

Trigonometric Equations

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We did see some equations that are identities, $\sin^2 x + \cos^2 x = 1$
- they are *true for every value of the variable* for which the expression is defined.

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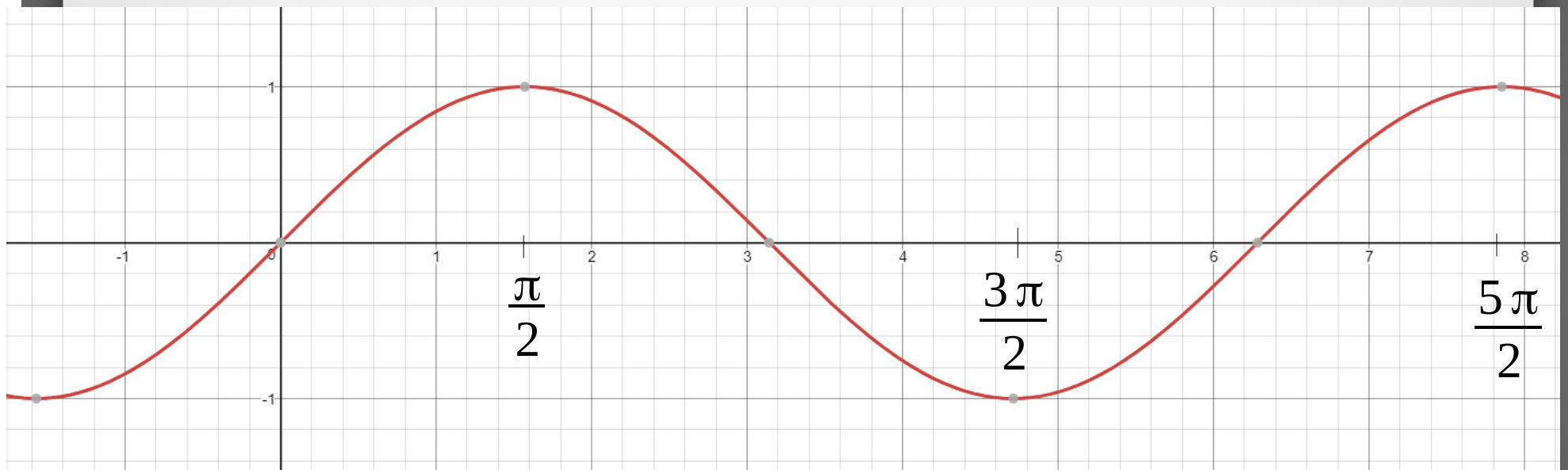
However, there are some equations that are *true for only some values of the variable*.

Trigonometric Equations

Example: $\sin x = 1$

Trigonometric Equations

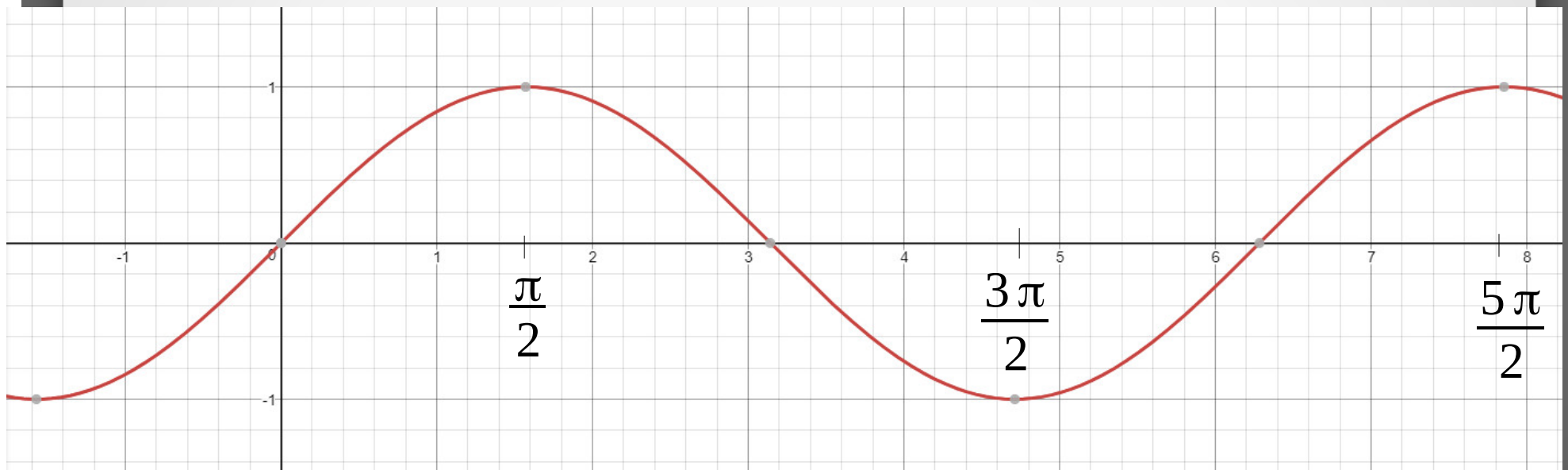
Example: $\sin x = 1$



$\sin x = 1$, when $x = \frac{\pi}{2}$

Trigonometric Equations

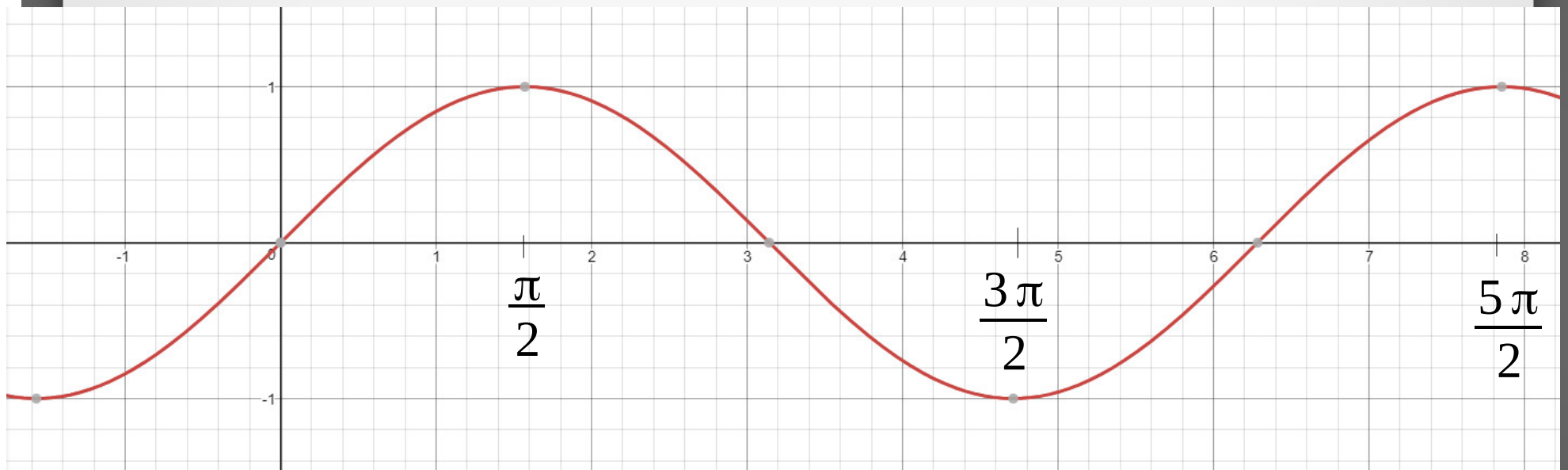
Example: $\sin x = 1$



$$\sin x = 1, \text{ when } x = \frac{\pi}{2}, \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$$

Trigonometric Equations

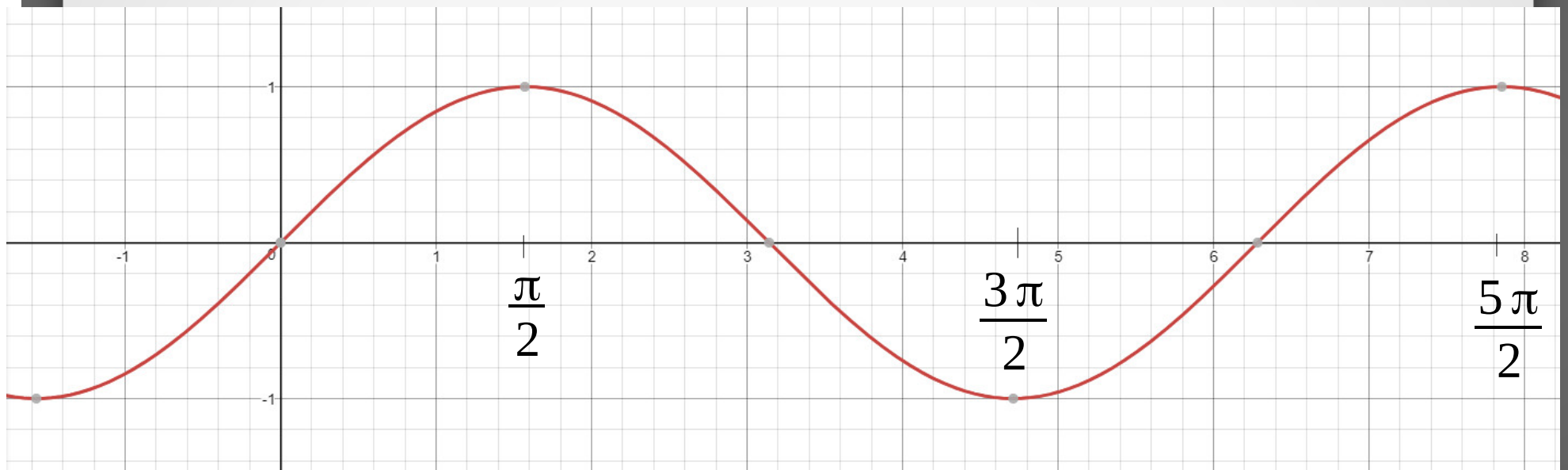
Example: $\sin x = 1$



$$\sin x = 1, \text{ when } x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}, \dots$$

Trigonometric Equations

Example: $\sin x = 1$



$\sin x = 1$, when $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}, \dots$

or $x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$

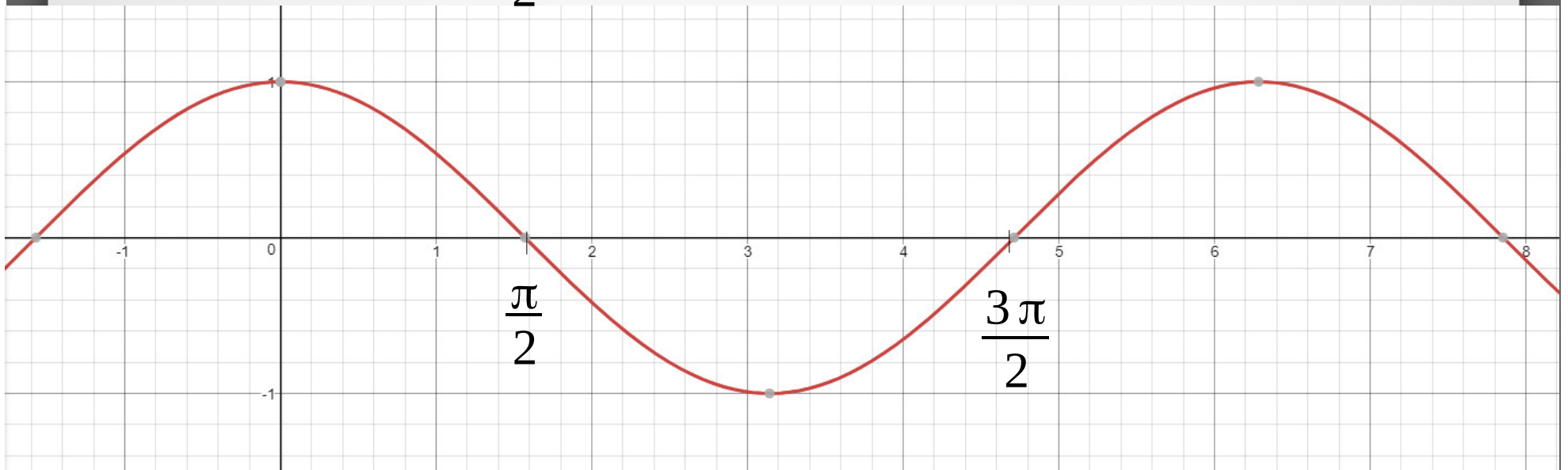
period

Trigonometric Equations

Example: $\cos x = \frac{\sqrt{3}}{2}$

Trigonometric Equations

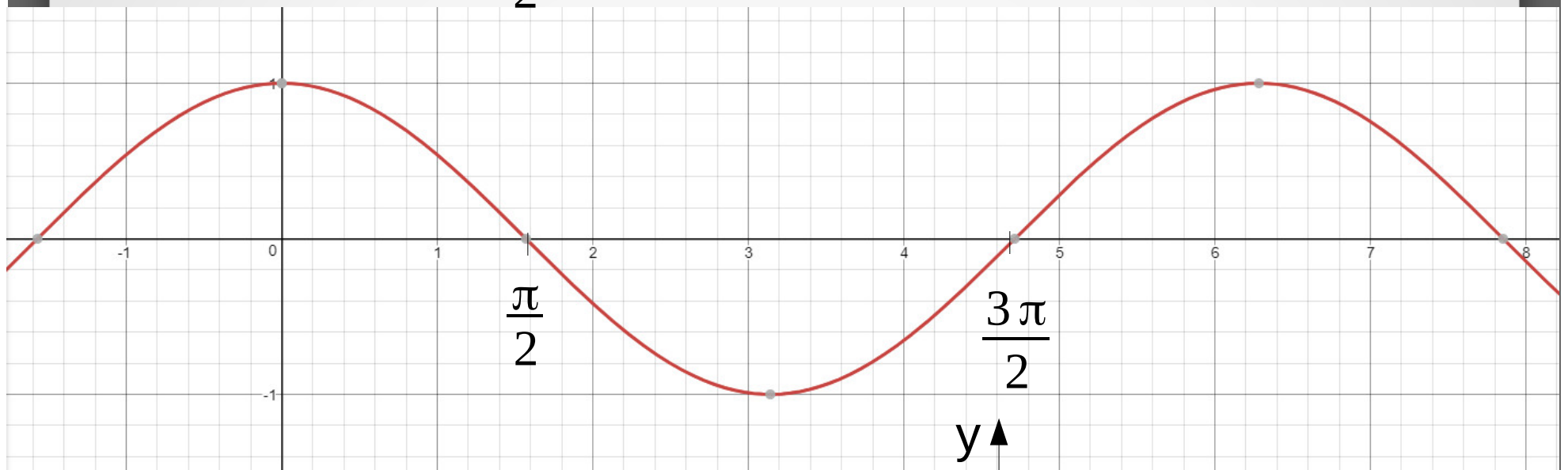
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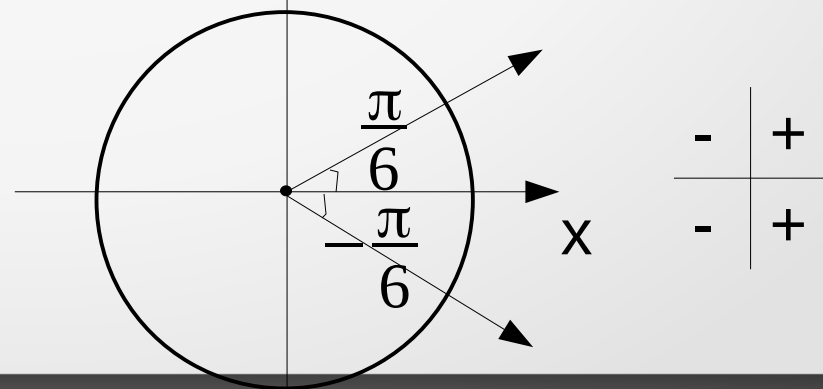
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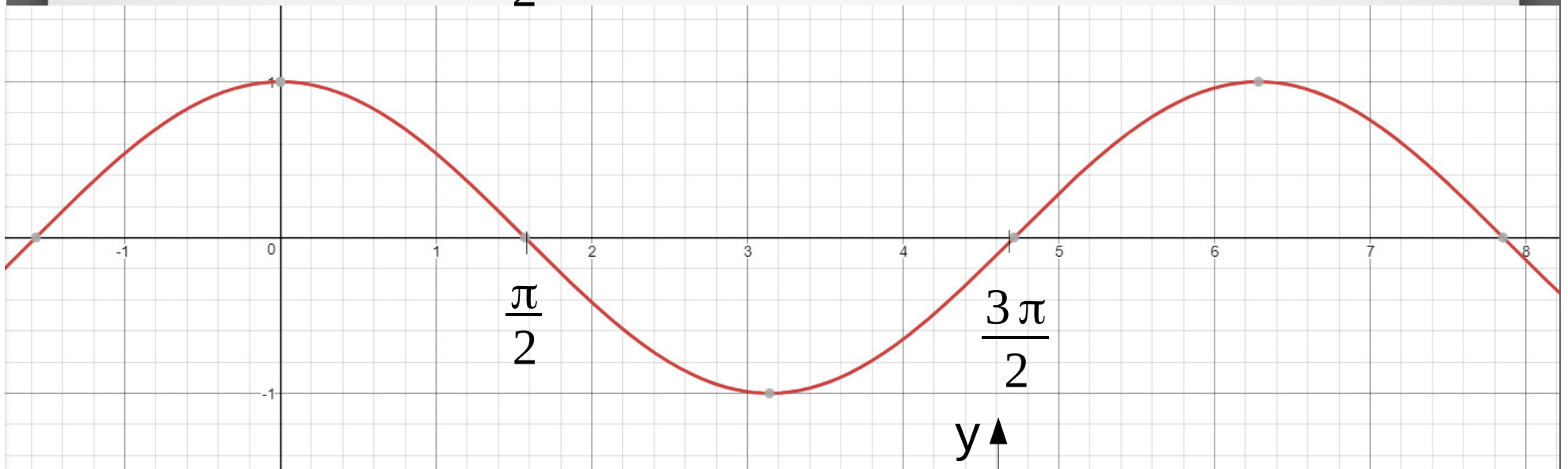


$$x = \pm \frac{\pi}{6}, \dots$$



Trigonometric Equations

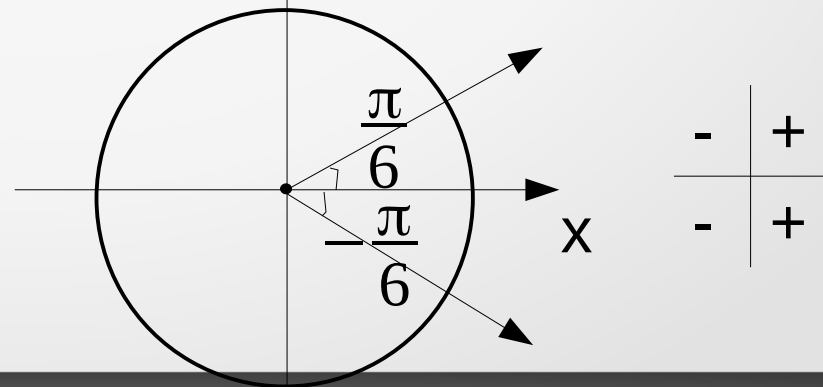
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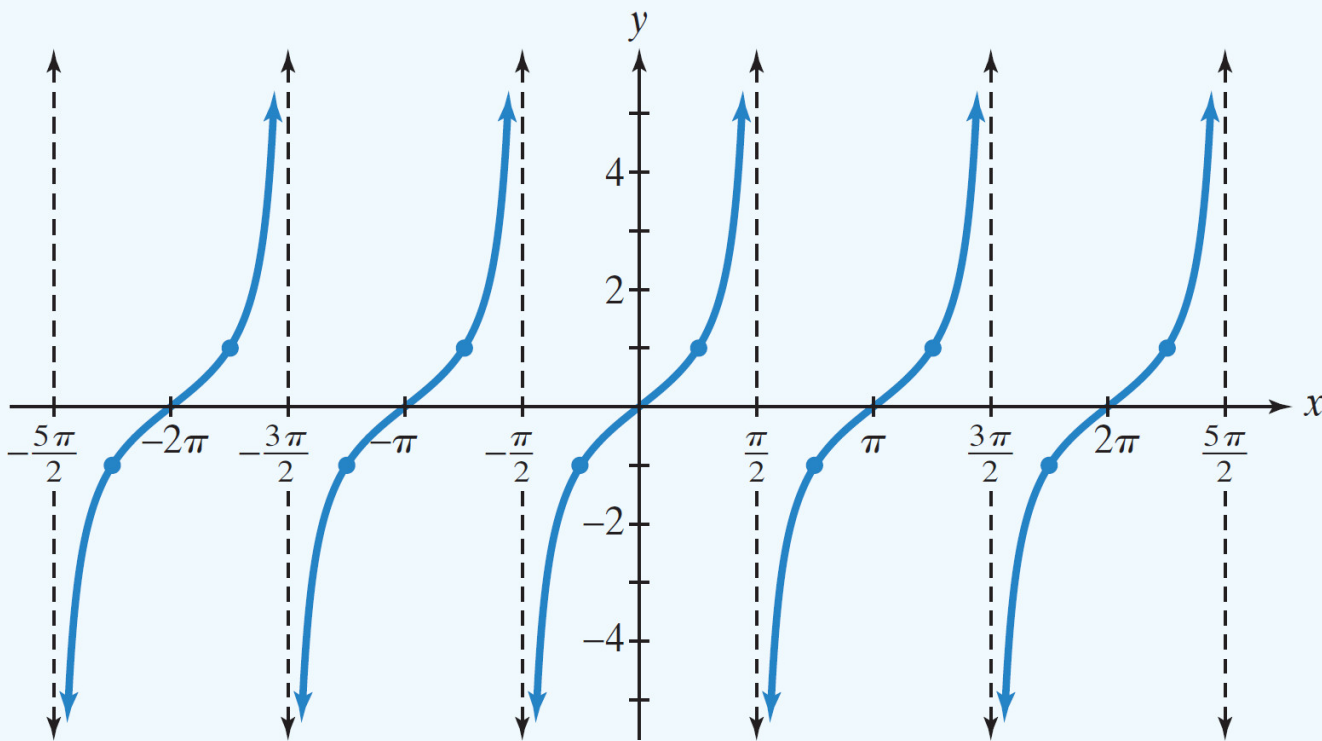


Trigonometric Equations

Example: $\tan x = \sqrt{3}$

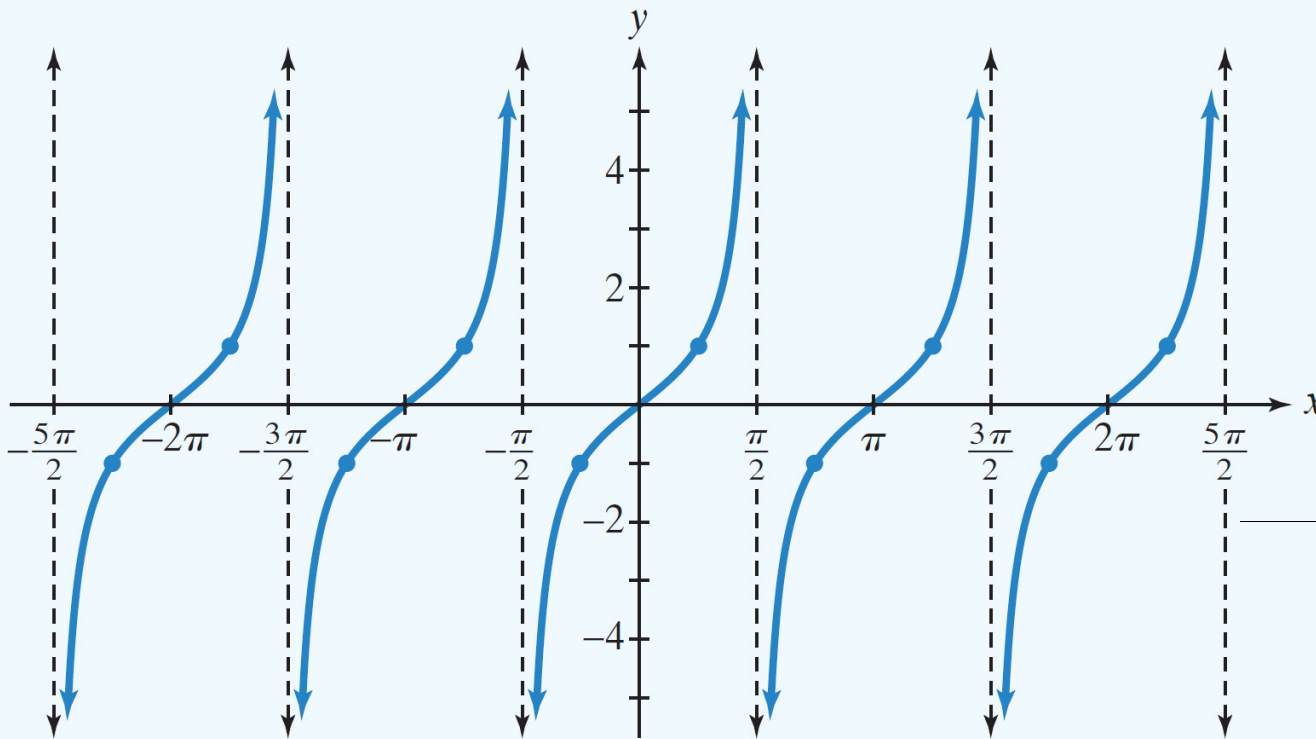
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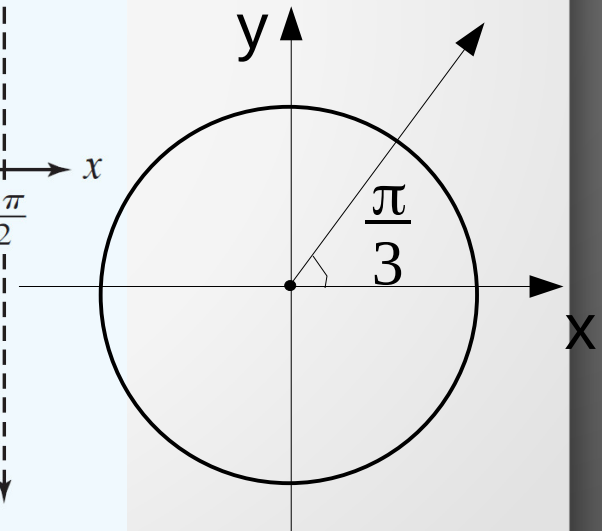


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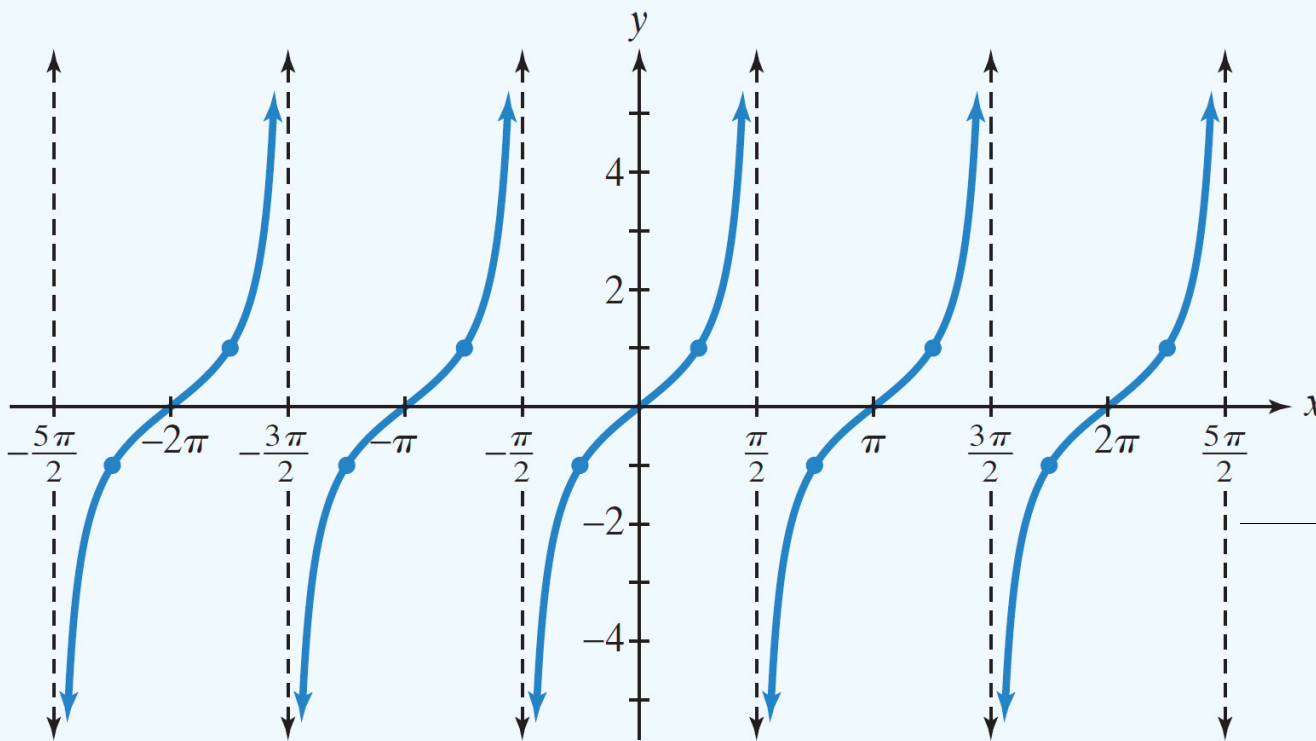
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+	-



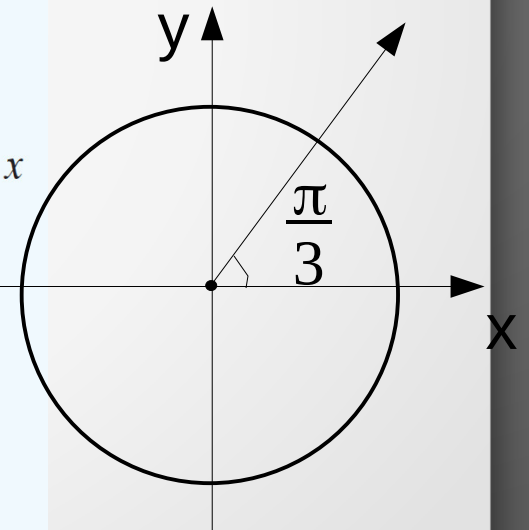
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-	+
+	-



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← period

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Example: $5 \sin \theta + 1 = 3 \sin \theta$

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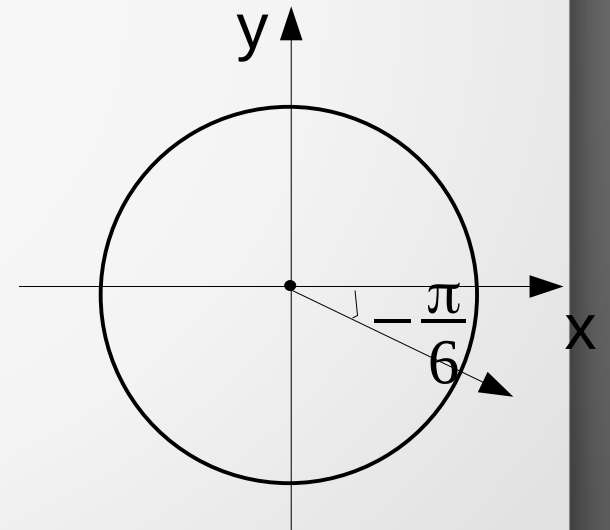
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+	+
-	-



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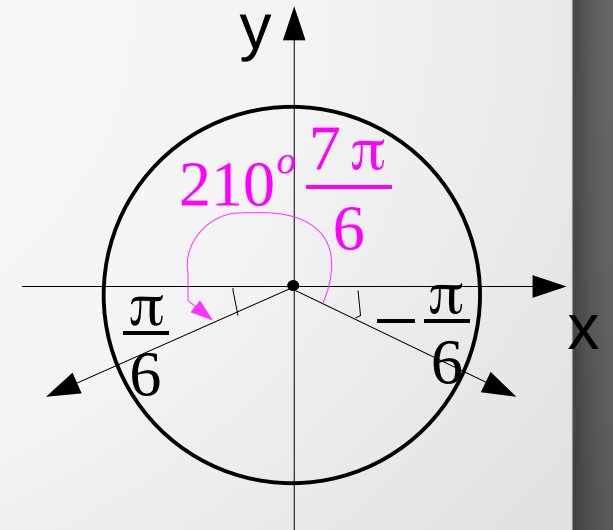
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+	+
-	-



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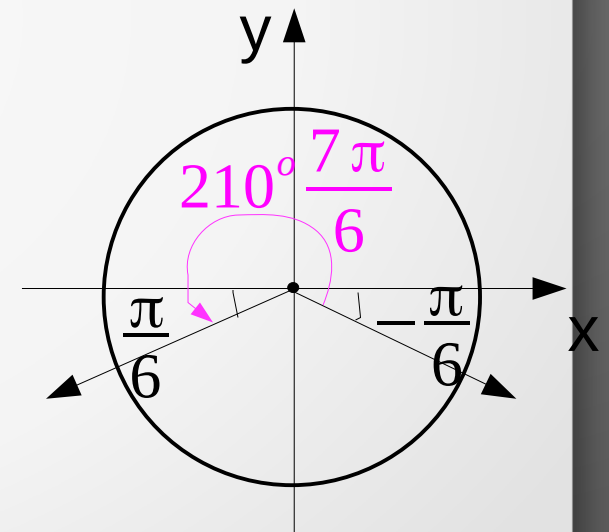
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$$\theta = \frac{11\pi}{6} + 2\pi n \quad \text{or} \quad \theta = \frac{7\pi}{6} + 2\pi n, n \in \mathbb{Z}$$

+	+
-	-



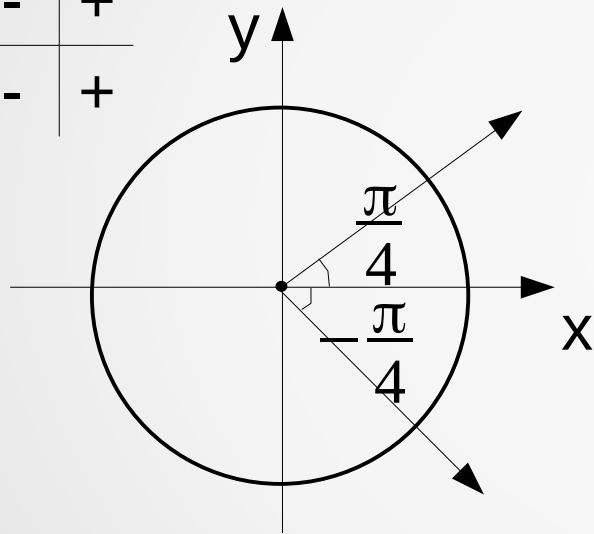
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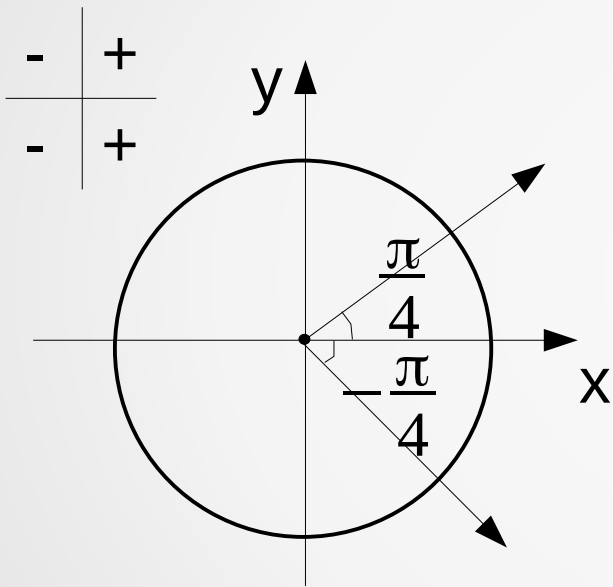
-	+
-	+



$$2x = \frac{\pi}{4} + 2\pi n \quad \text{or} \quad 2x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$$

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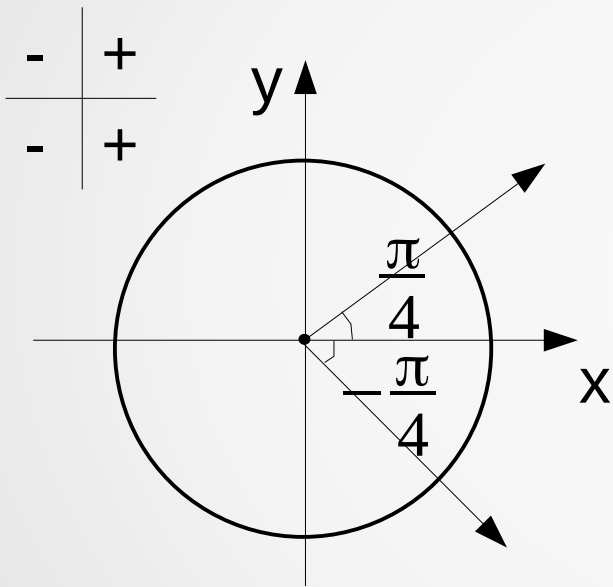


$$2x = \frac{\pi}{4} + 2\pi n \quad \text{or} \quad 2x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$$

$$x = \frac{\pi}{8} + \pi n \quad \text{or} \quad x = \frac{7\pi}{8} + \pi n, n \in \mathbb{Z}$$

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$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

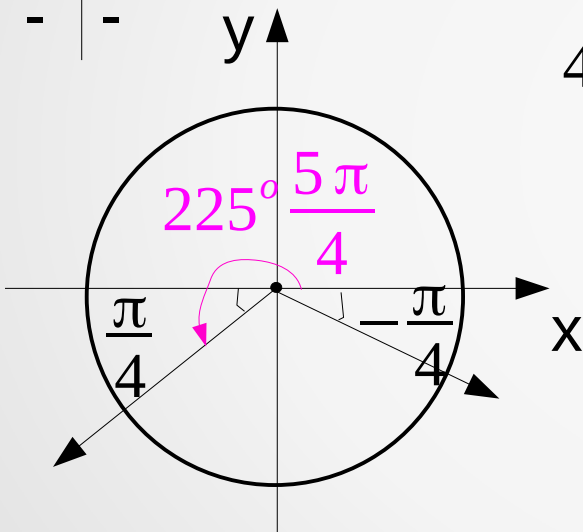
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+	+
-	-

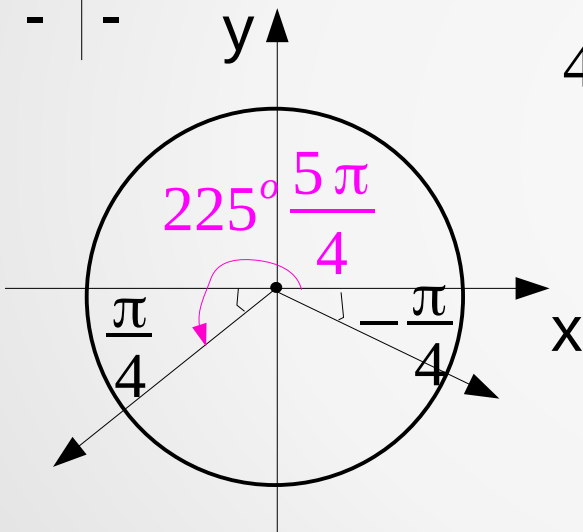


$$4x = \frac{5\pi}{4} + 2\pi n \quad \text{or} \quad 4x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$$

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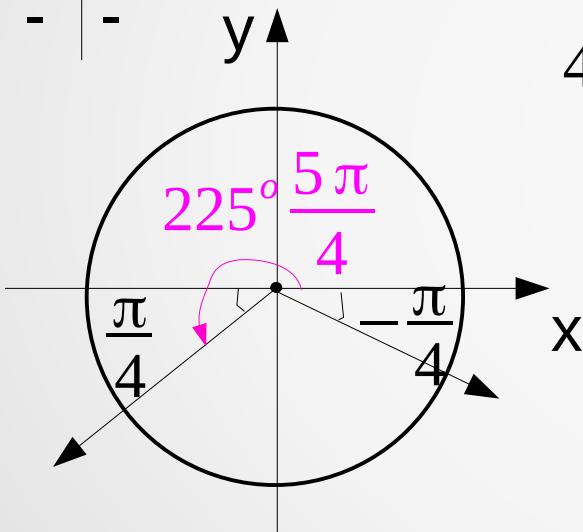
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$$x = \frac{5\pi}{16} + \frac{\pi n}{2} \quad \text{or} \quad x = \frac{7\pi}{16} + \frac{\pi n}{2}, n \in \mathbb{Z}$$

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Trigonometric Equations

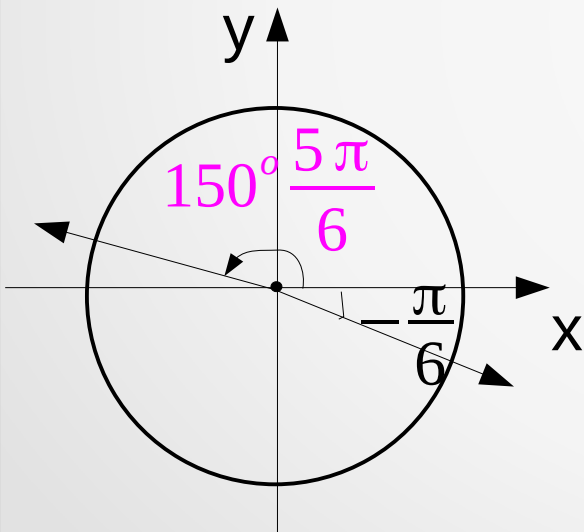
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-	+
+	-

$$\frac{3\theta}{2} = \frac{5\pi}{6} + \pi n \quad \text{or} \quad \frac{3\theta}{2} = \frac{11\pi}{6} + \pi n, n \in \mathbb{Z}$$



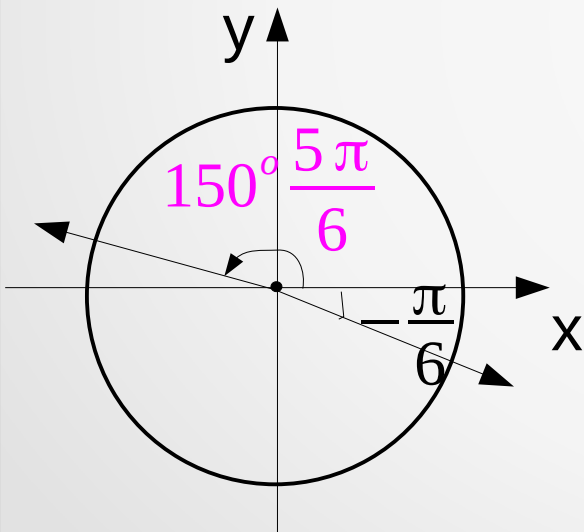
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-	+
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Trigonometric Equations

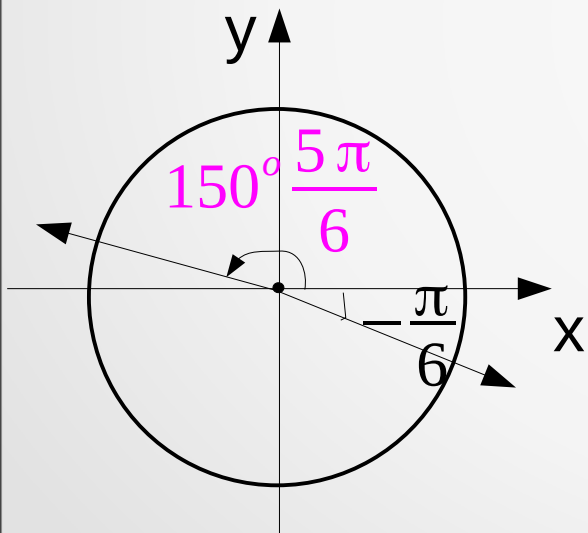
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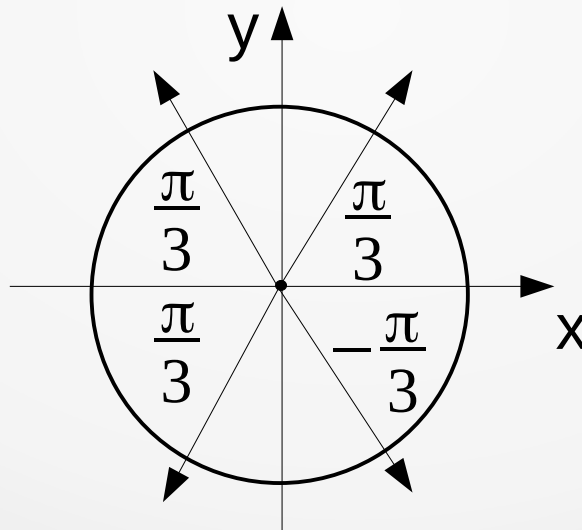
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$$\sin^2 x = \frac{3}{4}$$

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+	+
-	-



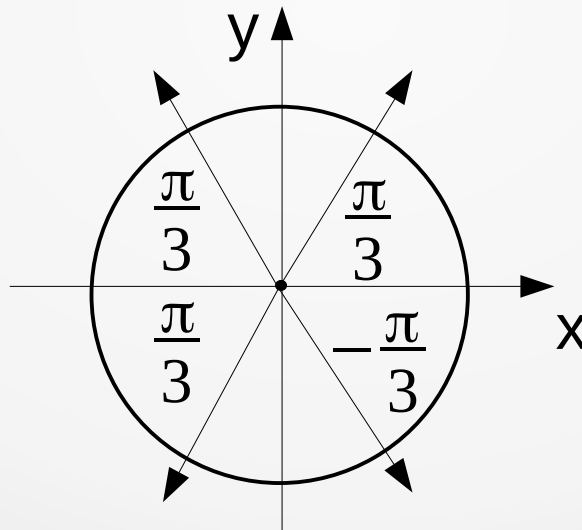
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$$\sin x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

+	+
-	-



$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

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therefore, we need to
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~~$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$~~

$$\tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

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$$\cos x = 0$$

$$1 - 2 \sin x = 0$$

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$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

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$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

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Homework assignment

1) zyBooks: *review* Section 7.5

or

Textbook: *review* Section 5.5