Learning objectives

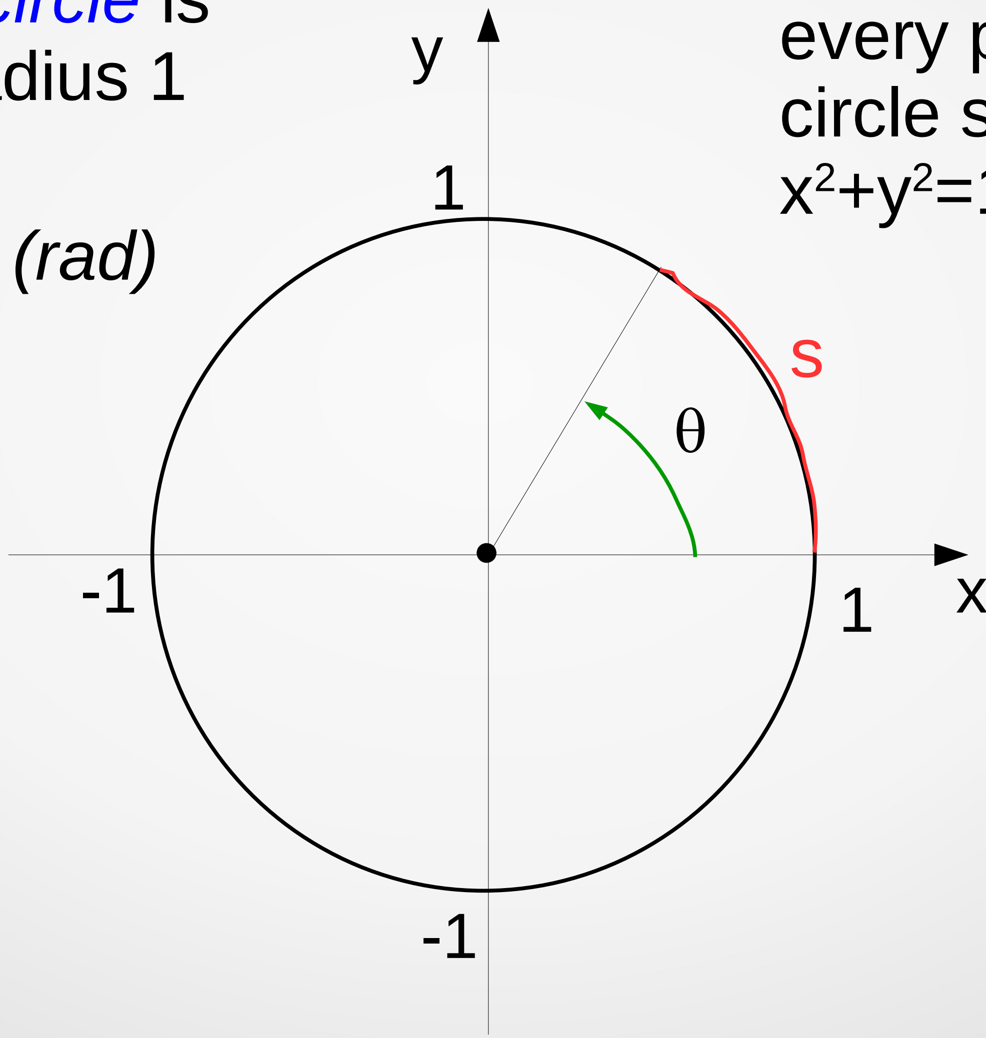
In this section we will:

- Find function values for the sine and cosine of 30° or ($\frac{\pi}{6}$), 45° or ($\frac{\pi}{4}$) and 60°or ($\frac{\pi}{3}$).
- Identify the domain and range of sine and cosine functions.
- Use reference angles to evaluate trigonometric functions.

Trigonometric Functions: The Unit Circle

[Def] a *unit circle* is a circle of radius 1

$$\theta = \frac{s}{r} = \frac{s}{1} = s \text{ (rad)}$$



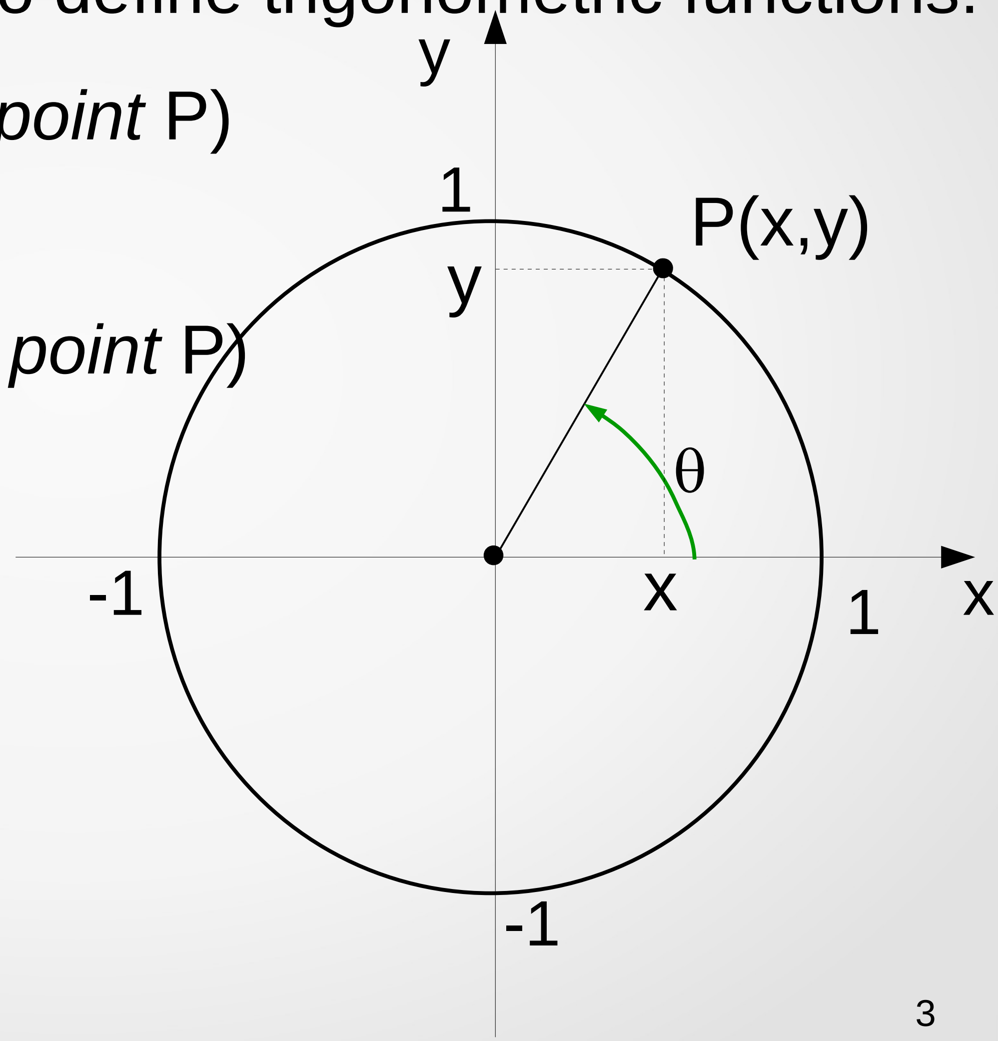
every point on the circle satisfies to $x^2+y^2=1$

Trigonometric Functions: The Unit Circle

We are using a *unit circle* to define trigonometric functions:

$\sin\theta = y$ (*y-coordinate of point* P)
"sine of theta"

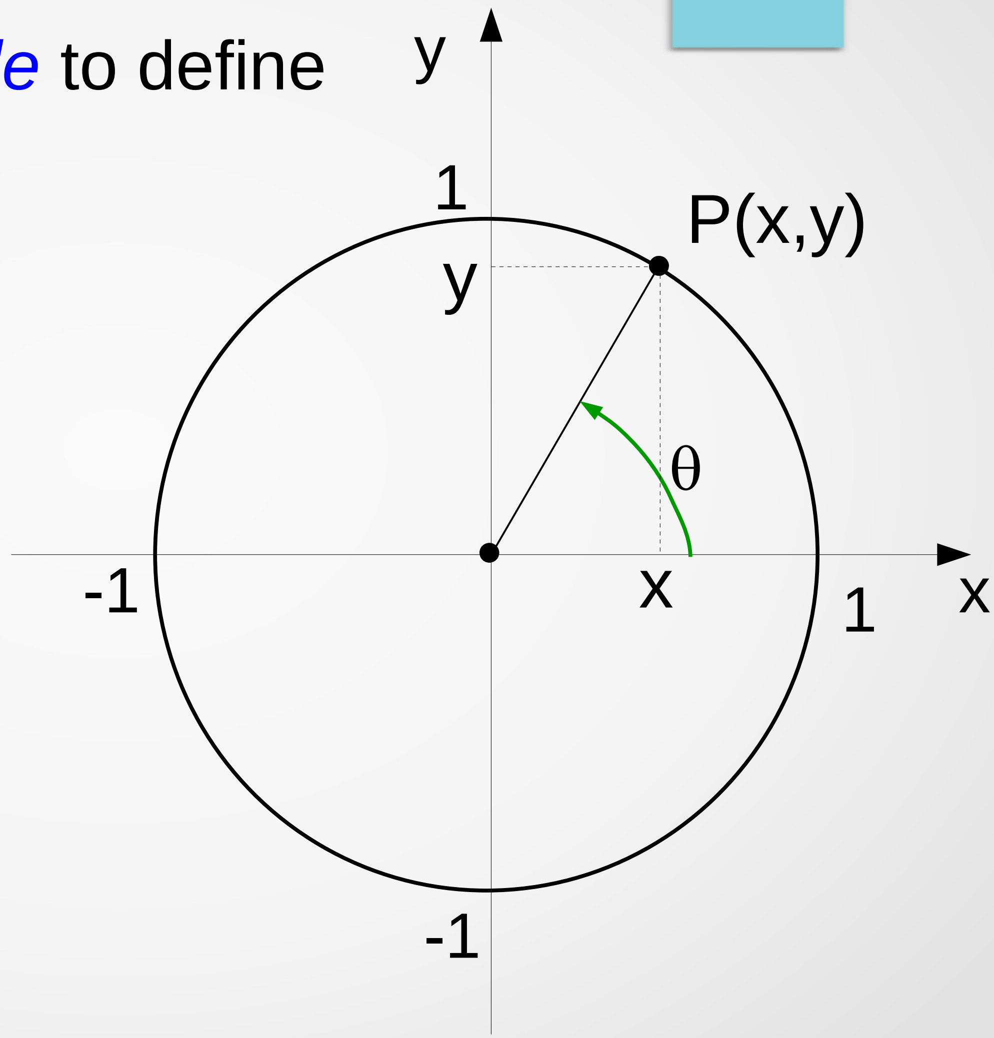
$\cos\theta = x$ (*x-coordinate of point* P)
"cosine of theta"



Trigonometric Functions: The Unit Circle

We are using a *unit circle* to define trigonometric functions:

$\sin \theta = y$
$\cos \theta = x$



Sometimes there functions are called *circular functions*.

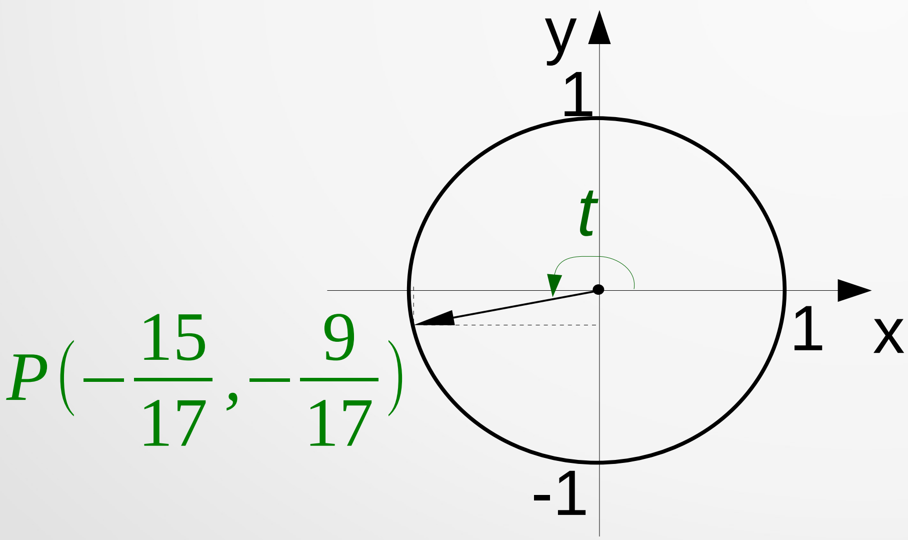
Trigonometric Functions: The Unit Circle

Example 1: for the point $P(-\frac{15}{17}, -\frac{9}{17})$, find the values of the sine and cosine at angle *t*, measured in radians

using a *unit circle* to define trigonometric functions:

sin θ = y

cos θ = x



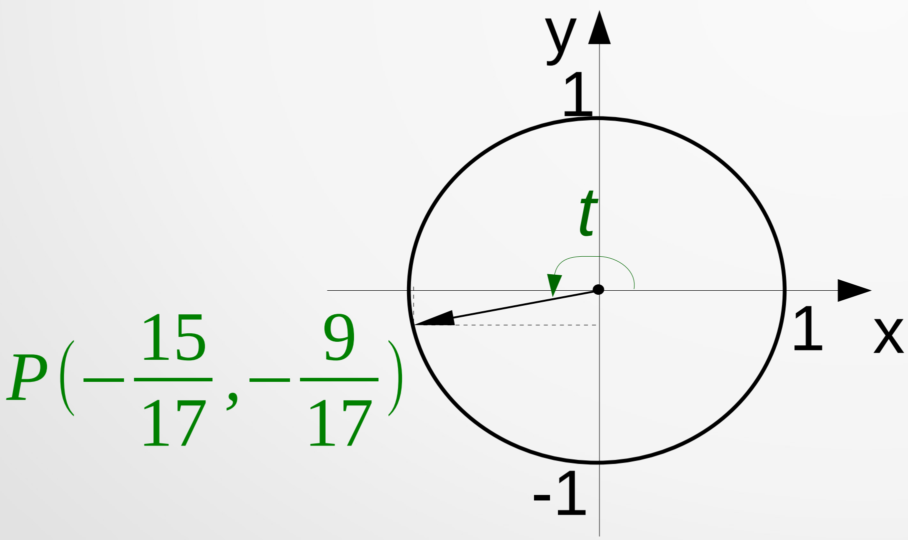
Trigonometric Functions: The Unit Circle

Example 1: for the point $P(-\frac{15}{17}, -\frac{9}{17})$, find the values of the sine and cosine at angle *t*, measured in radians

using a *unit circle* to define trigonometric functions:

$\sin\theta = y = -\frac{9}{17}$

$\cos\theta = x = -\frac{15}{17}$



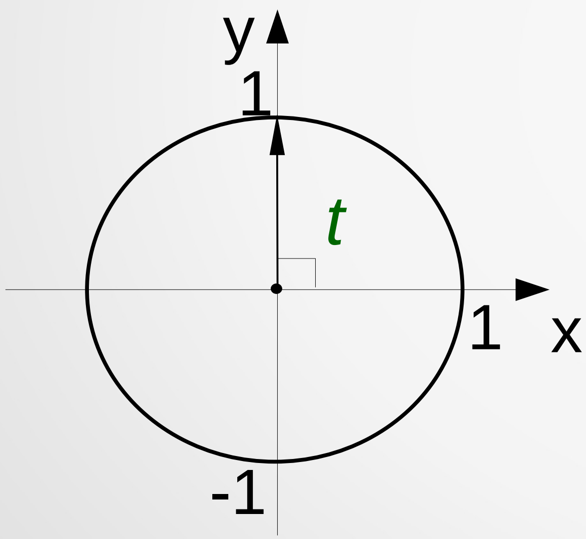
Trigonometric Functions: The Unit Circle

Example 2: find the values of the sine and cosine at angle $t = \frac{\pi}{2}$

using a *unit circle* to define trigonometric functions:

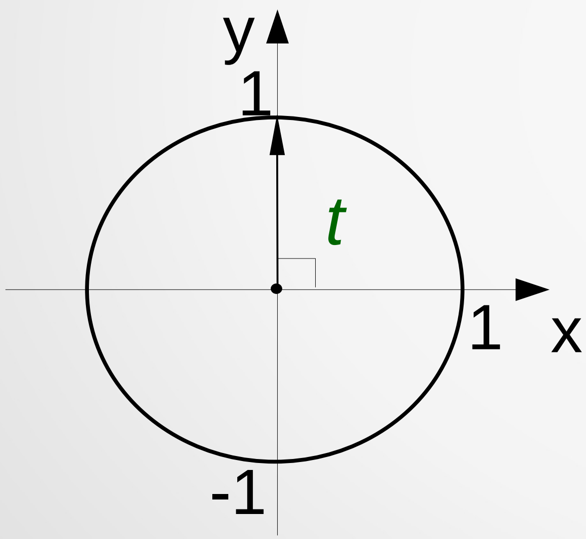
$\sin\theta = y$

$\cos\theta = x$



Trigonometric Functions: The Unit Circle

Example 2: find the values of the sine and cosine at angle $t = \frac{\pi}{2}$



using a *unit circle* to define trigonometric functions:

$\sin \theta = y = 1$

$\cos \theta = x = 0$

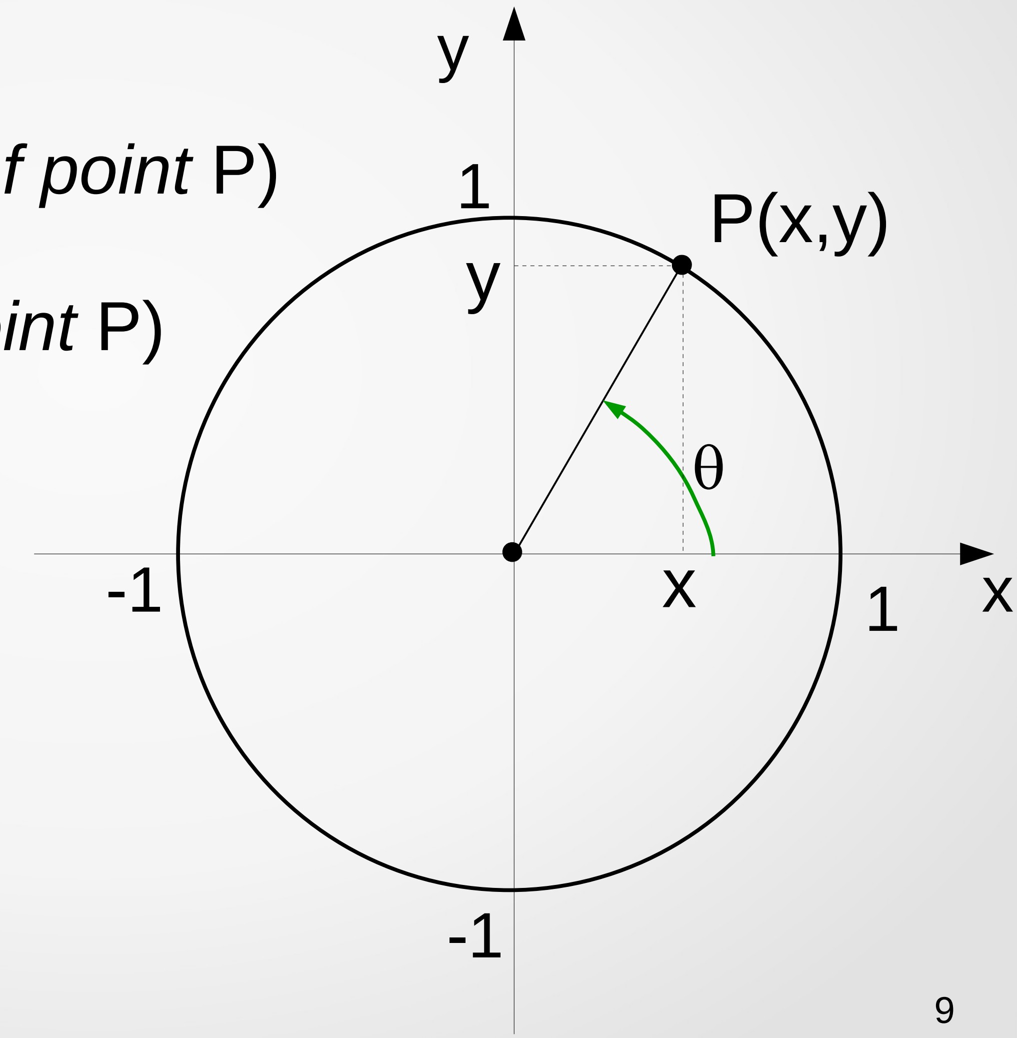
Trigonometric Functions: The Unit Circle

Domain and range of sin, cos functions

$\sin\theta = y$ (*y-coordinate of point* P)

$\cos\theta = x$ (*x-coord. of point* P)

domain: $(-\infty,\infty)$
range: $[-1,1]$

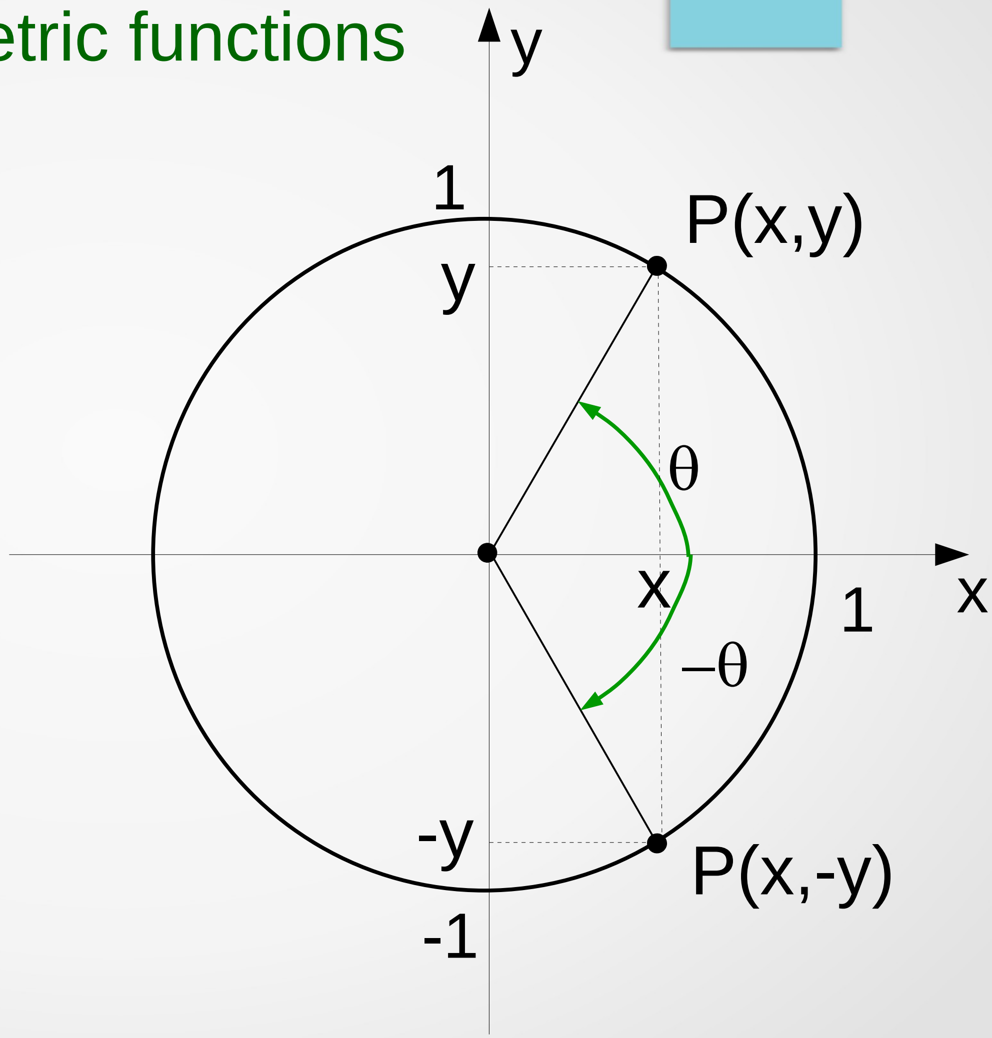


Trigonometric Functions: The Unit Circle

Even and odd trigonometric functions

sin θ = y   sin(-θ) =

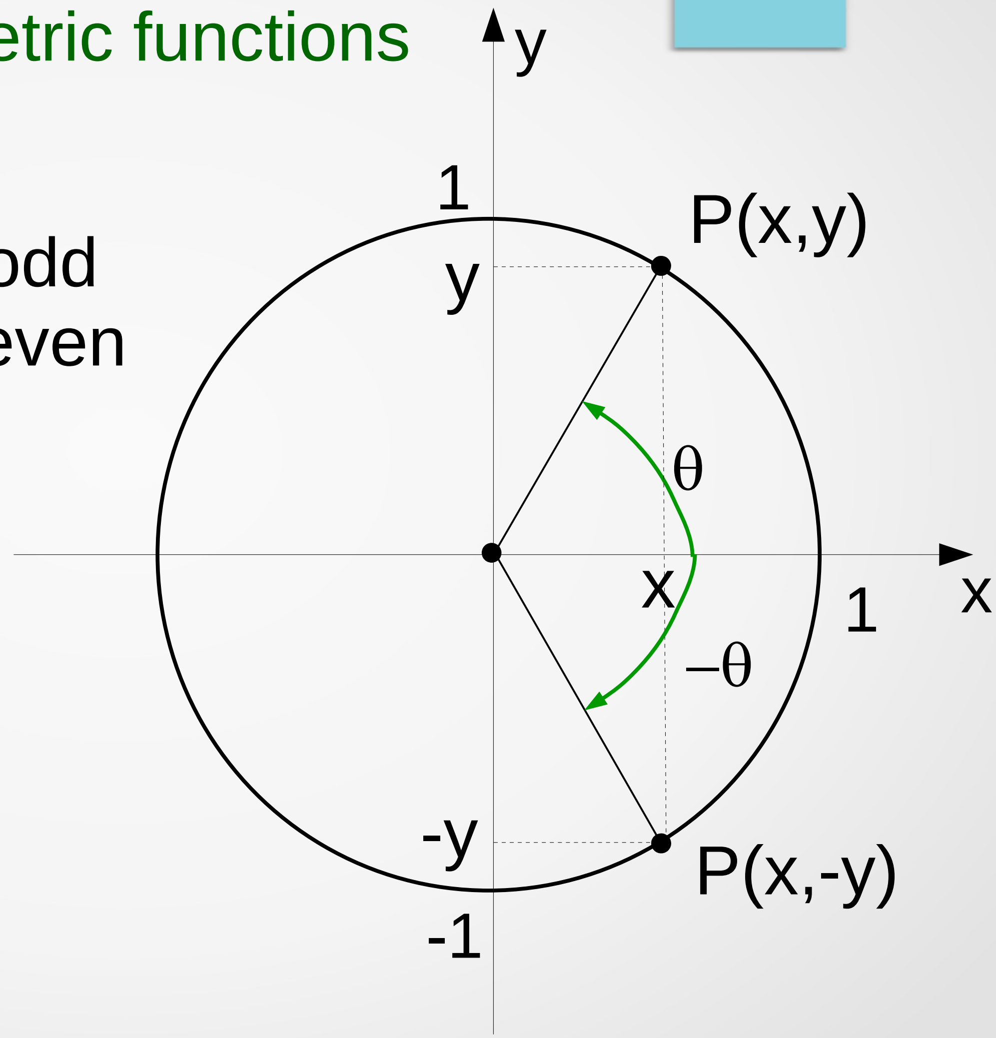
cos θ = x   cos(-θ) =



Trigonometric Functions: The Unit Circle

Even and odd trigonometric functions

sin θ = y	sin(-θ) = -y	odd
cos θ = x	cos(-θ) = x	even



Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$$\sin\left(\frac{\pi}{4}\right)$$

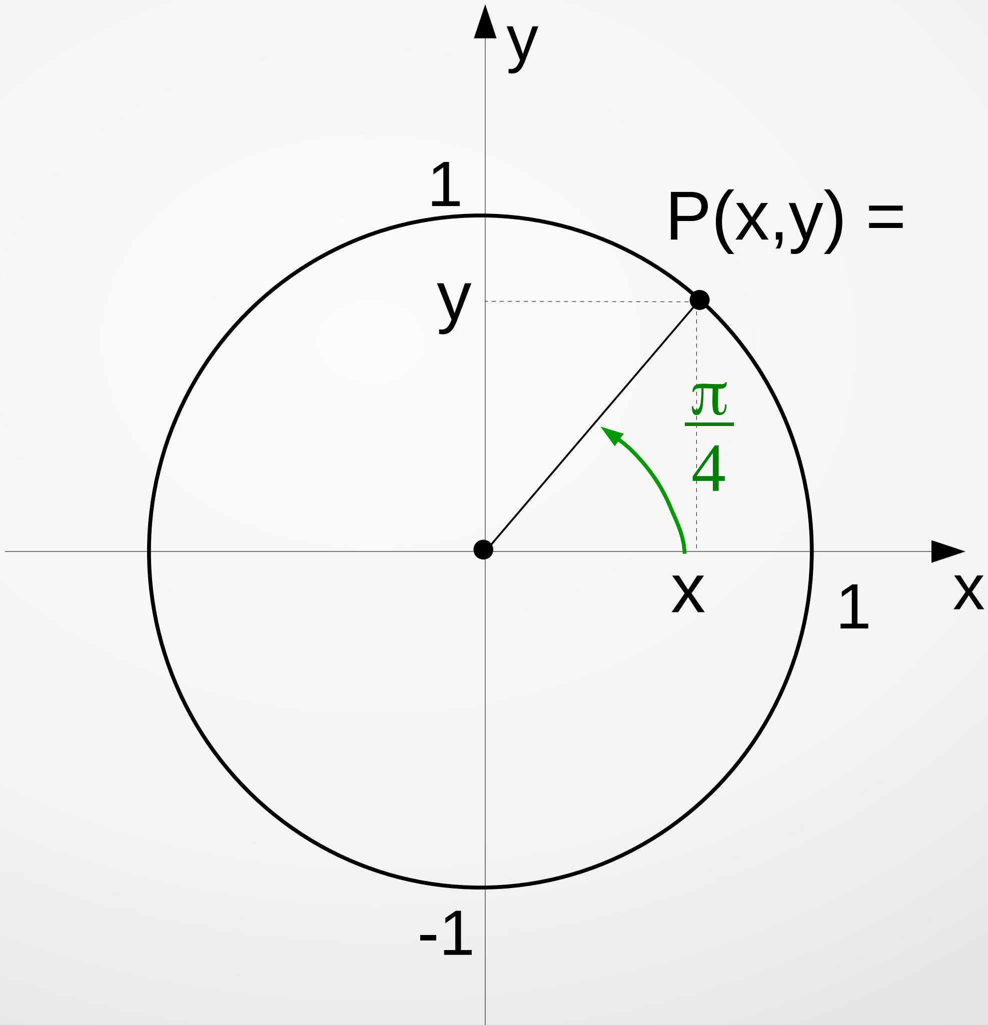
$$\cos\left(\frac{\pi}{4}\right)$$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{4}\right)$



Trigonometric Functions: The Unit Circle

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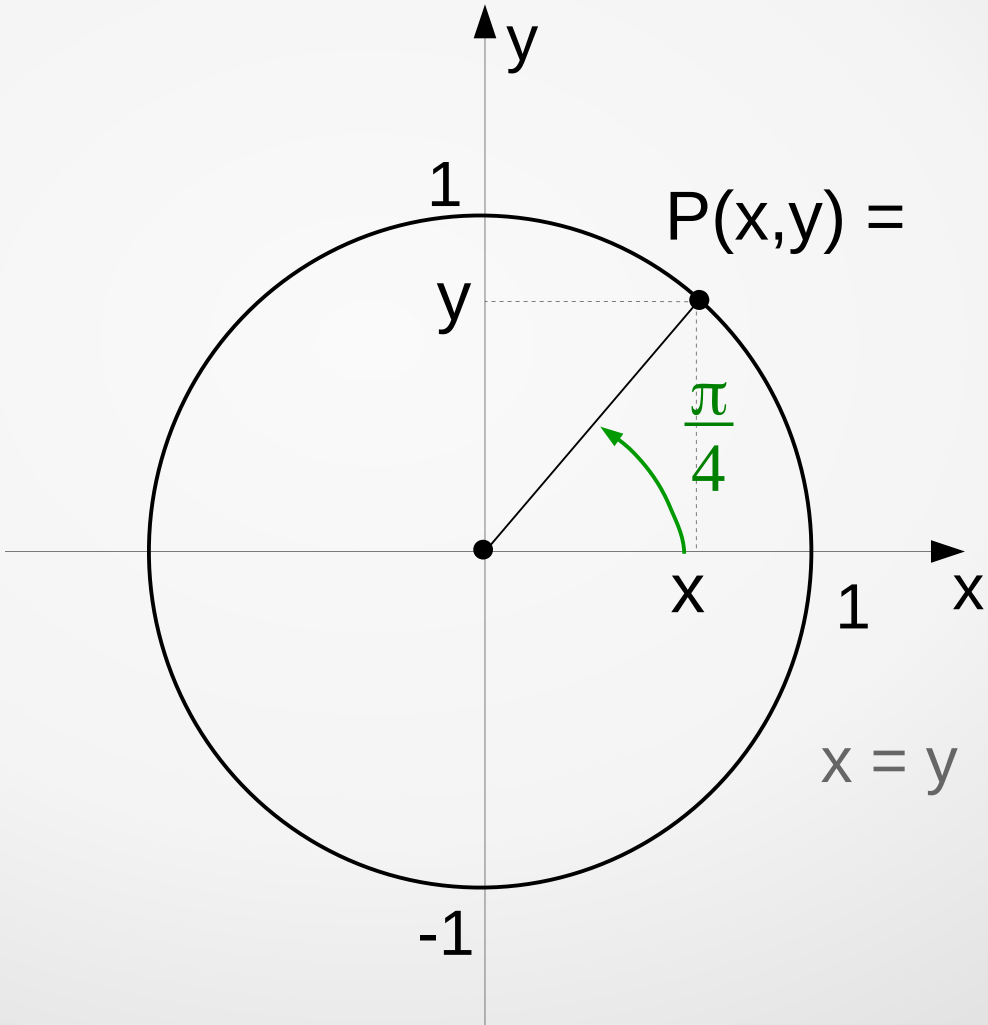

Trigonometric Functions: The Unit Circle

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$\sin\left(\frac{\pi}{4}\right)$

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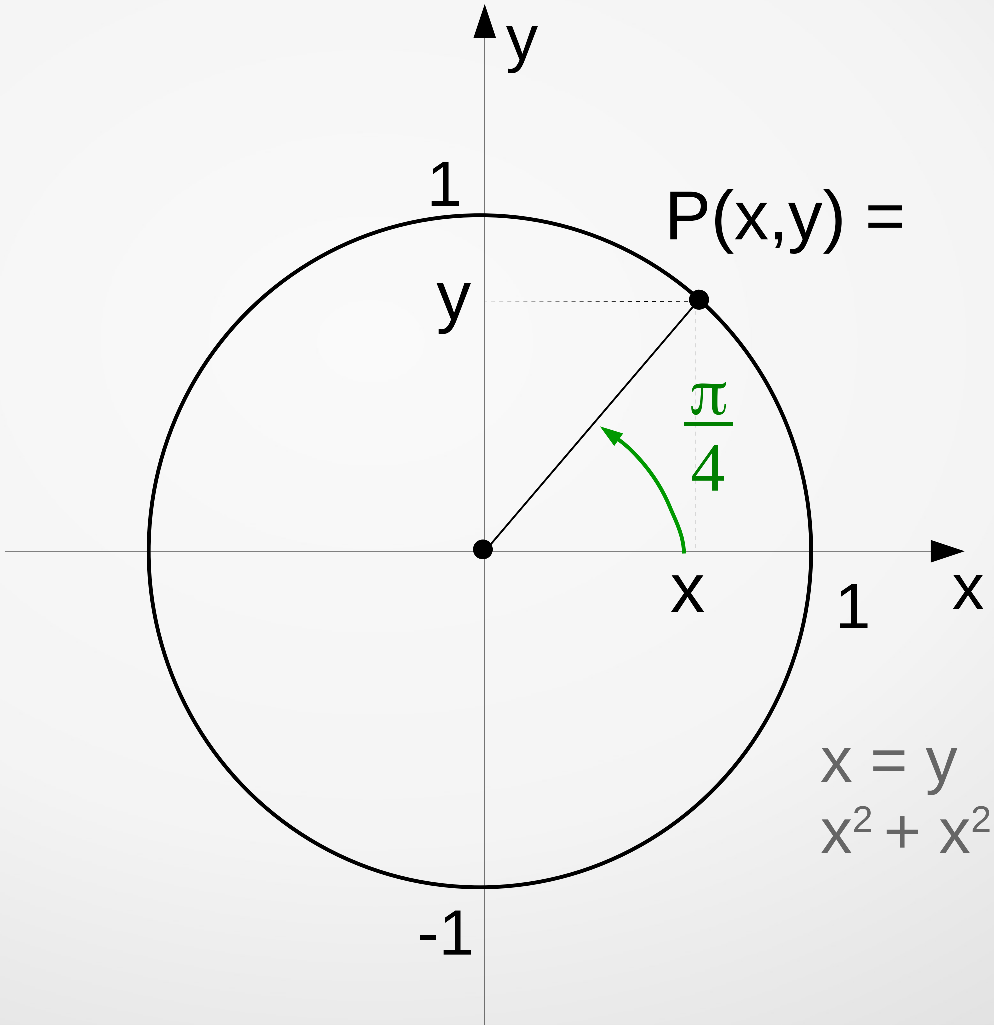


$x = y$

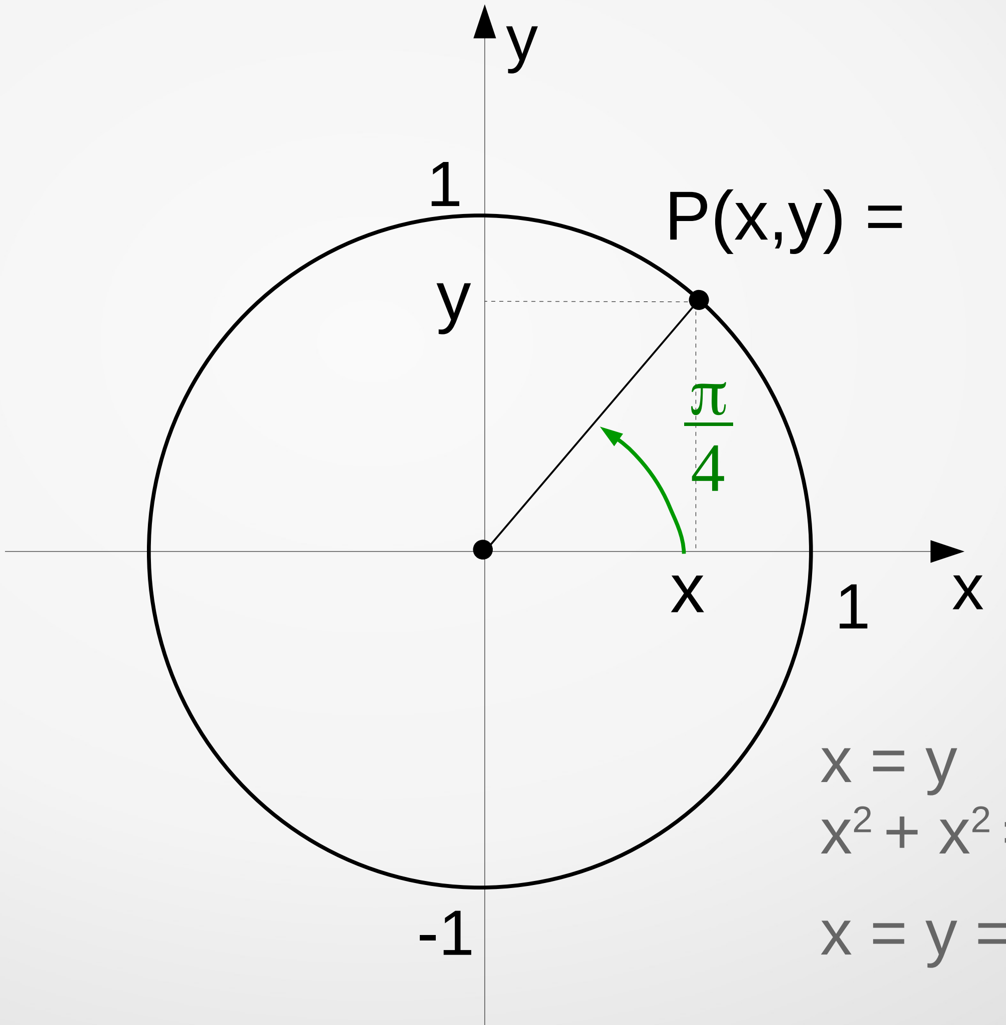
$x^2 + x^2 = 1$ …

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{4}\right)$



x = y

$x^2 + x^2 = 1 \ldots$

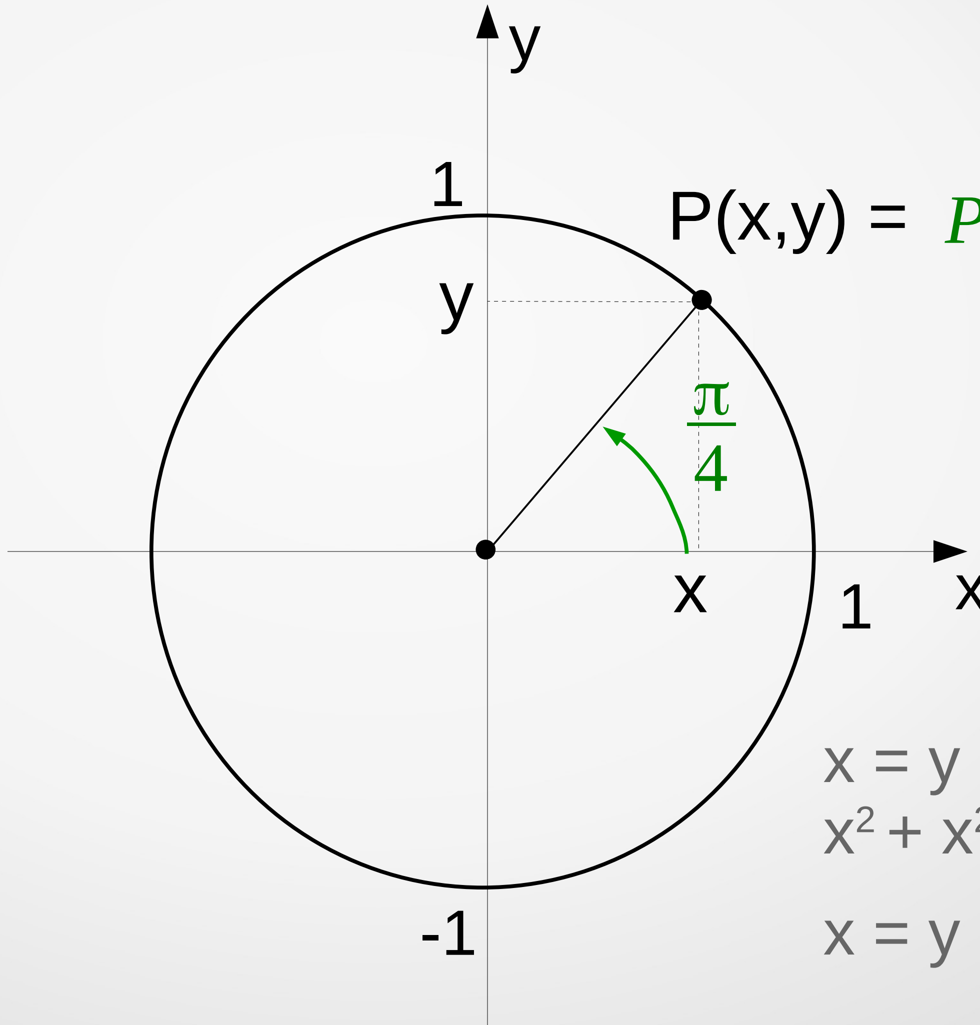
$x = y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{4}\right)$



P(x,y) = $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$x = y$

$x^2 + x^2 = 1 \ldots$

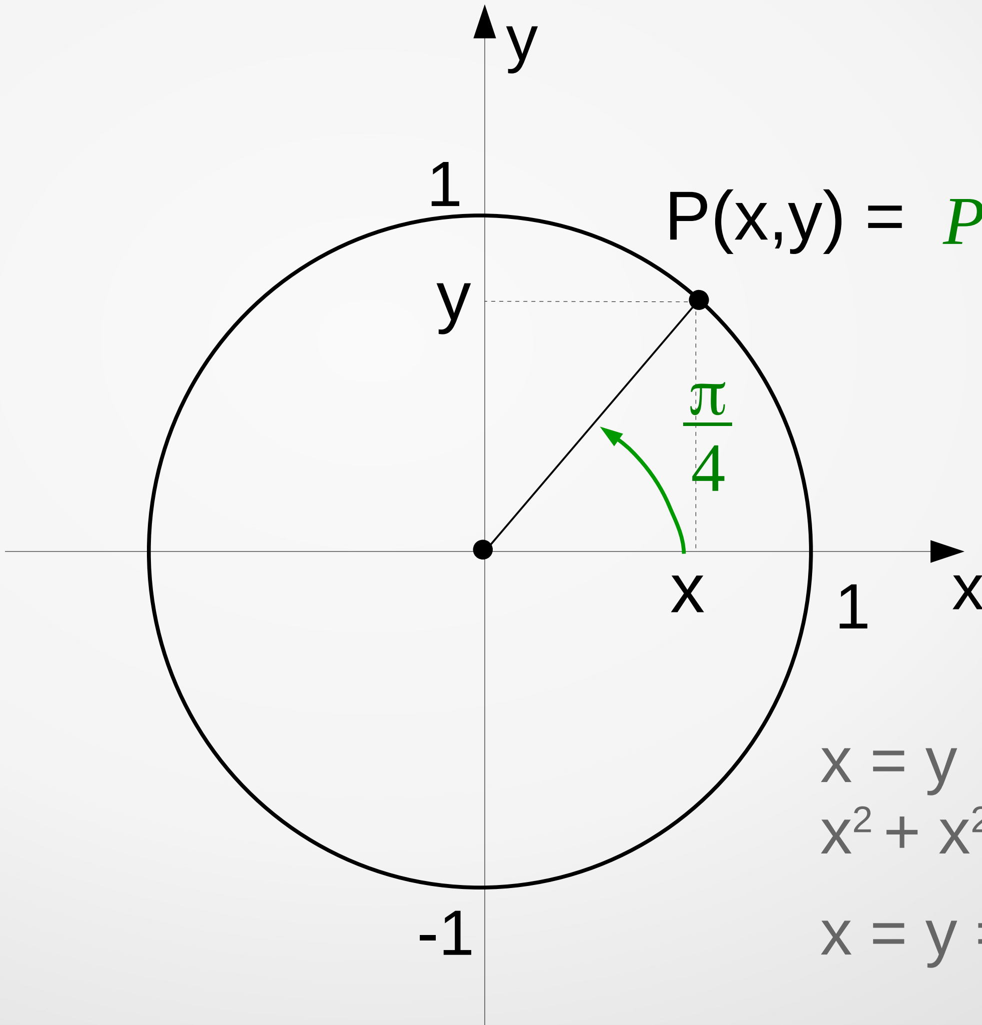
$x = y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$



P(x,y) = $P\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$x = y$

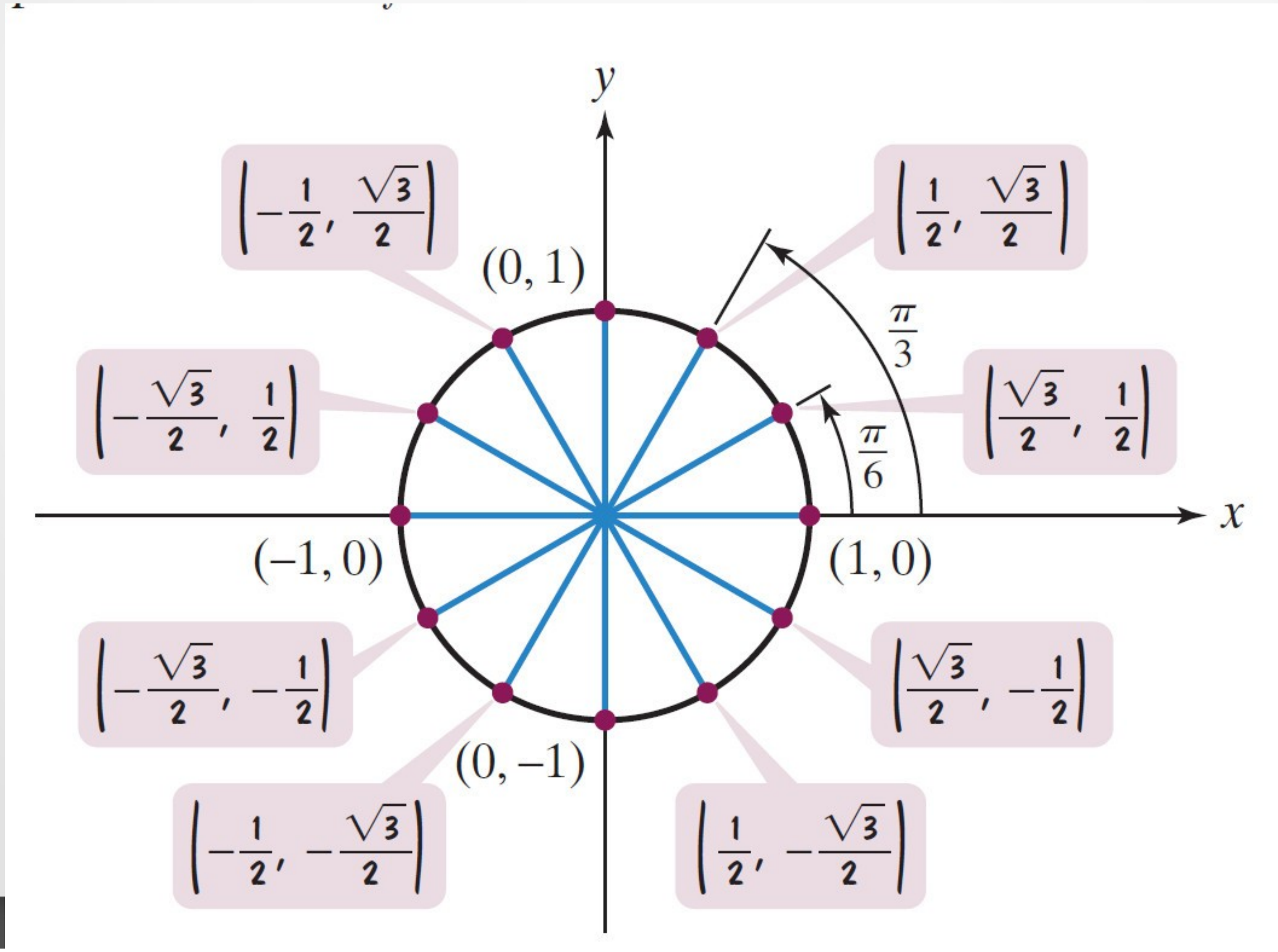
$x^2 + x^2 = 1$ …

$x = y = \frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Trigonometric Functions at some other angles

can be found using this picture of the unit circle



Trigonometric Functions: The Unit Circle

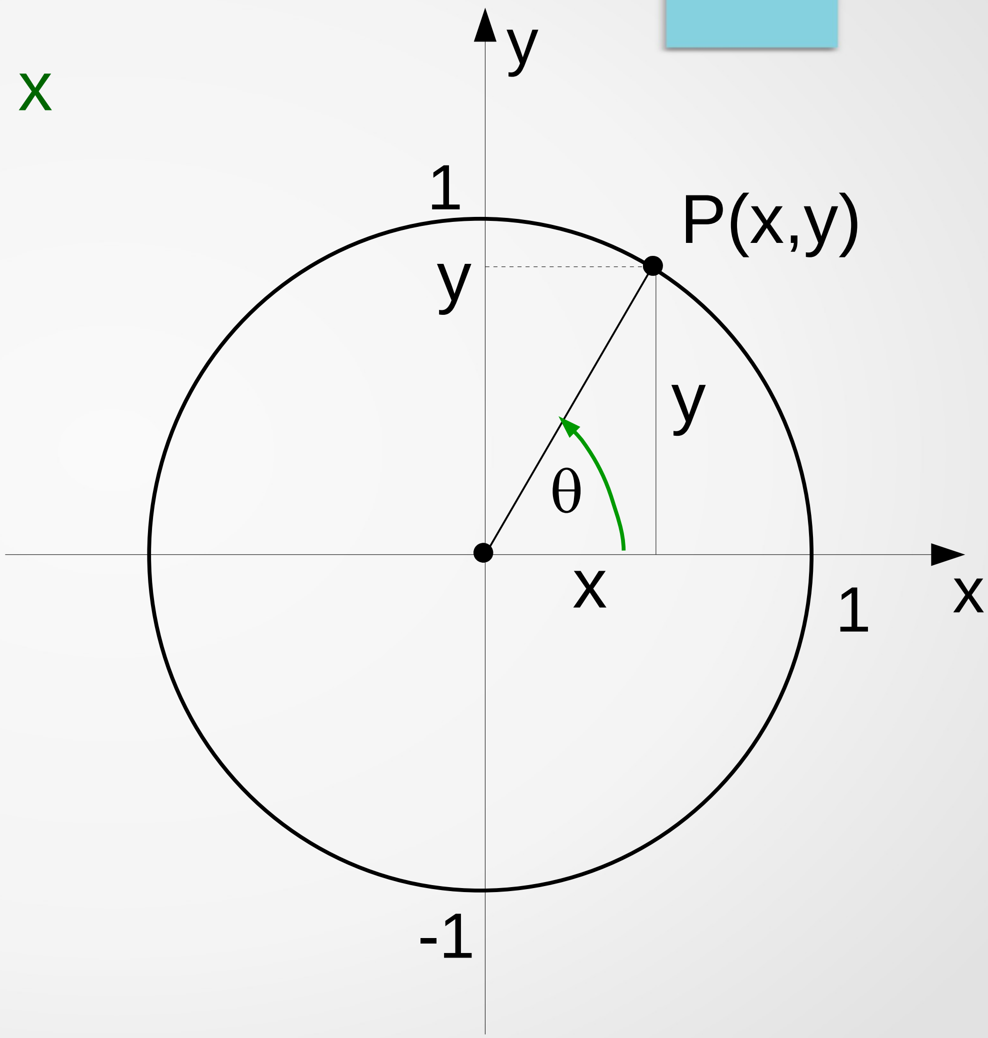
Trigonometric Functions at some other angles

θ	0°	$\frac{\pi}{6}$ (30°), $\frac{5\pi}{6}$ (150°)	$\frac{7\pi}{6}$ (210°), $\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°), $\frac{3\pi}{4}$ (135°)	$\frac{5\pi}{4}$ (225°), $\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°), $\frac{2\pi}{3}$ (120°)	$\frac{4\pi}{3}$ (240°), $\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Sinθ	0	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	1	0	-1
θ	0°	$\frac{\pi}{6}$ (30°), $\frac{11\pi}{6}$ (330°)	$\frac{5\pi}{6}$ (150°), $\frac{7\pi}{6}$ (210°)	$\frac{\pi}{4}$ (45°), $\frac{7\pi}{4}$ (315°)	$\frac{3\pi}{4}$ (135°), $\frac{5\pi}{4}$ (225°)	$\frac{\pi}{3}$ (60°), $\frac{5\pi}{3}$ (300°)	$\frac{2\pi}{3}$ (120°), $\frac{4\pi}{3}$ (240°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{-1}{2}$	0	-1	0
θ	0°	$\frac{\pi}{6}$ (30°), $\frac{7\pi}{6}$ (210°)	$\frac{5\pi}{6}$ (150°), $\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°), $\frac{5\pi}{4}$ (225°)	$\frac{3\pi}{4}$ (135°), $\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°), $\frac{4\pi}{3}$ (240°)	$\frac{2\pi}{3}$ (120°), $\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Tanθ	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	1	-1	$\sqrt{3}$	$-\sqrt{3}$	*	0	*
Cotθ	*	$\sqrt{3}$	$-\sqrt{3}$	1	-1	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	*	0
θ	0°	30°, 150°	210°, 330°	45°, 135°	225°, 315°	60°, 120°	240°, 300°	90°	180°	270°
Cosecθ	*	2	-2	$\sqrt{2}$	$-\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{-2}{\sqrt{3}}$	1	*	-1
θ	0°	30°, 330°	150°, 240°	45°, 315°	135°, 225°	60°, 300°	120°, 240°	90°	180°	270°
Secθ	1	$\frac{2}{\sqrt{3}}$	$\frac{-2}{\sqrt{3}}$	$\sqrt{2}$	$-\sqrt{2}$	2	-2	*	-1	*

available on our web-page in the Notices

Pythagorean identity

sin θ = y   and   cos θ = x

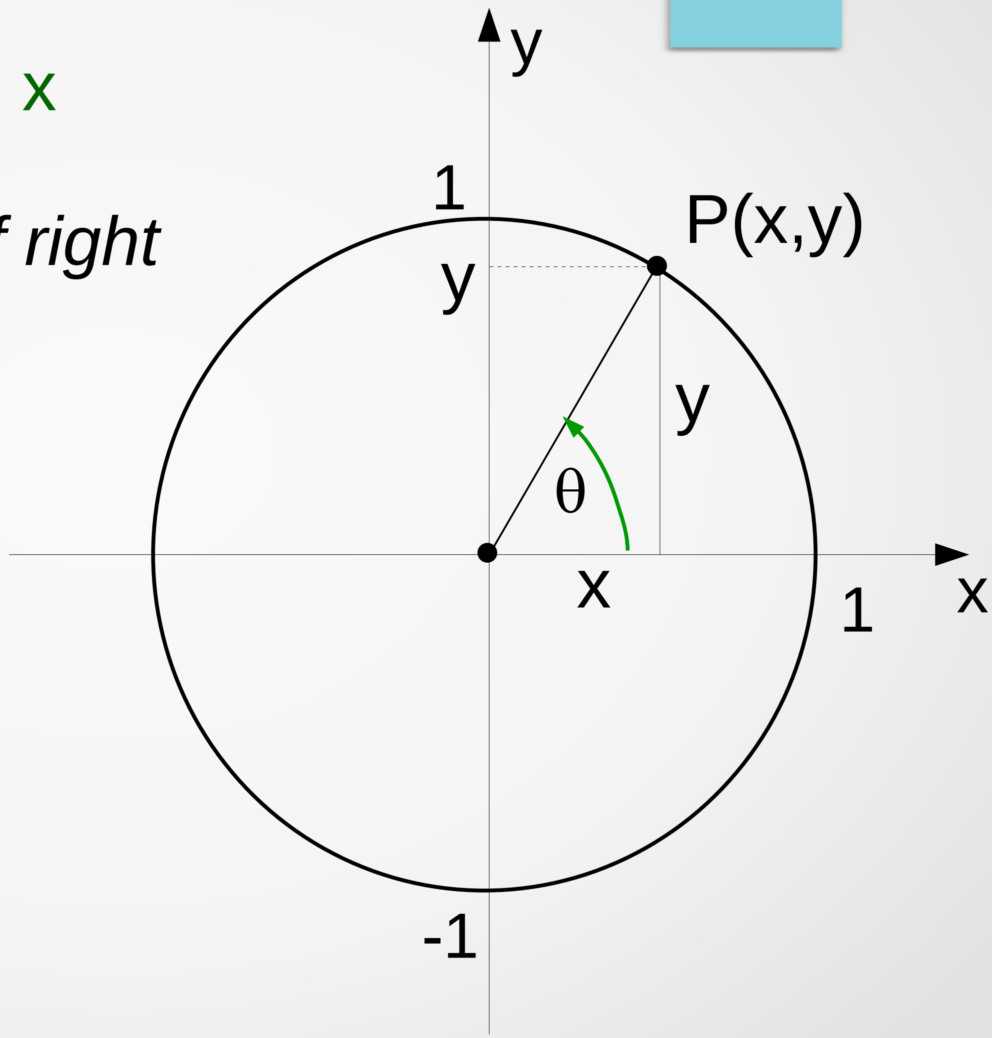


Pythagorean identity

$\sin\theta = y$ and $\cos\theta = x$

Pythagorean property of right triangles:

$a^2 + b^2 = c^2$

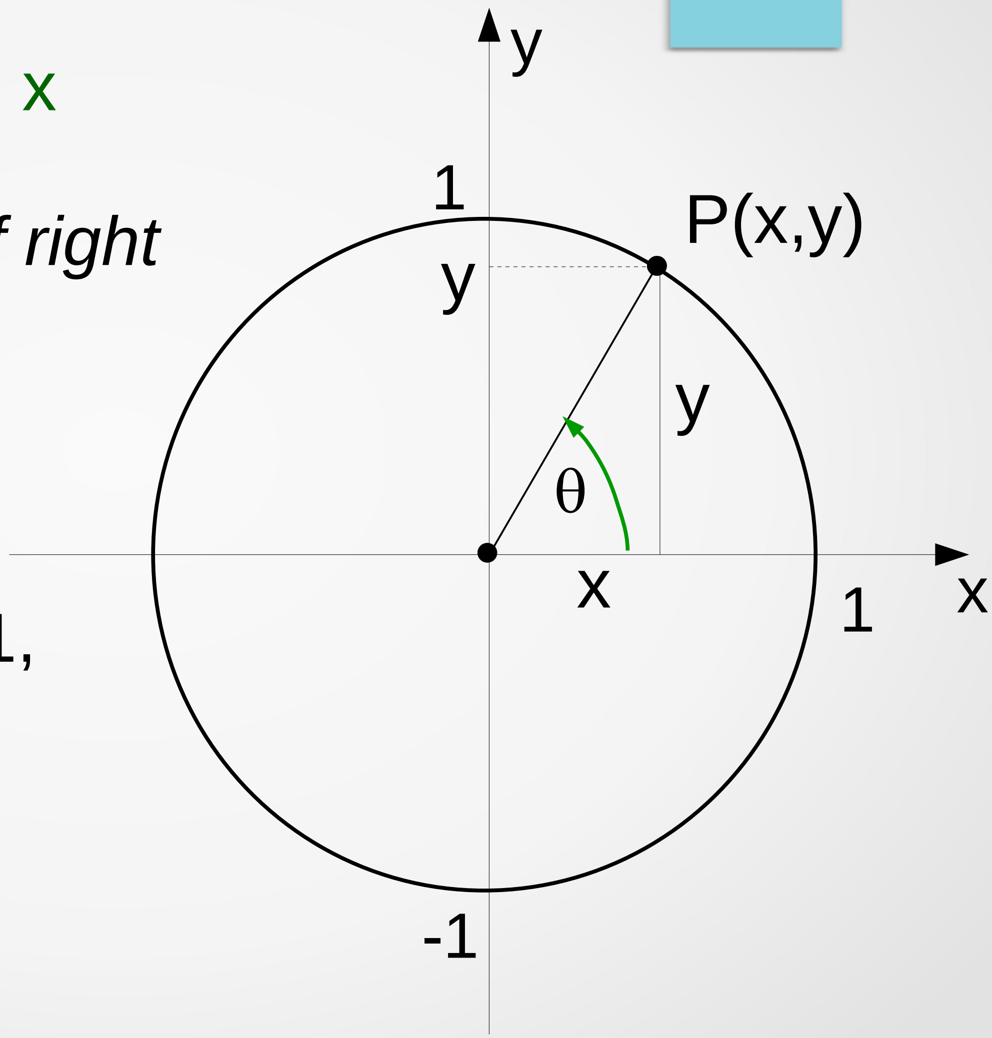


Pythagorean identity

sin θ = y   and   cos θ = x

Pythagorean property of right triangles:
$a^2 + b^2 = c^2$

If a = x = cos θ and
  b = y = sin θ, and c = 1,
then



Pythagorean identity

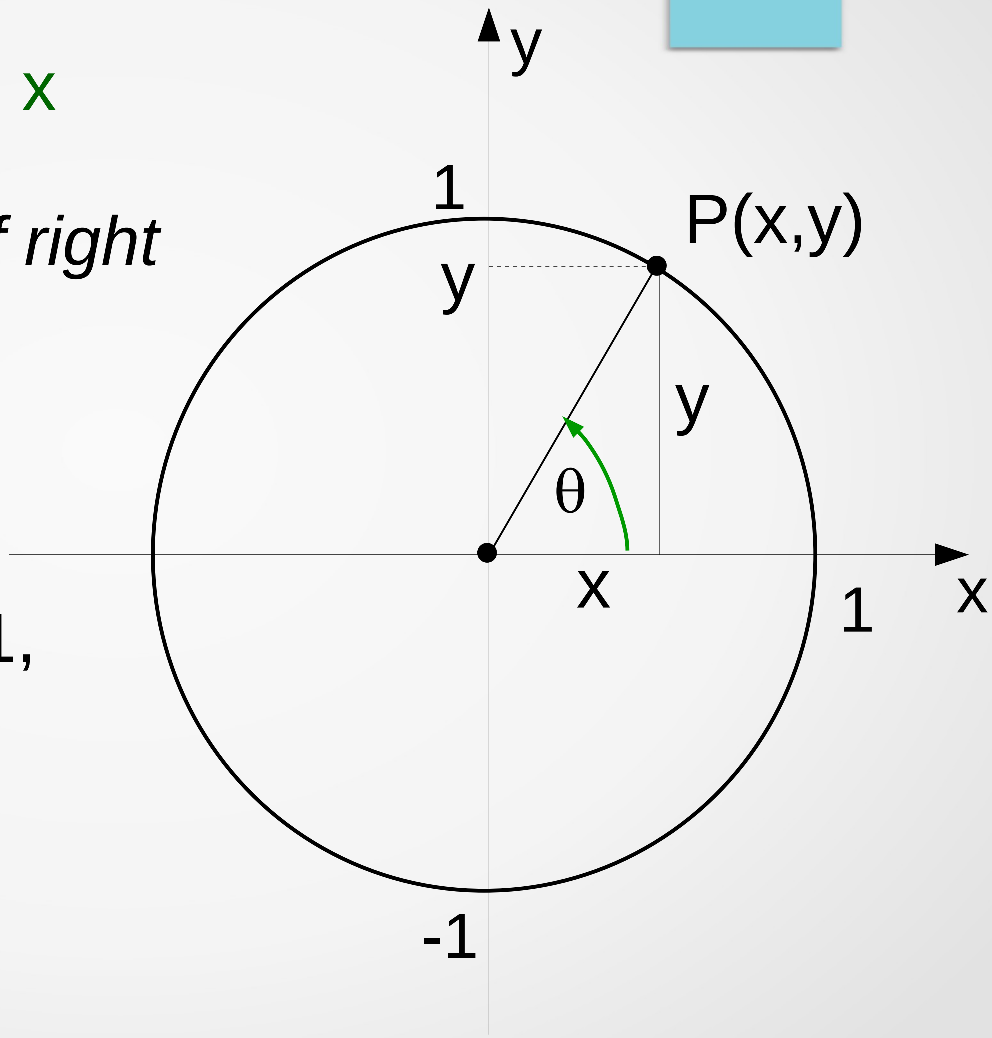
$\sin\theta = y$ and $\cos\theta = x$

Pythagorean property of right triangles:

$a^2 + b^2 = c^2$

If $a = x = \cos\theta$ and
$b = y = \sin\theta$, and $c = 1$,
then

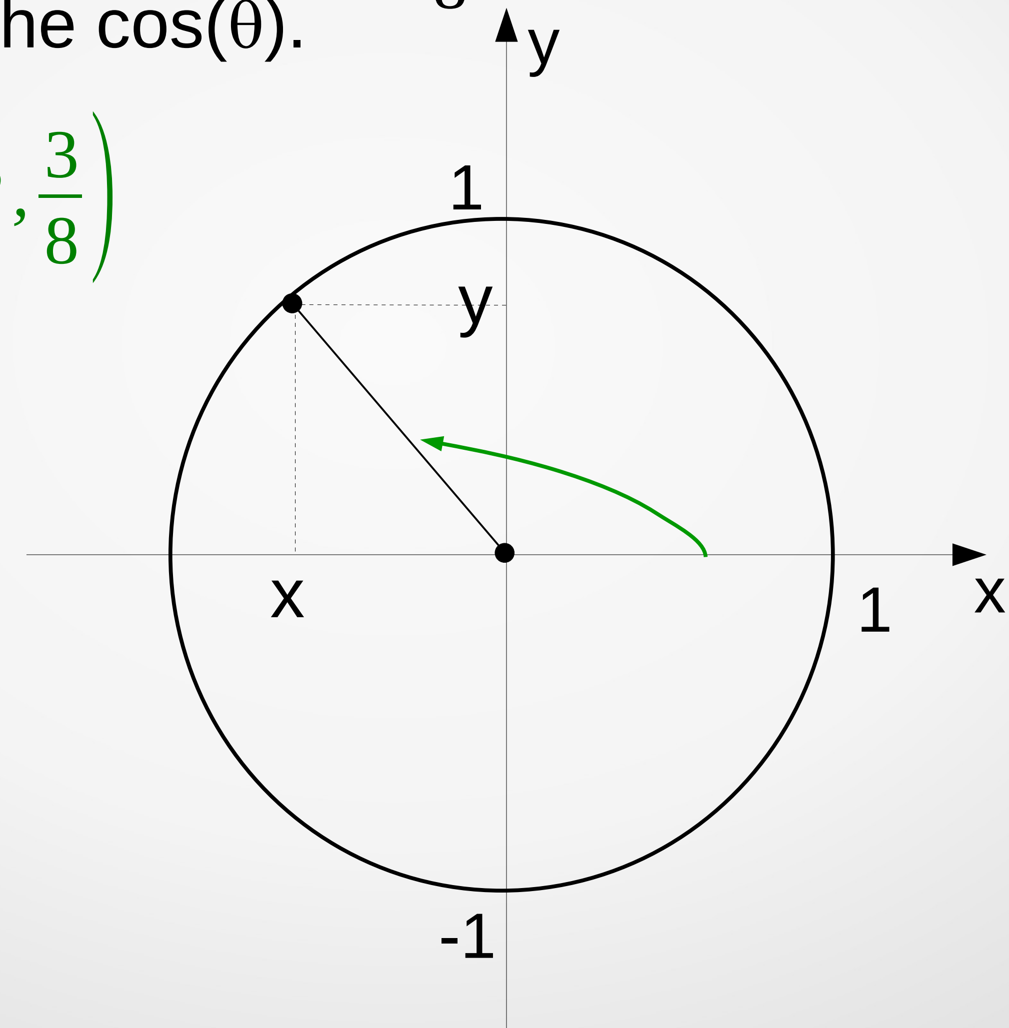
$\cos^2\theta + \sin^2\theta = 1$



Pythagorean identity

Example: Given that $\sin(\theta) = \frac{3}{8}$ and θ is in the second quadrant, find the $\cos(\theta)$.

P(x,y) = $P\left(?, \frac{3}{8}\right)$



$\cos^2\theta + \sin^2\theta = 1$

In-class work

Exercise: Given that $\cos(\theta) = \frac{1}{7}$ and θ is in the fourth quadrant, find the $\sin(\theta)$.

P(x,y) = $P\left(?, \frac{3}{8}\right)$

$\cos^2\theta + \sin^2\theta = 1$

Reference angles : see the textbook

Learning objectives

In this section we:

- Found function values for the sine and cosine of 30° or ($\frac{\pi}{6}$), 45° or ($\frac{\pi}{4}$) and 60°or ($\frac{\pi}{3}$).
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