Exponential and Logarithmic equations

Today we will continue using *logarithmic properties* along with the *exponential properties* you already know to solve *exponential* and *logarithmic equations*.

Exponential and Logarithmic equations

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$$b^M = b^N$$

- use logarithms:

$$\log(b^M) = \log(b^N)$$

$$M \cdot \cancel{\log} b = N \cdot \cancel{\log} b$$

$M = N$ then finish solving the equation

Exponential and Logarithmic equations

Examples: Let's solve

1) $3^{2x+1} = 27$

2) $5^{2-x} = 1 / 125$

3) $9^x = \dfrac{1}{\sqrt[3]{3}}$

In-class practice

Exercise 1: Solve

1) $2^{4x-2}=64$

2) $2(3)^{x+4}=\frac{2}{81}$

3) $16^{x-1}=\sqrt{2}$

Exponential and Logarithmic equations

What if we cannot "express each side as a power of the same base" ?

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Example: $10^x = 8.07$

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- isolate the exponential expression (already done)
- take common (log) or natural (ln) logarithm of both sides of the equation

$$\log(10^x) = \log 8.07$$

- simplify using properties of logarithms

$$x \log 10 = \log 8.07 \qquad \rightarrow \qquad x = \log 8.07$$

- solve for the variable (already done)

$$x = \log 8.07 \approx 0.91$$

Exponential and Logarithmic equations

Examples: Let's solve

1) $4e^{7x} = 10{,}273$

2) $e^{4x-5} - 7 = 11{,}243$

3) $e^{4x} - 3e^{2x} - 18 = 0$

4) $7 \cdot 5^{2x-3} = 315$

In-class practice

Exercise 2: Solve

1) $4\,(7)^{2x} = 204$

2) $3^{x+4} = 7^{2x-1}$

3) $e^{2x} - e^{x} - 6 = 0$

Exponential and Logarithmic equations

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Examples: $\log_3(x-1)=5$ $\quad$ $\log(x+2)-\log x=\log\left(\frac{1}{x}\right)$

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1) re-write in exponential form

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1) re-write in exponential form

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2) express each side as a single logarithm with coefficient 1 and with the same bases

$\log_b M = \log_b N$

then $M = N$, solve it and check solutions!

Exponential and Logarithmic equations

Examples: Let's solve

1) $\log_5 x = 3$

2) $7 + 3\ln(x+1) = 6$

3) $2\log_3(x+4) = \log_3 9 + 2$

$$\log_b(MN) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^p) = p\log_b M,\ p \in R$$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b b^x = x$

$$b^{\log_b x} = x$$

Exponential and Logarithmic equations

4) $\log\sqrt{x+4}=1$

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5) $\log_2 (x-3) + \log_2 x – \log_2 (x+2) = 2$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b b^x = x$

$$b^{\log_b x}=x$$

6) $\log_2 (x-1) – \log_2 (x+3) = \log_2 (1 / x)$

In-class practice: solve the given equations

(a) $3+4\ln(2x)=15$

(b) $\log_5 x + \log_5 (4x-1) = 1$

(c) log (x+7) – log 3 = log (7x+1)

(d) $\ln(x+4)-\ln(x+1)=\ln x$

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