Plan for today

Today we will discuss:

- right triangle trigonometry

Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

$$\sin\theta = y = \frac{y}{1} \qquad \cos\theta = x = \frac{x}{1}$$

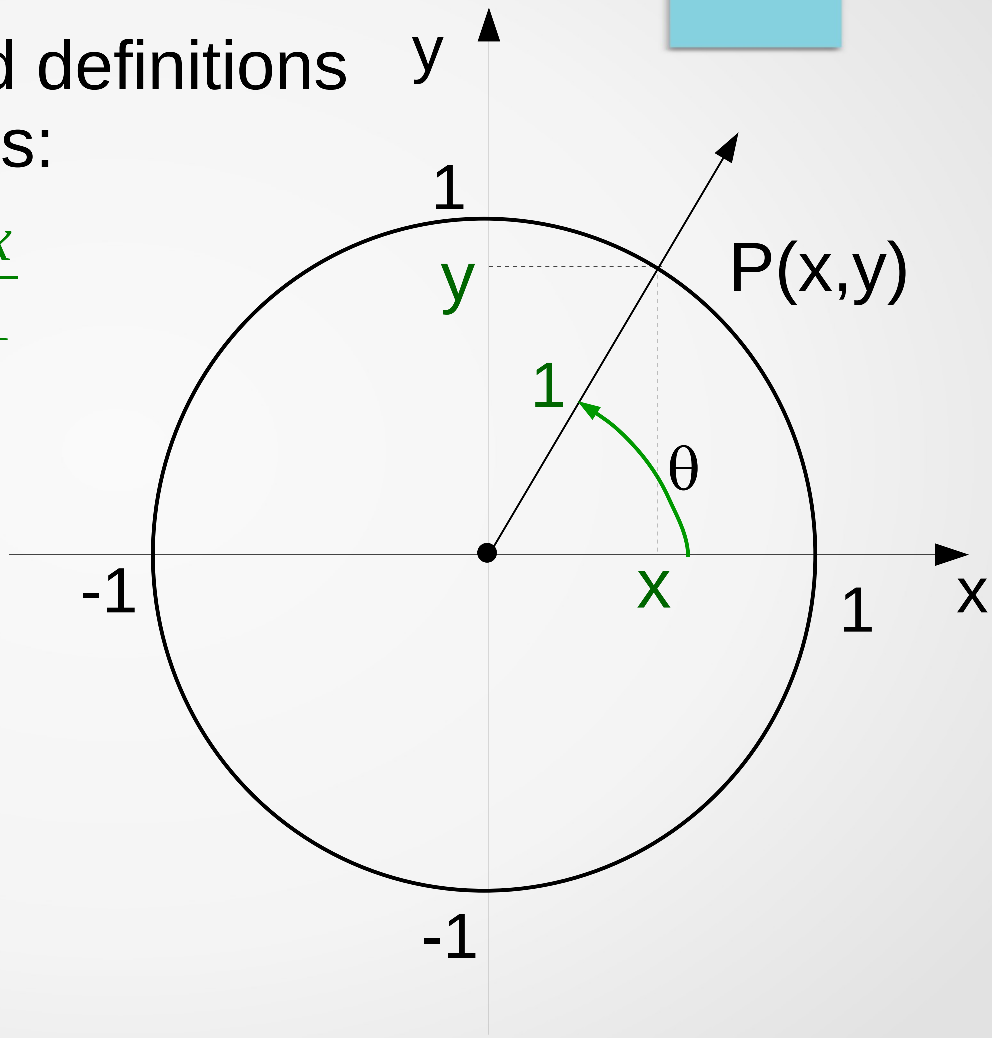
radius = 1

$$\tan\theta = \frac{y}{x}, x \neq 0$$

$$\cot\theta = \frac{x}{y}, y \neq 0$$

$$csc\,\theta = \frac{1}{y}, y \neq 0$$

$$sec\,\theta = \frac{1}{x}, x \neq 0$$



Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

$$\sin\theta = y = \frac{y}{1} \qquad \cos\theta = x = \frac{x}{1}$$

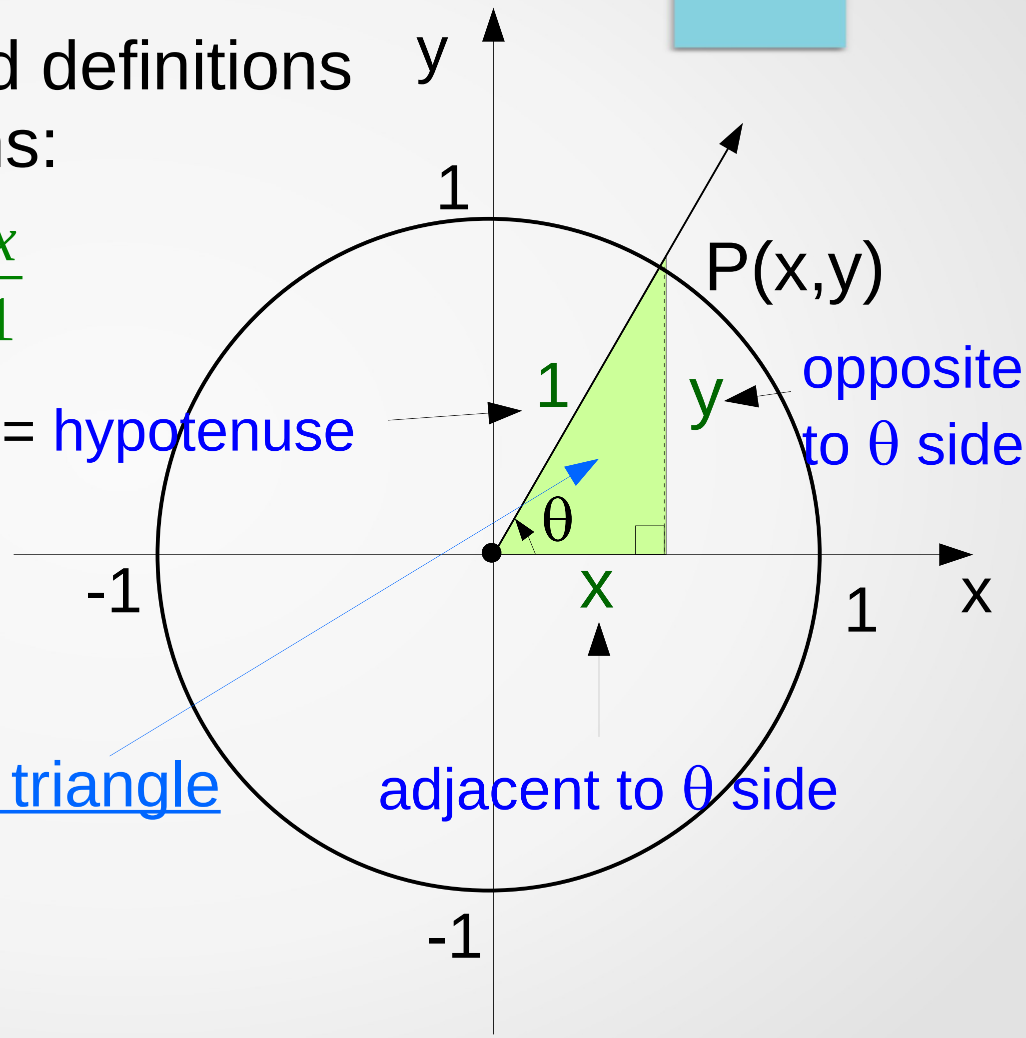
$$\tan\theta = \frac{y}{x}, x \neq 0$$

$$\cot\theta = \frac{x}{y}, y \neq 0$$

$$csc\,\theta = \frac{1}{y}, y \neq 0$$

$$sec\,\theta = \frac{1}{x}, x \neq 0$$

radius = 1 = hypotenuse



Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

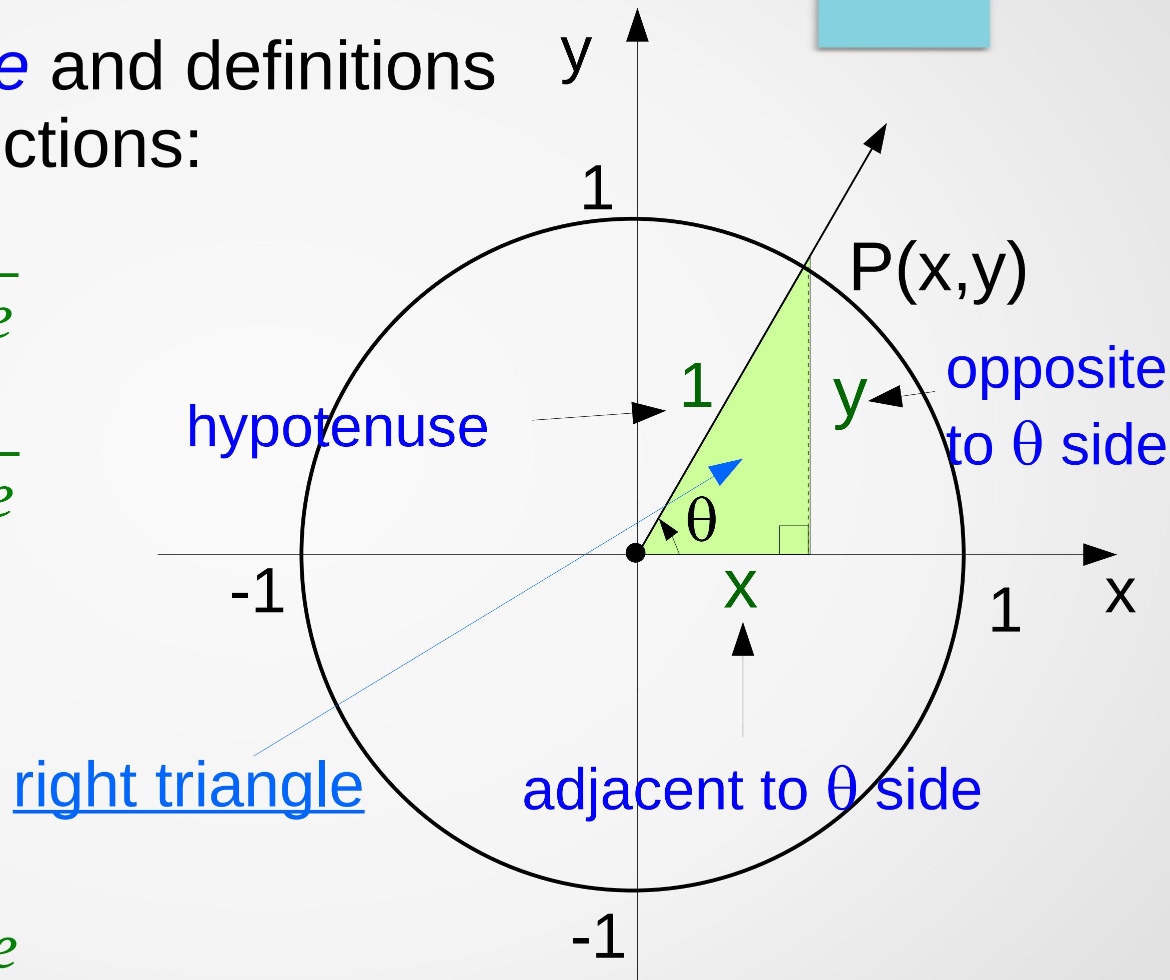
$$\sin\theta = \frac{y}{1} = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{x}{1} = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{y}{x} = \frac{opposite}{adjacent}$$

$$\cot\theta = \frac{x}{y} = \frac{adjacent}{opposite}$$

$$\csc\theta = \frac{1}{y} = \frac{hypotenuse}{opposite}$$

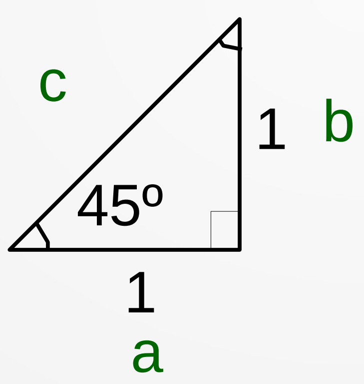


$$\sec\theta = \frac{1}{x} = \frac{hypotenuse}{adjacent}$$

Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:



Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

by *Pythagorean theorem*:

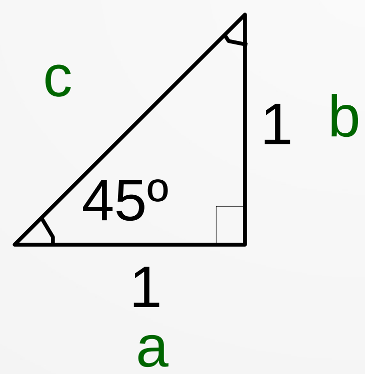
$a^2 + b^2 = c^2$

hence

$1^2 + 1^2 = c^2$

and finally

$c = \sqrt{2}$



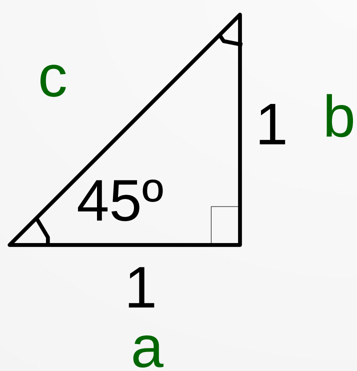
Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

$$\sin 45^o =$$

by *Pythagorean theorem*:



$$\cos 45^o =$$

$a^2 + b^2 = c^2$

hence

$1^2 + 1^2 = c^2$

$$\tan 45^o =$$

and finally

$c = \sqrt{2}$

$$\cot 45^o =$$

$$csc\, 45^o =$$

$$sec\, 45^o =$$

Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

by *Pythagorean theorem*:

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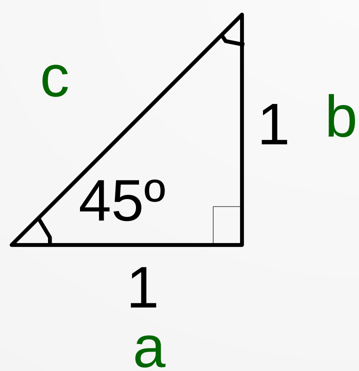


$$\sin 45^o = \frac{opposite}{hypotenuse} = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^o = \frac{adjacent}{hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^o = \frac{opposite}{adjacent} = \frac{b}{a} = \frac{1}{1} = 1$$

$$\cot 45^o = \frac{adjacent}{opposite} = \frac{a}{b} = \frac{1}{1} = 1$$

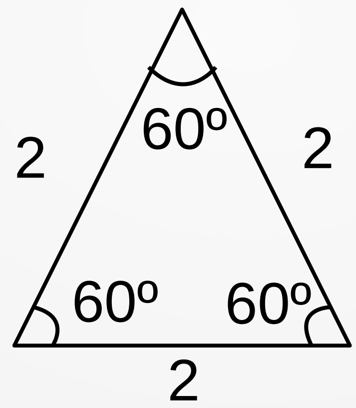
$$csc\ 45^o = \frac{hypotenuse}{opposite} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$sec\ 45^o = \frac{hypotenuse}{adjacent} = \sqrt{2}$$

Trigonometric Functions

Function values for some special angles

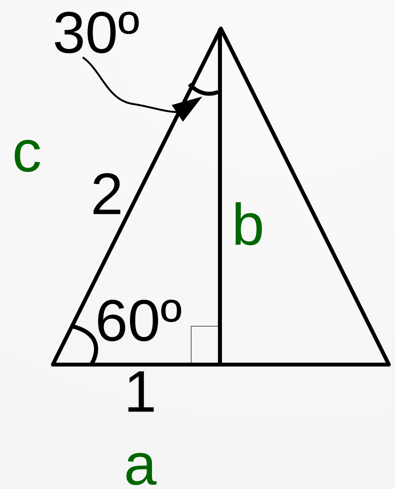
Consider an *equilateral triangle* with all equal sides of length 2:



Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:



Trigonometric Functions

Function values for some special angles

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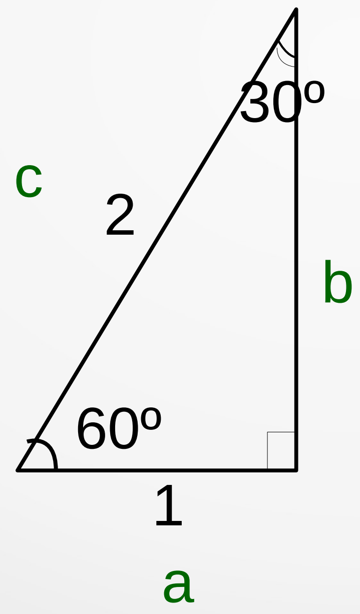


Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

by *Pythagorean theorem*:

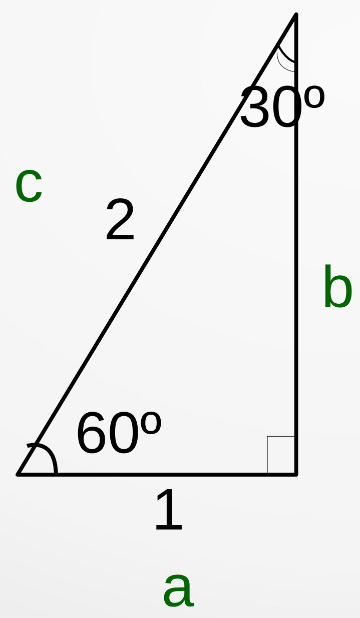
$a^2 + b^2 = c^2$

hence

$1^2 + b^2 = 2^2$

and finally

$b = \sqrt{3}$



Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

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hence

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and finally

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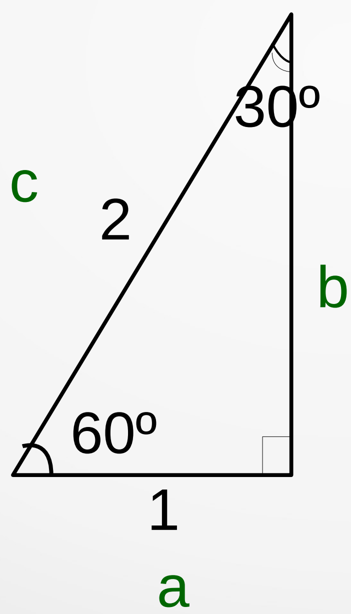

$\sin 30^o =$

$\sin 60^o =$

$\cos 30^o =$

$\cos 60^o =$

...

Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

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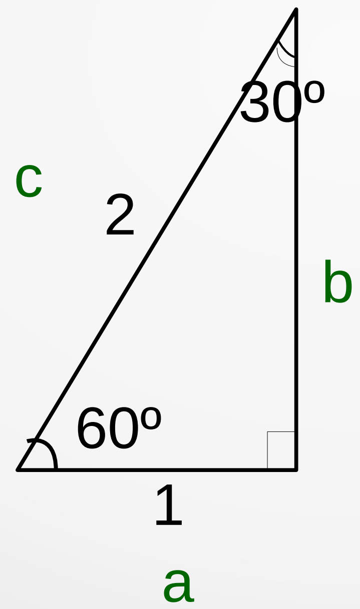


$$\sin 30^o = \frac{opposite}{hypotenuse} = \frac{a}{c} = \frac{1}{2}$$

$$\sin 60^o = \frac{opposite}{hypotenuse} = \frac{b}{c} = \frac{\sqrt{3}}{2}$$

$$\cos 30^o = \frac{adjacent}{hypotenuse} = \frac{b}{c} = \frac{\sqrt{3}}{2}$$

$$\cos 60^o = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \frac{1}{2}$$

...

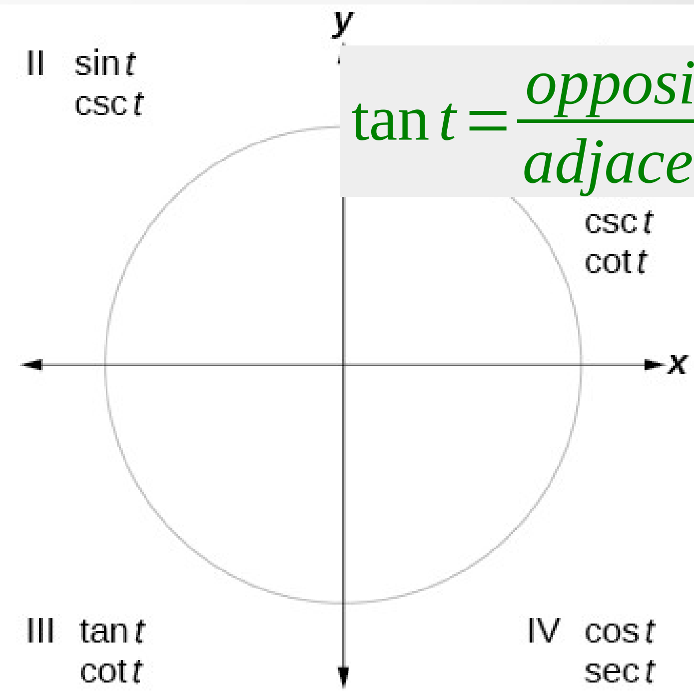
Trigonometric Functions

Example: If the terminal point determined by t is (-5,-4), then find `sin(t), cos(t)` and `tan(t)`.

$$\sin t = \frac{opposite}{hypotenuse}$$

$$\cos t = \frac{adjacent}{hypotenuse}$$

$$\tan t = \frac{opposite}{adjacent}$$



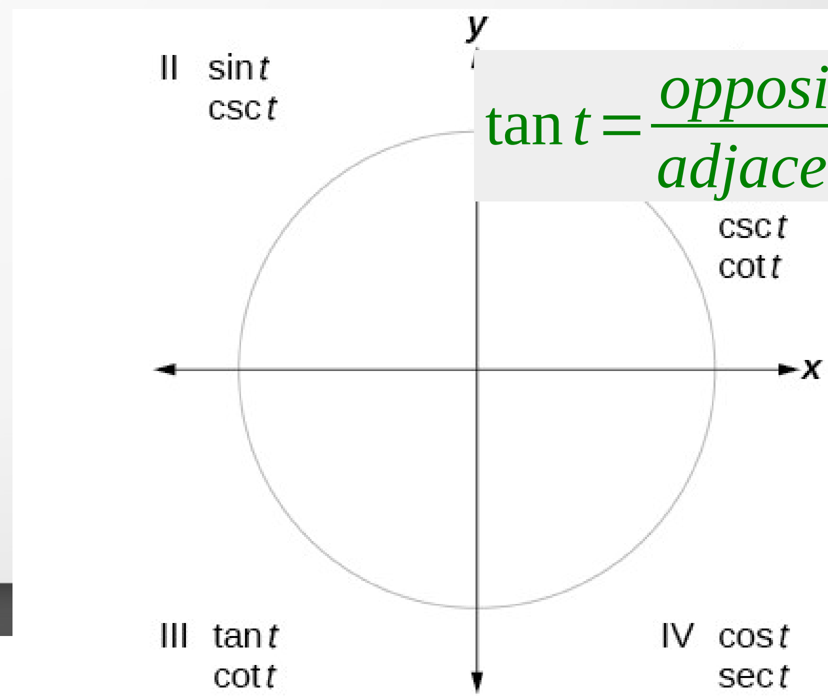
In-class practice

Example: If the terminal point determined by t is (4,-3), find `sin(t)`.

$$\sin t = \frac{opposite}{hypotenuse}$$

$$\cos t = \frac{adjacent}{hypotenuse}$$

$$\tan t = \frac{opposite}{adjacent}$$



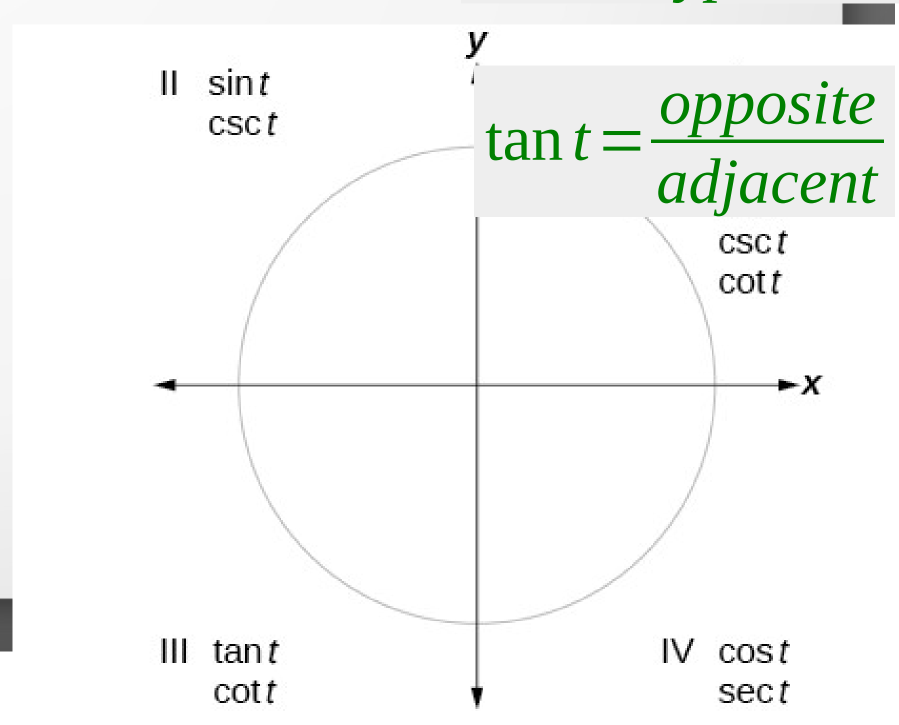
In-class practice

Example: If the terminal point determined by t is (4,-3), find `cos(t)`.

$$\sin t = \frac{opposite}{hypotenuse}$$

$$\cos t = \frac{adjacent}{hypotenuse}$$

$$\tan t = \frac{opposite}{adjacent}$$



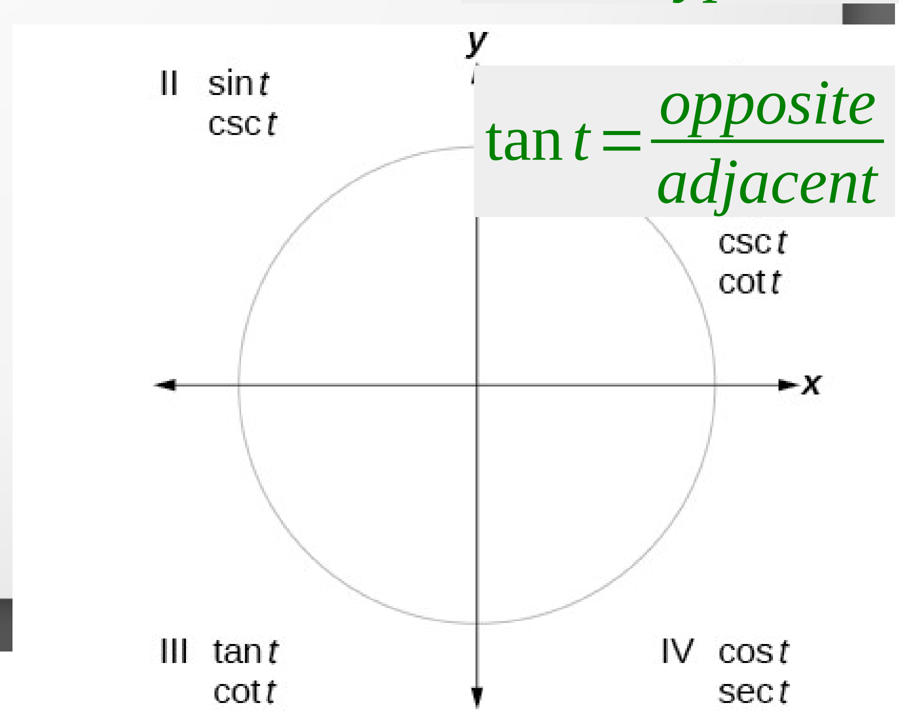
In-class practice

Example: If the terminal point determined by t is (4,-3), find `tan(t)`.

$$\sin t = \frac{opposite}{hypotenuse}$$

$$\cos t = \frac{adjacent}{hypotenuse}$$

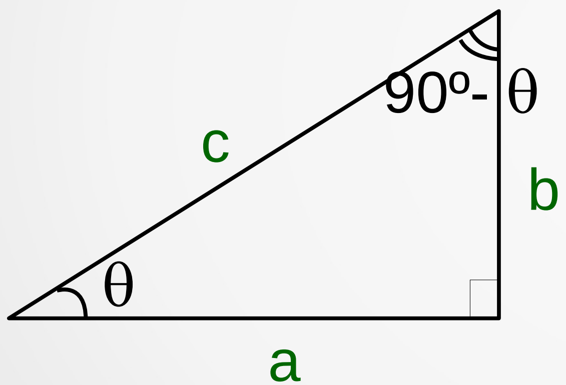
$$\tan t = \frac{opposite}{adjacent}$$



Trigonometric Functions

Trigonometric Functions and Complements

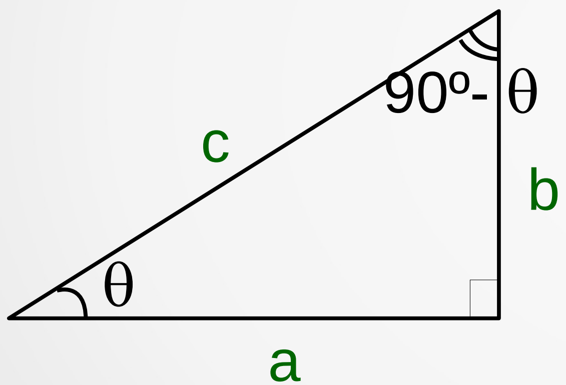
Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$



Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$

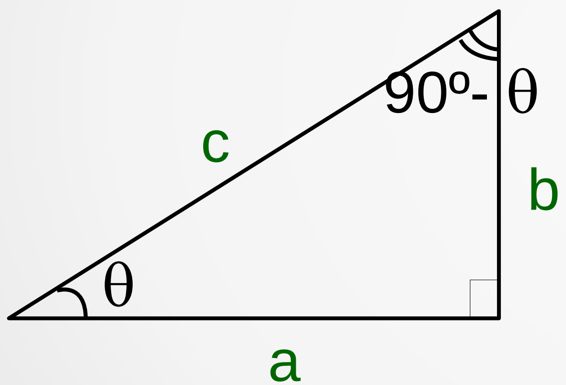


$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos\left(90^{o} - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$



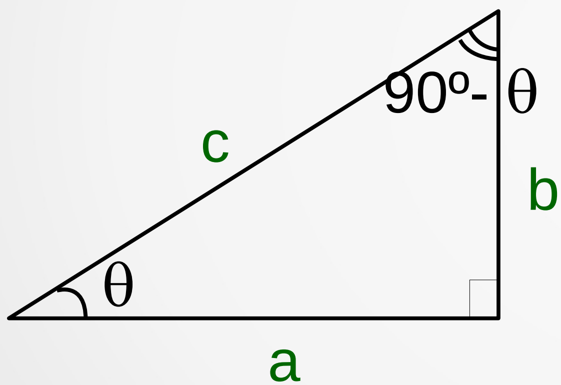
$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \sin\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos(90^o - \theta)$$

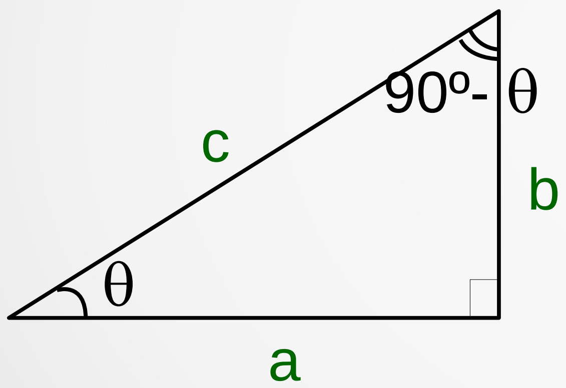
$$\cos(90^o - \theta) = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \sin(90^o - \theta)$$

Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos\left(90^o - \theta\right)$$

$$\cos\left(90^o - \theta\right) = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

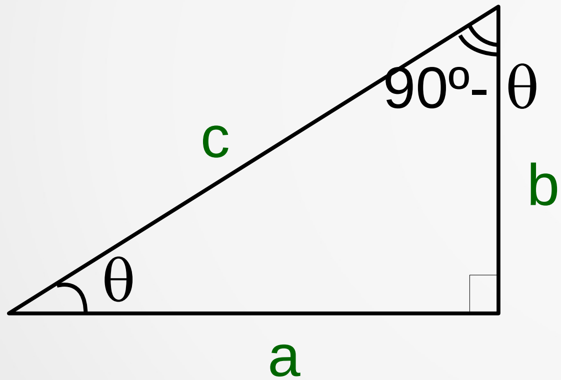
$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \sin\left(90^o - \theta\right)$$

$$\sin\left(90^o - \theta\right) = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90º or $\frac{\pi}{2}$



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos\left(90^{o} - \theta\right)$$

$$\cos\left(90^{o} - \theta\right) = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \sin\left(90^{o} - \theta\right)$$

$$\sin\left(90^{o} - \theta\right) = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

Similarly for tan, cot, sec, and csc

Trigonometric Functions

Trigonometric Functions and Complements

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

$$sec\,\theta = csc\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

$$sec\,\theta = csc\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º)

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

$$sec\,\theta = csc\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º) = sin 30º = ½

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

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Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º) = sin 30º = ½

- If we know that tan 17º ≈ 0.3057 and asked to find cot 73º

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

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Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º) = sin 30º = ½

- If we know that tan 17º ≈ 0.3057 and asked to find cot 73º, then

cot 73º = cot (90-17º)

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

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Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º) = sin 30º = ½

- If we know that tan 17º ≈ 0.3057 and asked to find cot 73º, then

cot 73º = cot (90-17º) = tan 17º ≈ 0.3057

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

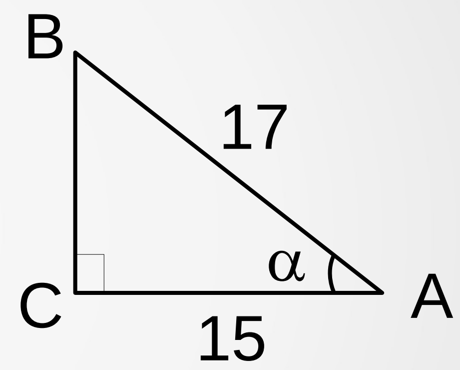
$$sec\,\theta = csc\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(1) Use *Pythagorean Theorem* to find the length of the missing side, then find the value of each of the six trigonometric functions.



Trigonometric Functions

Trigonometric Functions and Complements

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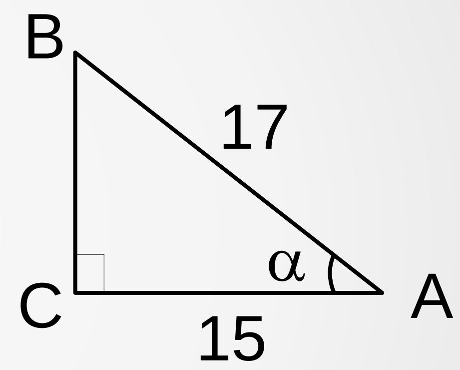

$$\sin\alpha = \frac{opposite}{hypotenuse}$$

$$csc\,\theta = \frac{hypotenuse}{opposite}$$

$$\cos\alpha = \frac{adjacent}{hypotenuse}$$

$$sec\,\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\cot\theta = \frac{adjacent}{opposite}$$

Trigonometric Functions

Trigonometric Functions and Complements

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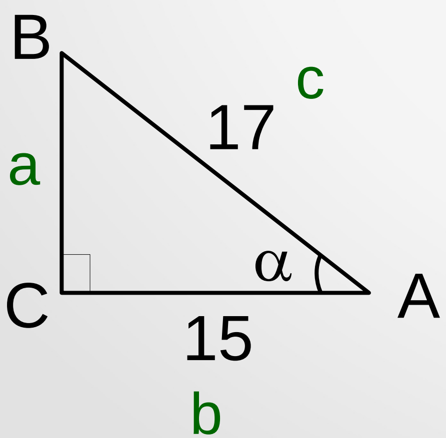
(1) Use *Pythagorean Theorem* to find the length of the missing side, then find the value of each of the six trigonometric functions.



$$a^2 + b^2 = c^2$$

$$a^2 + 15^2 = 17^2$$

$$a =$$

$$\sin\alpha = \frac{opposite}{hypotenuse}$$

$$\cos\alpha = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\cot\theta = \frac{adjacent}{opposite}$$

$$\csc\theta = \frac{hypotenuse}{opposite}$$

$$\sec\theta = \frac{hypotenuse}{adjacent}$$

Trigonometric Functions

Trigonometric Functions and Complements

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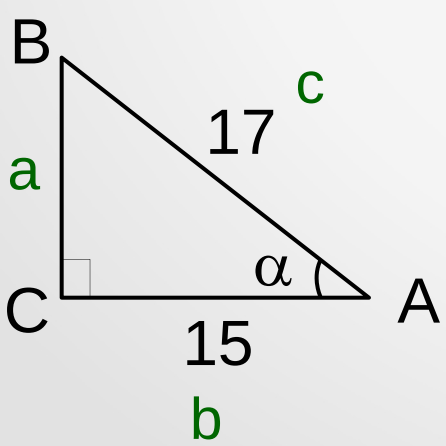


$$a^2 + b^2 = c^2$$

$$a^2 + 15^2 = 17^2$$

$$a = \sqrt{289 - 225} = \sqrt{64} = 8$$

$$\sin\alpha = \frac{opposite}{hypotenuse} = \frac{8}{17}$$

$$\cos\alpha = \frac{adjacent}{hypotenuse} = \frac{15}{17}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{8}{15}$$

$$\cot\theta = \frac{adjacent}{opposite} = \frac{15}{8}$$

$$\csc\theta = \frac{hypotenuse}{opposite} = \frac{17}{8}$$

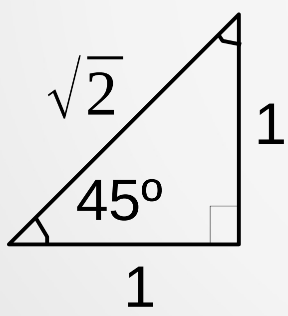
$$\sec\theta = \frac{hypotenuse}{adjacent} = \frac{17}{15}$$

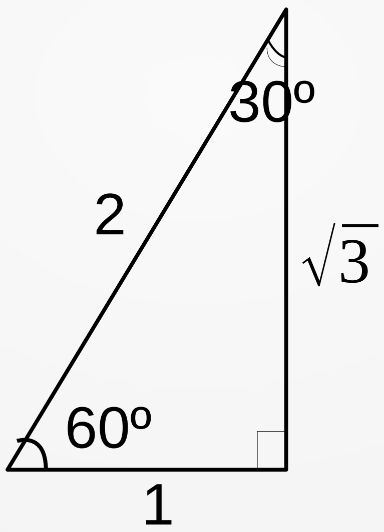
Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(2) Use the given triangles to evaluate the given expressions (simplify your answer)





a) $\sin 30^o$ b) $\cot \frac{\pi}{4}$ c) $\cos \frac{\pi}{6} \, sec \frac{\pi}{6} - \cot \frac{\pi}{6}$

d) $2 \tan 60^0 + \cos 45^o \tan 30^o$

Trigonometric Functions

Trigonometric Functions and Complements

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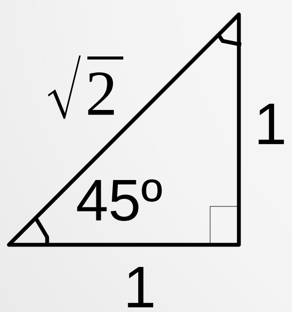
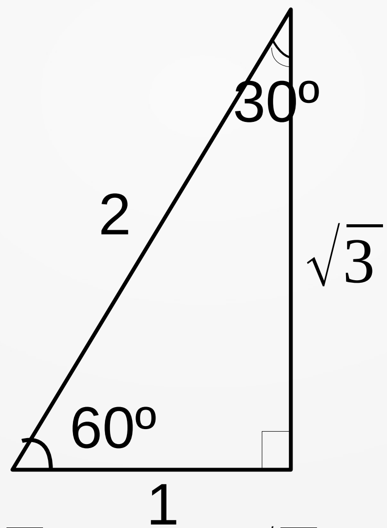
(2) Use the given triangles to evaluate the given expressions (simplify your answer)





a) $\sin 30^o = \frac{1}{2}$

b) $\cot \frac{\pi}{4} = \frac{1}{1} = 1$

c) $\cos\frac{\pi}{6}\, sec\frac{\pi}{6} - \cot\frac{\pi}{6} = \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{1} = 1 - \sqrt{3}$

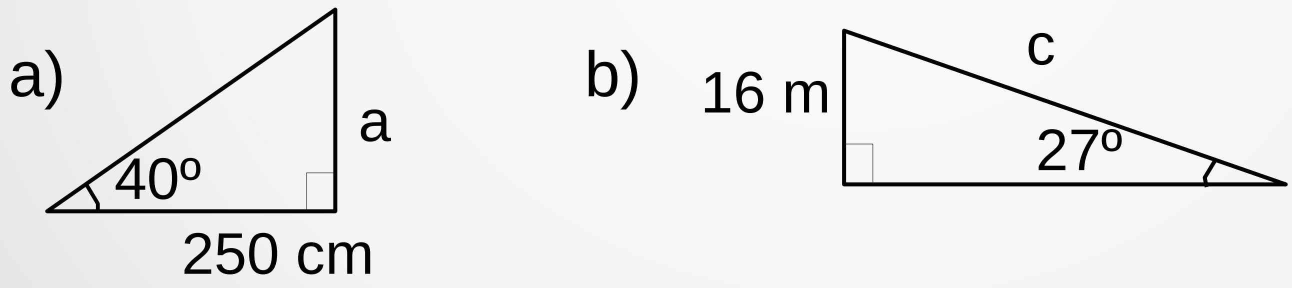
d) $2\tan 60^0 + \cos 45^o \tan 30^o = 2 \times \frac{\sqrt{3}}{1} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} = 2\sqrt{3} + \frac{1}{\sqrt{6}} = 2\sqrt{3} + \frac{\sqrt{6}}{6}$

Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(3) Find the length of the side marked with a lower case letter. Round the answer to the nearest whole number (use calculator)



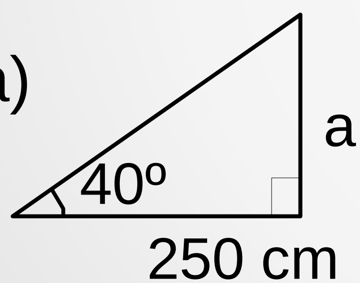
Trigonometric Functions

Trigonometric Functions and Complements

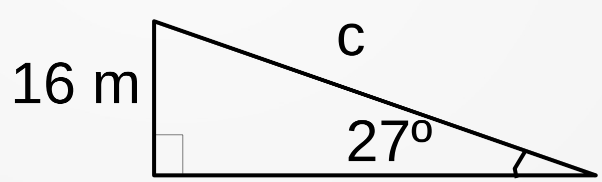
Examples: $\tan\theta = \dfrac{opposite}{adjacent}$ $\sin\alpha = \dfrac{opposite}{hypotenuse}$

(3) Find the length of the side marked with a lower case letter. Round the answer to the nearest whole number (use calculator)

a)



b)



Trigonometric Functions

Trigonometric Functions and Complements

Examples:

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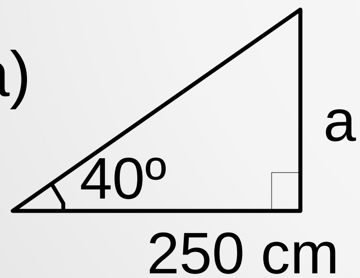
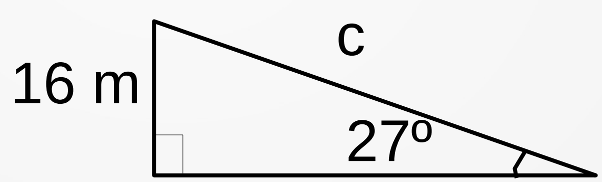


b)



$$\tan 40^o = \frac{a}{250}$$

$$a = 250 \tan 40^o \approx 210\,cm$$

$$\sin 27^o = \frac{16}{c}$$

$$c = \frac{16}{\sin 27^o} \approx 35\,m$$

Plan for today

Today we will discussed:

- right triangle trigonometry