

Sum and Difference Formulas

Sum and difference formulas for sines and cosines:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

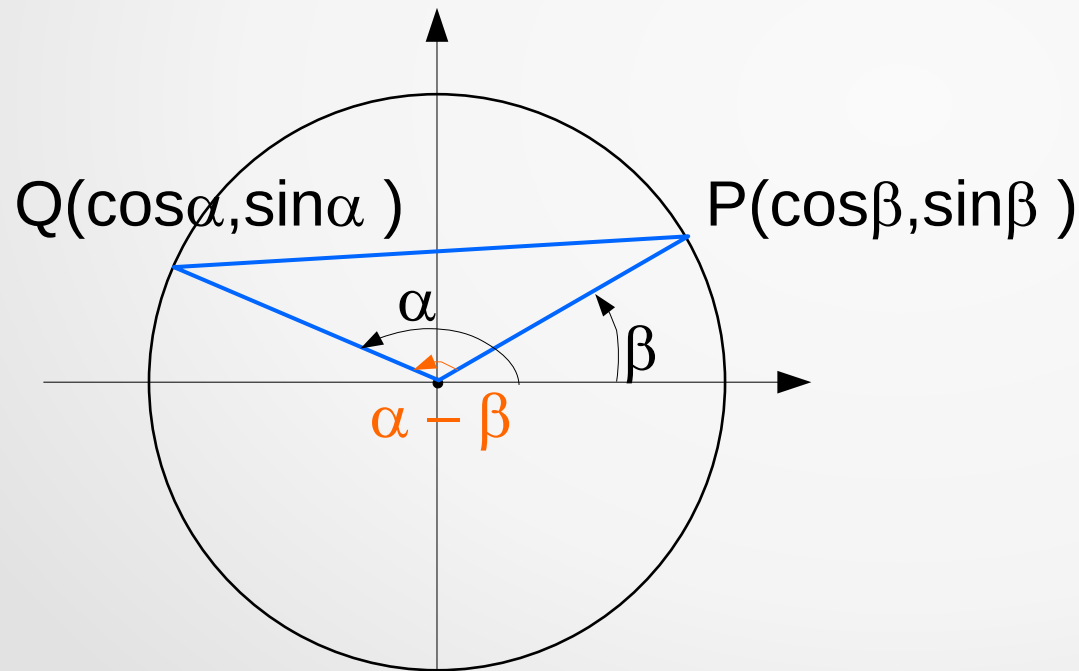
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Sum and Difference Formulas

Derivation of the
cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

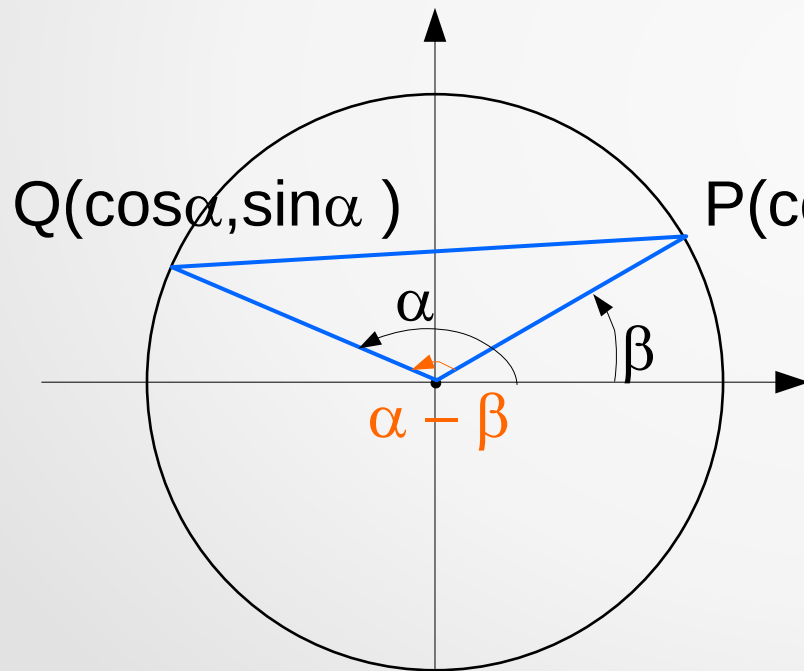


unit circle, i.e. $x^2 + y^2 = 1$

Sum and Difference Formulas

Derivation of the
cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



$$PQ = \sqrt{(\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2}$$

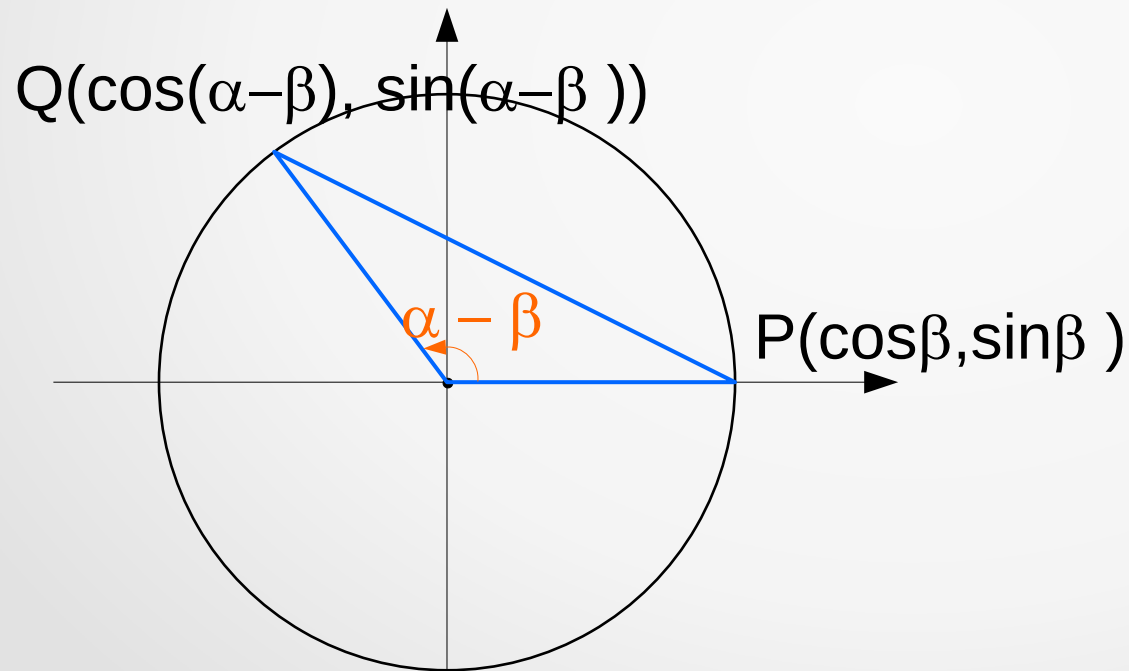
$$PQ = \sqrt{2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta}$$

unit circle, i.e. $x^2 + y^2 = 1$

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cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

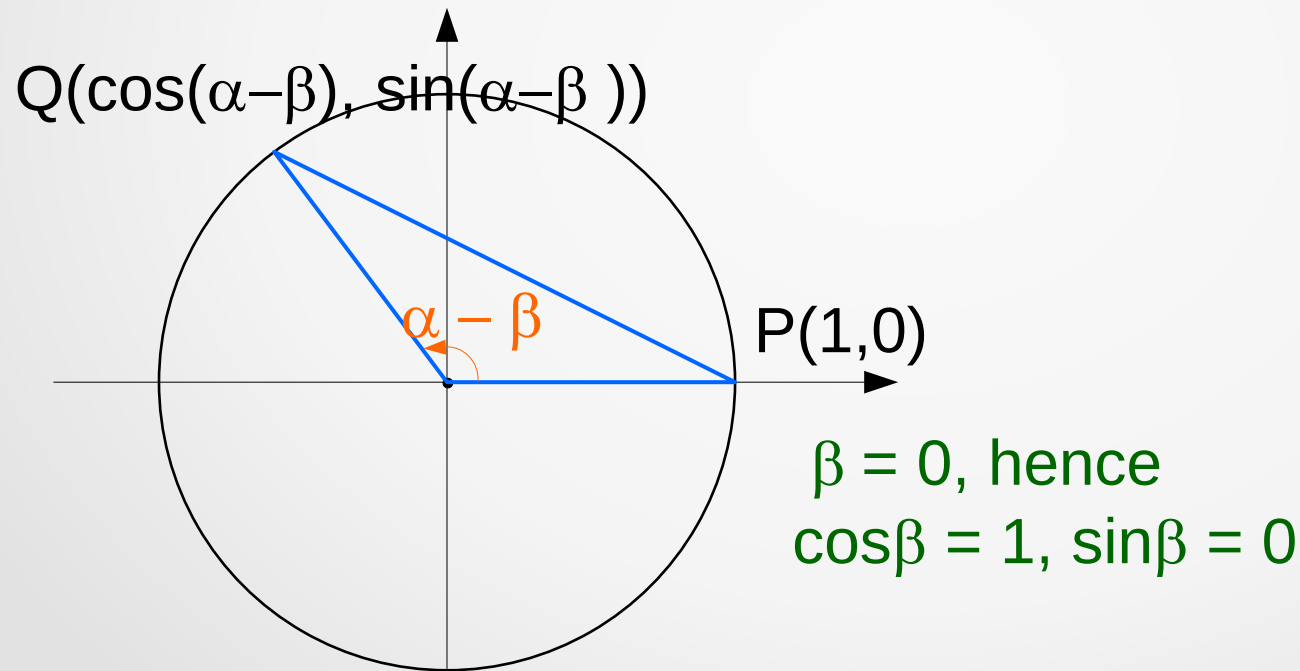


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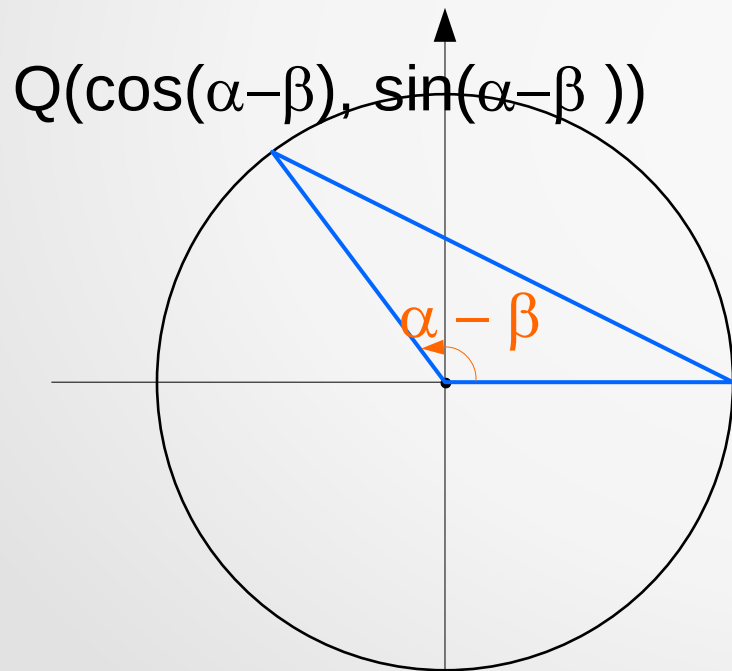


unit circle, i.e. $x^2 + y^2 = 1$

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cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



P(1,0)

$$PQ = \sqrt{(\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2}$$

$$PQ = \sqrt{2 - 2\cos(\alpha - \beta)}$$

unit circle, i.e. $x^2 + y^2 = 1$

Sum and Difference Formulas

Derivation of the
cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

We ended up with two expressions for PQ:

$$PQ = \sqrt{2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta}$$

$$PQ = \sqrt{2 - 2 \cos(\alpha - \beta)}$$

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Derivation of the
cos of the difference of two angles:

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

We ended up with two expressions for PQ:

$$PQ = \sqrt{2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta}$$

$$PQ = \sqrt{2 - 2 \cos(\alpha - \beta)}$$

they must be equal:

$$\sqrt{2 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta} = \sqrt{2 - 2 \cos(\alpha - \beta)}$$

$$-2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta = -2 \cos(\alpha - \beta)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Sum and Difference Formulas

Examples

Find exact values of each expression

1) $\cos(120^\circ - 45^\circ)$

2) $\cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ$

3) $\sin 75^\circ$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

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Sum and Difference Formulas

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Find exact values of each expression

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Find exact values of each expression

$$\begin{aligned} 1) \quad \cos(120^\circ - 45^\circ) &= \cos 120^\circ \cos 45^\circ + \sin 120^\circ \sin 45^\circ = \\ &= -\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ = -\frac{1}{2} \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} \end{aligned}$$

$$2) \quad \cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ$$

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$$2) \cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ = \cos(50^\circ - 5^\circ)$$

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$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

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$$2) \cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ = \cos(50^\circ - 5^\circ) = \cos 45^\circ$$

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$$3) \quad \sin 75^\circ = \sin(30^\circ + 45^\circ)$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

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$$2) \quad \cos 50^\circ \cos 5^\circ + \sin 50^\circ \sin 5^\circ = \cos(50^\circ - 5^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$3) \quad \sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

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$$\begin{aligned} 3) \sin 75^\circ &= \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ = \\ &= \frac{1}{2} \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

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Sum and Difference Formulas

Sum and difference formulas for tangent:

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Sum and Difference Formulas

Examples:

Find exact values of each expression

1) $\tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right)$

2) $\frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$

3) $\tan 105^\circ$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Sum and Difference Formulas

Examples:

Find exact values of each expression

$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{6}}$$

$$2) \frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$$

$$3) \tan 105^\circ$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

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Sum and Difference Formulas

Examples:

Find exact values of each expression

$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{6}} = \frac{-\sqrt{3} - \frac{\sqrt{3}}{3}}{1 + (-\sqrt{3})\frac{\sqrt{3}}{3}}$$

$$2) \frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$$

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$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

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$$2) \frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$$

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$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

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$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan\frac{2\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{2\pi}{3}\tan\frac{\pi}{6}} = \frac{-\sqrt{3} - \frac{\sqrt{3}}{3}}{1 + (-\sqrt{3})\frac{\sqrt{3}}{3}} = \frac{-\frac{2\sqrt{3}}{3}}{0} = \text{undefined}$$

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$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{6}} = \frac{-\sqrt{3} - \frac{\sqrt{3}}{3}}{1 + (-\sqrt{3}) \frac{\sqrt{3}}{3}} = \frac{-\frac{2\sqrt{3}}{3}}{0} = \text{undefined}$$

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$$3) \tan 105^\circ = \tan(45^\circ + 60^\circ) = \frac{\tan 45^\circ + \tan 60^\circ}{1 - \tan 45^\circ \tan 60^\circ} = \frac{1 + \sqrt{3}}{1 - 1 \cdot \sqrt{3}} = \frac{(1 + \sqrt{3}) \cdot (1 + \sqrt{3})}{(1 - \sqrt{3}) \cdot (1 + \sqrt{3})}$$

$$= \frac{(1 + \sqrt{3})^2}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

Sum and Difference Formulas

Examples:

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Find exact values of each expression

$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{6}} = \frac{-\sqrt{3} - \frac{\sqrt{3}}{3}}{1 + (-\sqrt{3})\frac{\sqrt{3}}{3}} = \frac{-\frac{2\sqrt{3}}{3}}{0} = \text{undefined}$$

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$$= \frac{(1 + \sqrt{3})^2}{1 - 3}$$

Sum and Difference Formulas

Examples:

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Find exact values of each expression

$$1) \tan\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{6}} = \frac{-\sqrt{3} - \frac{\sqrt{3}}{3}}{1 + (-\sqrt{3})\frac{\sqrt{3}}{3}} = \frac{-\frac{2\sqrt{3}}{3}}{0} = \text{undefined}$$

$$2) \frac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ} = \tan(50^\circ - 20^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$3) \tan 105^\circ = \tan(45^\circ + 60^\circ) = \frac{\tan 45^\circ + \tan 60^\circ}{1 - \tan 45^\circ \tan 60^\circ} = \frac{1 + \sqrt{3}}{1 - 1 \cdot \sqrt{3}} = \frac{(1 + \sqrt{3}) \cdot (1 + \sqrt{3})}{(1 - \sqrt{3}) \cdot (1 + \sqrt{3})}$$

$$= \frac{(1 + \sqrt{3})^2}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

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$$= \frac{(1 + \sqrt{3})^2}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

Sum and Difference Formulas

More Examples:

Verify each identity:

$$1) \cos\left(x - \frac{5\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

$$2) \cos(\pi - x) = -\cos x$$

$$3) \tan(\pi - x) = -\tan x$$

$$4) \sin\left(x + \frac{3\pi}{2}\right) = -\cos x$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Homework assignment

1) zyBooks: *review* Section 7.1 and 7.2

or

Textbook: *review* Sections 5.1 and 5.2

2) We will have **Quiz 19 (last quiz)** based on today's topics in the beginning of our next meeting.

3) WeBWorK: **HW 19** (due date is in one week)