

Verifying Trigonometric Identities

Fundamental Identities

$$\sin t = \frac{1}{\csc t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

even/odd identities:

$$\sin(-t) = -\sin t$$

$$\cos(-t) = \cos t$$

$$\tan(-t) = -\tan t$$

$$\csc(-t) = -\csc t$$

$$\sec(-t) = \sec t$$

$$\cot(-t) = -\cot t$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\cos x \csc x = \cot x$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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$$\sin t = \frac{1}{\csc t}$$

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Example: verify that $\cos x \csc x \stackrel{?}{=} \cot x$

$$\cos x \frac{1}{\sin x} \stackrel{?}{=} \cot x$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

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Example: verify that $\cos x \csc x \stackrel{?}{=} \cot x$

$$\cos x \frac{1}{\sin x} \stackrel{?}{=} \cot x$$

$$\frac{\cos x}{\sin x} \stackrel{?}{=} \cot x$$

rewriting

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

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Example: verify that $\cos x \csc x \stackrel{?}{=} \cot x$

$$\cos x \frac{1}{\sin x} \stackrel{?}{=} \cot x$$

$$\frac{\cos x}{\sin x} \stackrel{?}{=} \cot x$$

$$\cot x = \cot x$$

rewriting

The identity is verified

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

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Fundamental Identities

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Example: verify that $\cos^2 x - \sin^2 x \stackrel{?}{=} 2\cos^2 x - 1$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

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Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\cos^2 x - \sin^2 x \stackrel{?}{=} 2\cos^2 x - 1$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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Example: verify that $\cos^2 x - \sin^2 x \stackrel{?}{=} 2 \cos^2 x - 1$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x - (1 - \cos^2 x) \stackrel{?}{=} 2 \cos^2 x - 1$$

$$2 \cos^2 x - 1 = 2 \cos^2 x - 1$$

The identity is verified

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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Verifying Trigonometric Identities

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We can use fundamental identities to verify other identities.

Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

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We can use fundamental identities to verify other identities.

Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

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Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

LCD for the left side is $\cos x \sin x$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

LCD for the left side is $\cos x \sin x$

$$\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

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$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

LCD for the left side is $\cos x \sin x$

$$\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

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$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

LCD for the left side is $\cos x \sin x$

$$\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x} \quad \frac{1}{\cos x} \frac{1}{\sin x} = \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\sin^2 t + \cos^2 t = 1$$

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Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

LCD for the left side is $\cos x \sin x$

$$\frac{\sin x \cdot \sin x}{\cos x \cdot \sin x} + \frac{\cos x \cdot \cos x}{\sin x \cdot \cos x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \stackrel{?}{=} \frac{1}{\cos x} \frac{1}{\sin x}$$

$$\sin^2 t + \cos^2 t = 1$$

$$\frac{1}{\cos x} \frac{1}{\sin x} = \frac{1}{\cos x} \frac{1}{\sin x} \quad \checkmark$$

The identity is verified

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

Verifying Trigonometric Identities

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We can use fundamental identities to verify other identities.

Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

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We can use fundamental identities to verify other identities.

Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cos t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cos t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

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Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cot t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + \cos^2 t + \sin^2 t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

open parentheses

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

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Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cot t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + \cos^2 t + \sin^2 t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + 1}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

open parentheses

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

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Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cot t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + \cos^2 t + \sin^2 t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cancel{\cot t + 1}}{\sin t(\cancel{1 + \cos t})} \stackrel{?}{=} \csc t$$

assuming
 $\cos t \neq -1$

$$\frac{1}{\sin t} \stackrel{?}{=} \csc t$$

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

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Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cot t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + \cos^2 t + \sin^2 t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + 1}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{1}{\sin t} \stackrel{?}{=} \csc t$$

$$\csc t = \csc t$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\sin^2 t + \cos^2 t = 1$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\cot t + \frac{\sin t}{1 + \cos t} = \csc t$

$$\frac{\cot t}{\sin t} + \frac{\sin t}{1 + \cos t} \stackrel{?}{=} \csc t$$

$$\frac{\cot t(1 + \cos t) + \sin t \cdot \sin t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + \cos^2 t + \sin^2 t}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{\cot t + 1}{\sin t(1 + \cos t)} \stackrel{?}{=} \csc t$$

$$\frac{1}{\sin t} \stackrel{?}{=} \csc t$$

$$\csc t = \csc t$$



The identity is verified

$$\sin^2 t + \cos^2 t = 1$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

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We can use fundamental identities to verify other identities.

Example: verify that $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1}$$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

Example: verify that

$$\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1}$$

$$\rightarrow \frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} - \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}$$

Verifying Trigonometric Identities

Fundamental Identities

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

We can use fundamental identities to verify other identities.

Example: verify that

$$\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1}$$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \rightarrow \frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} - \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}$$

$$\frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} - \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}$$

$$\frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}$$

$$\frac{\frac{\cos x - \sin x}{\sin x \cos x}}{\frac{\cos x + \sin x}{\cos x \sin x}} \stackrel{?}{=} \frac{\frac{\cos x - \sin x}{\sin x}}{\frac{\cos x + \sin x}{\sin x}}$$

$$\frac{\frac{\cos x - \sin x}{\sin x \cos x}}{\frac{\cos x + \sin x}{\cos x \sin x}} \rightarrow \frac{\frac{\cos x - \sin x}{\sin x}}{\frac{\cos x + \sin x}{\sin x}}$$

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$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

We can use fundamental identities to verify other identities.

Example: verify that $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1} \rightarrow \frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} - \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}$$

$$\frac{\frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\sin x} \cancel{\cos x}}}{\frac{\cancel{\cos x} + \cancel{\sin x}}{\cancel{\cos x} \cancel{\sin x}}} \stackrel{?}{=} \frac{\frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\sin x}}}{\frac{\cancel{\cos x} + \cancel{\sin x}}{\cancel{\sin x}}}$$

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$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

We can use fundamental identities to verify other identities.

Example: verify that $\frac{\csc x - \sec x}{\csc x + \sec x} = \frac{\cot x - 1}{\cot x + 1}$

$$\frac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{1}{\sin x} + \frac{1}{\cos x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - 1}{\frac{\cos x}{\sin x} + 1} \rightarrow \frac{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} - \frac{1 \cdot \sin x}{\cos x \cdot \sin x}}{\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{1 \cdot \sin x}{\cos x \cdot \sin x}} \stackrel{?}{=} \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}$$

$$\frac{\frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\sin x} \cancel{\cos x}}}{\frac{\cancel{\cos x} + \cancel{\sin x}}{\cancel{\cos x} \cancel{\sin x}}} \stackrel{?}{=} \frac{\frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\sin x}}}{\frac{\cancel{\cos x} + \cancel{\sin x}}{\cancel{\sin x}}} \rightarrow \frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\cos x} + \cancel{\sin x}} = \frac{\cancel{\cos x} - \cancel{\sin x}}{\cancel{\cos x} + \cancel{\sin x}} \quad \checkmark$$

The identity is verified

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\tan^2 x + \cos^2 x + \sin^2 x = \sec^2 x$

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

$$\sin^2 t + \cos^2 t = 1$$

Verifying Trigonometric Identities

Fundamental Identities

We can use fundamental identities to verify other identities.

Example: verify that $\tan^2 x + \cos^2 x + \sin^2 x = \sec^2 x$

$$\tan^2 x + \cos^2 x + \sin^2 x = \tan^2 x + 1 = \sec^2 x \quad \checkmark$$

The identity is verified

$$\sin t = \frac{1}{\csc t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

$$\sin^2 t + \cos^2 t = 1$$

Verifying Trigonometric Identities

Practice

$$\sin t = \frac{1}{\csc t} \quad \cos t = \frac{1}{\sec t}$$

$$\csc t = \frac{1}{\sin t} \quad \sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

$$\sin(-t) = -\sin t$$

$$\csc(-t) = -\csc t$$

$$\cos(-t) = \cos t$$

$$\sec(-t) = \sec t$$

$$\tan(-t) = -\tan t$$

$$\cot(-t) = -\cot t$$