty, 1]$

$g(x) = \dfrac{1}{2+x}$ domain: $2+x \neq 0$ or $x \neq -2$

domain: D_g $(-\infty, -2) \cup (-2, \infty)$

(a) $(f+g)(x) = f(x) + g(x) = \boxed{\sqrt{1-x} + \dfrac{1}{2+x}}$

domain: $D_f \cap D_g$ =



$\boxed{(-\infty, -2) \cup (-2, 1]}$

(b) $(f-g)(x) = f(x) - g(x) = \boxed{\sqrt{1-x} - \dfrac{1}{2+x}}$

domain: $\boxed{(-\infty, -2) \cup (-2, 1]}$

(c) $(fg)(x) = f(x) \cdot g(x) = \sqrt{1-x} \cdot \dfrac{1}{2+x} = \boxed{\dfrac{\sqrt{1-x}}{2+x}}$

domain: $\boxed{(-\infty, -2) \cup (-2, 1]}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{1-x}}{\dfrac{1}{2+x}} = \boxed{(2+x)\sqrt{1-x}}$

domain: $D_f \cap D_g$ - values of x that make $g(x) = 0$

$g(x) \neq 0$ for any x, hence there is nothing to "exclud

hence, the domain of $\left(\dfrac{f}{g}\right)(x)$ is $\boxed{(-\infty, -2) \cup (-2, 1]}$

#12

(e) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2+x}\right) =$

$$= \sqrt{1 - \frac{1}{2+x}} = \sqrt{\frac{2+x}{2+x} - \frac{1}{2+x}} = \boxed{\sqrt{\frac{1+x}{2+x}}} \text{ or } \boxed{\frac{\sqrt{1+x}\,\sqrt{2+x}}{2+x}}$$

domain: 1) the domain of $g(x)$ D_g:

$(-\infty, -2) \cup (-2, \infty)$

2) then exclude those values of x from D_g that make $f(x)$ undefined .... too much to exclude

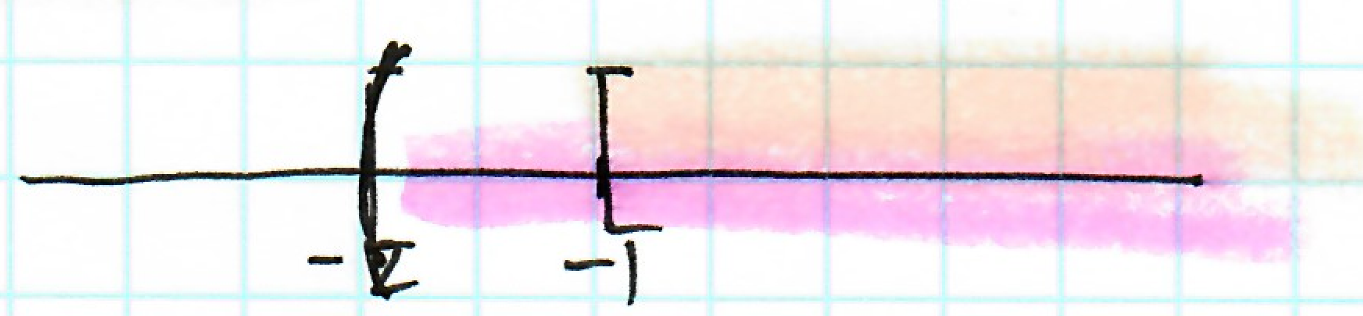
or

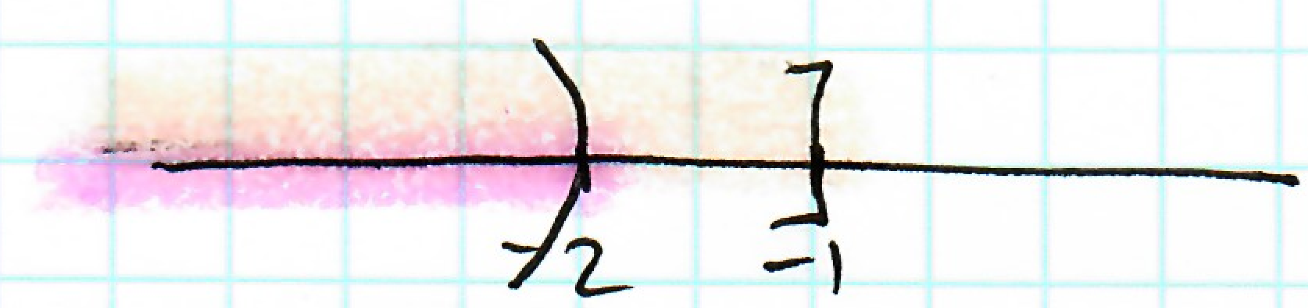
$$1 - \frac{1}{2+x} \geq 0 \qquad \frac{1+x}{2+x} \geq 0$$

2 cases:

$1+x \geq 0$ **and** $2+x > 0$ **or** $1+x \leq 0$ **and** $2+x < 0$

$x \geq -1$ and $x > -2$ $\qquad$ $x \leq -1$ $\quad$ $x < -2$





$[-1, \infty)$ $\qquad$ $(-\infty, -2)$

Hence, the domain: $\boxed{(-\infty, -2) \cup [-1, \infty)}$

(f) two ways to do it

① use (e): $(f \circ g)(x) = \sqrt{\frac{1+x}{2+x}}$, so $(f \circ g)(4) = \sqrt{\frac{1+4}{2+4}} =$

② use expressions of f and g: $(f \circ g)(4) = f(g(4)) =$

$f\left(\frac{1}{2+4}\right) = f\left(\frac{1}{6}\right) = \sqrt{1 - \frac{1}{6}} = \sqrt{\frac{6}{6} - \frac{1}{6}} = \boxed{\sqrt{\frac{5}{6}}}$

#12

(g) $(g \circ f)(x) = g(f(x)) = g(\sqrt{1-x}) =$

$= \boxed{\dfrac{1}{2+\sqrt{1-x}}}$

* if we are asked to "remove radical from the denominator":

$$\frac{1}{2+\sqrt{1-x}} \cdot \frac{(2-\sqrt{1-x})}{(2-\sqrt{1-x})} = \frac{2-\sqrt{1-x}}{4-(1-x)} = \boxed{\frac{2-\sqrt{1-x}}{3+x}}$$

$\parallel$

$(g \circ f)(x)$

domain: grab domain of $f(x)$ and remove all values of x, such that $f(x)$ is not from the domain of $g(x)$

or look at $\dfrac{1}{2+\sqrt{1-x}}$

- exclude cases when the denominator is 0:

$2+\sqrt{1-x} \neq 0$ but $\sqrt{1-x} \neq -2$.

- put the restriction for $\sqrt{\ }$: $1-x \geq 0$

$1 \geq x$ or $x \leq 1$

Hence, the domain: $\boxed{(-\infty, 1]}$

(h) 2 ways:

(1) use (g) $(g \circ f)(x) = \dfrac{1}{2+\sqrt{1-x}}$

$$(g \circ f)(-3) = \frac{1}{2+\sqrt{1-(-3)}} = \frac{1}{2+\sqrt{4}} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

(2) use expressions for $f(x)$ and $g(x)$: $(g \circ f)(4) = g(f(4$

$= g(\sqrt{1-(-3)}) = g(\sqrt{4}) = g(2) = \dfrac{1}{2+2} = \boxed{\dfrac{1}{4}}$