Logarithmic Functions

Learning objectives:

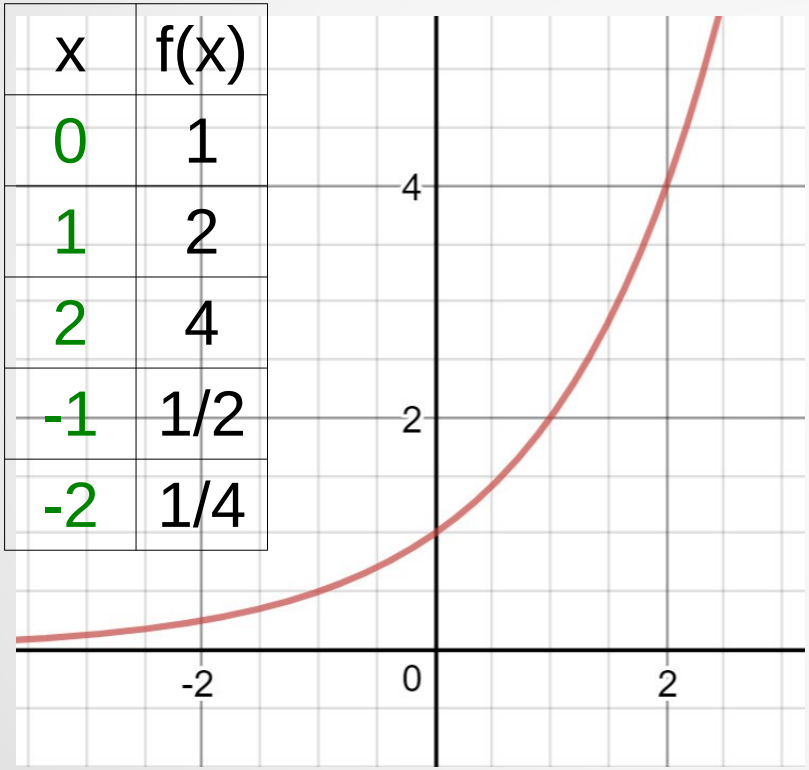
today we will:

- Identify the domain of a logarithmic function.
- Graph logarithmic functions.

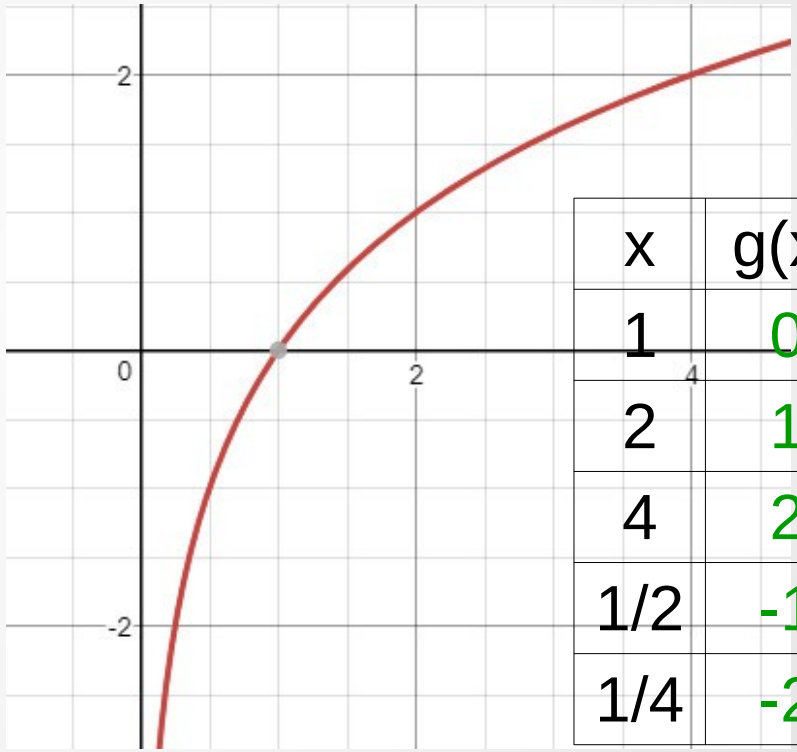
Graphs of Exponential and Logarithmic Functions

1) $f(x)=2^x$

x	f(x)
0	1
1	2
2	4
-1	1/2
-2	1/4



2) $g(x) = \log_2 x$



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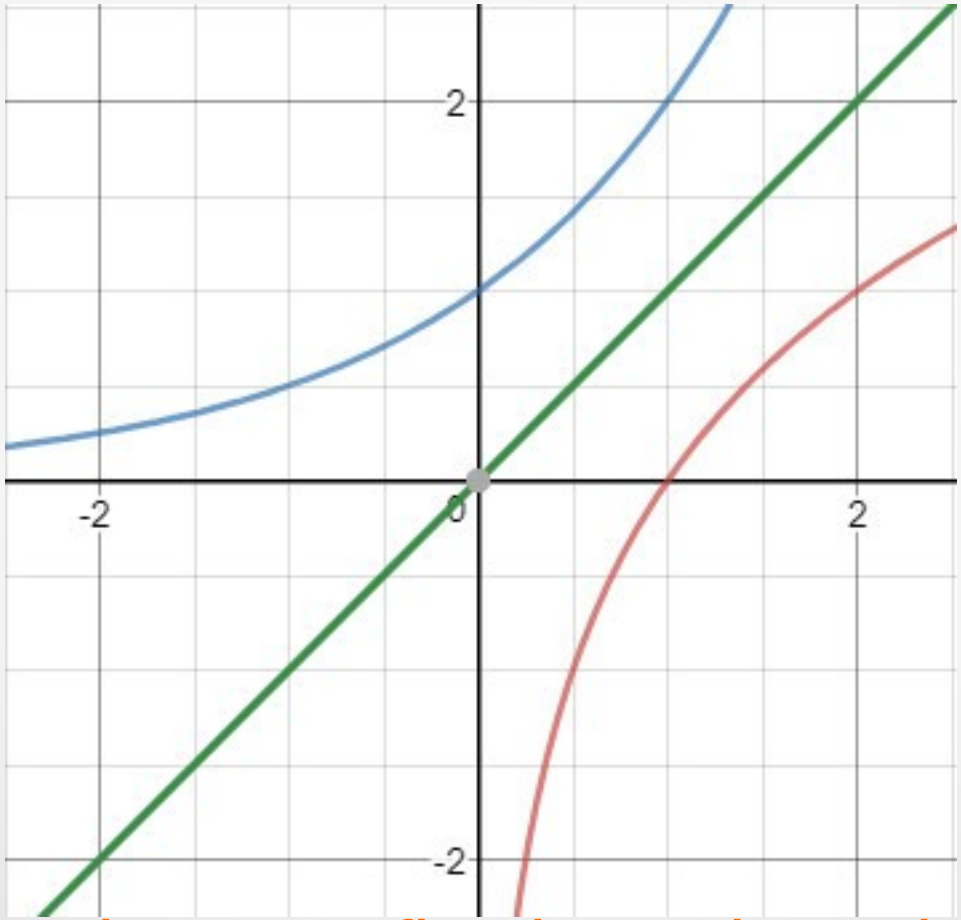

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Their graphs are reflections about the y=x line

Graphs of Exponential and Logarithmic Functions

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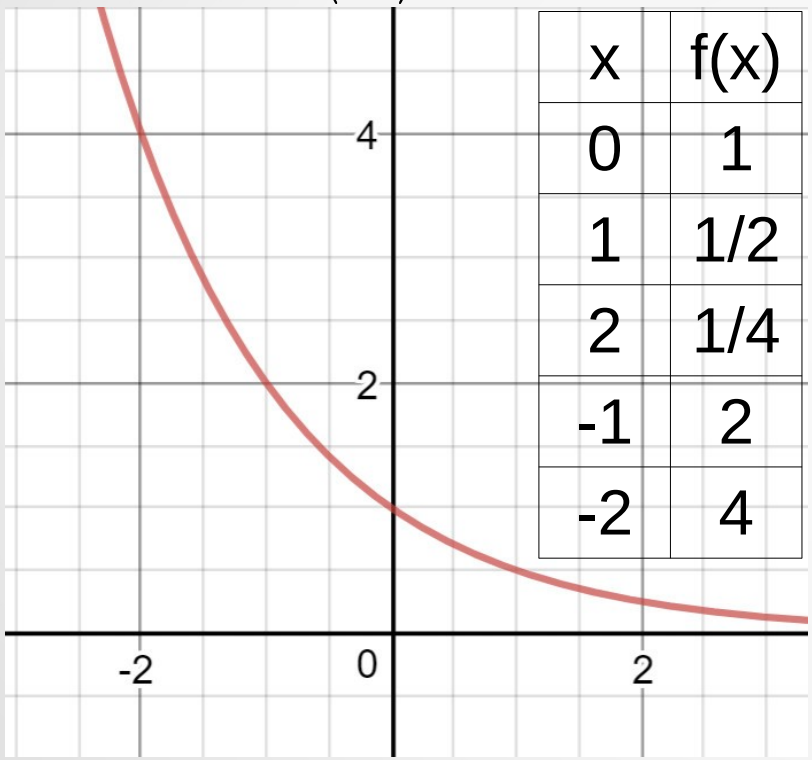

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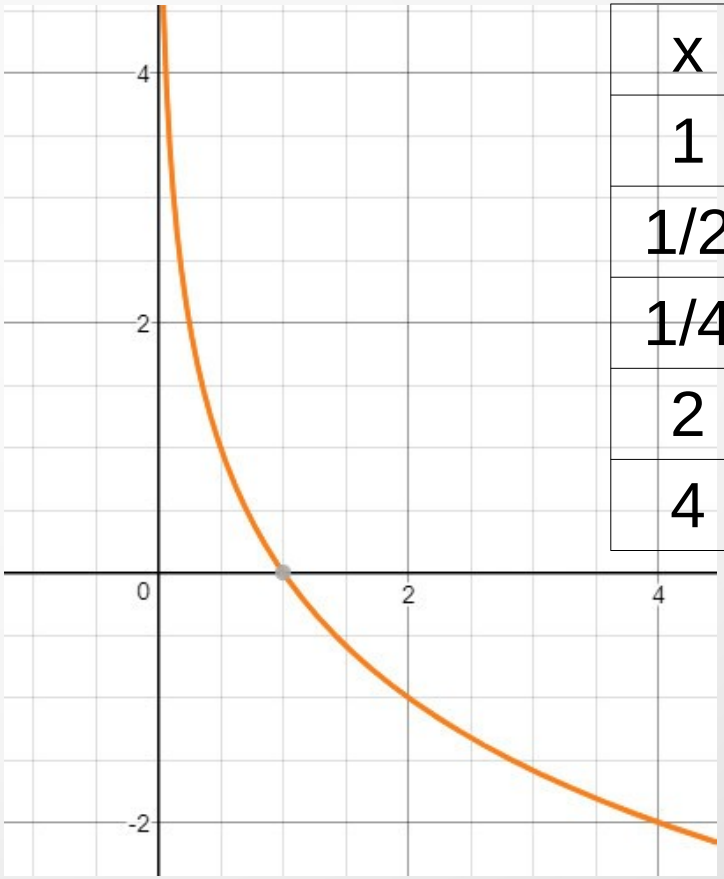

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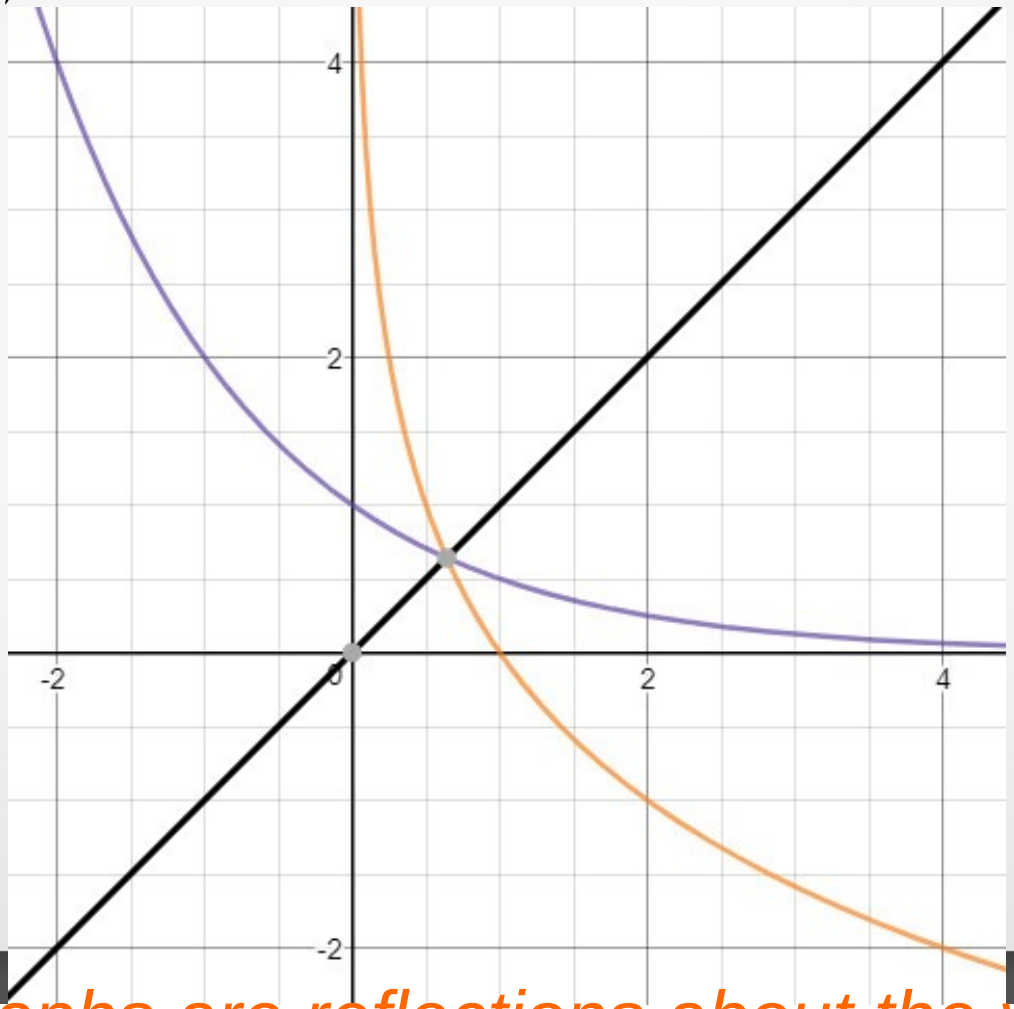

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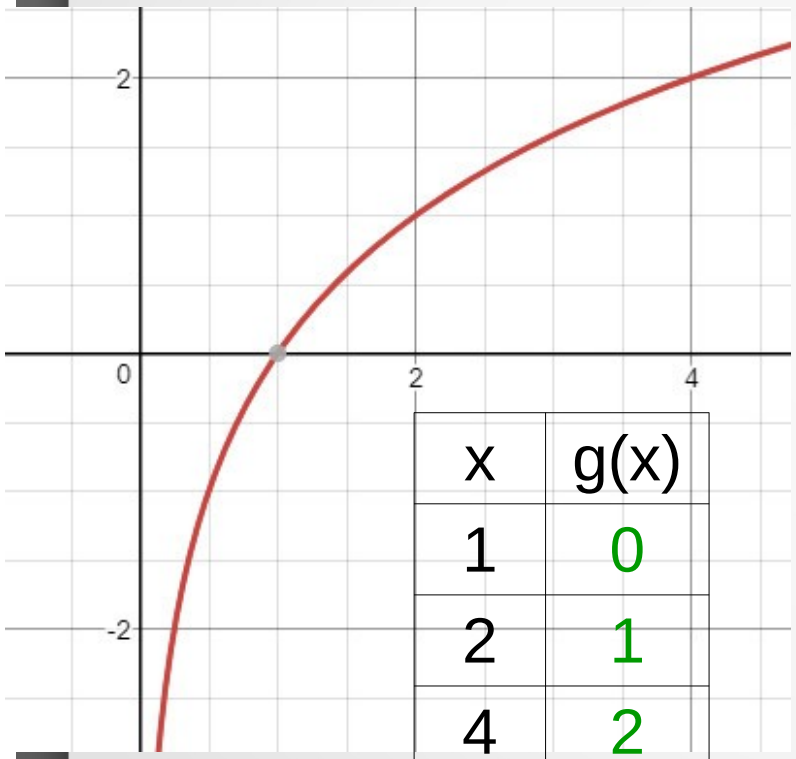

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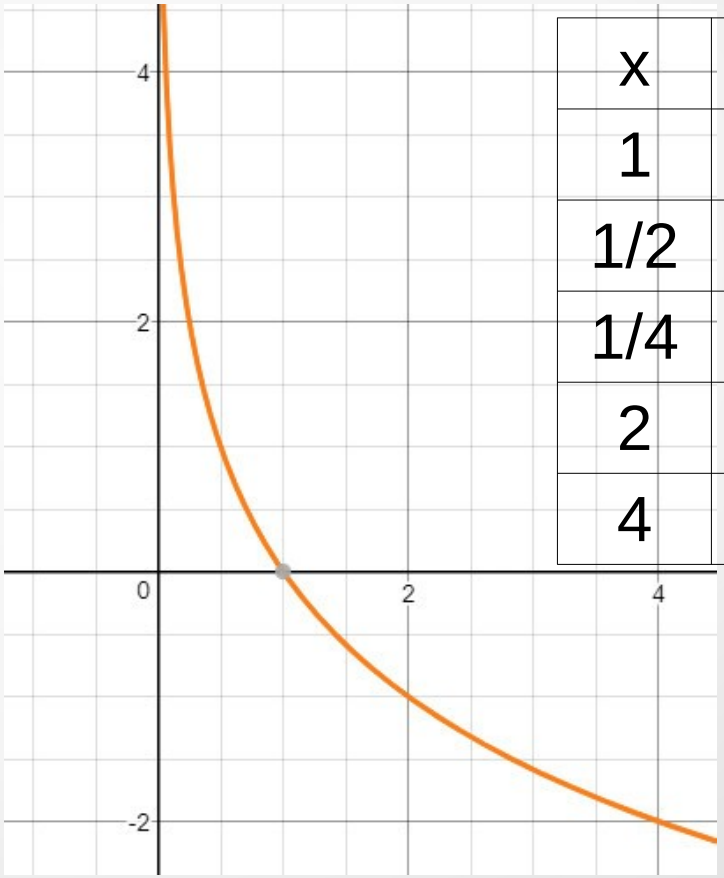

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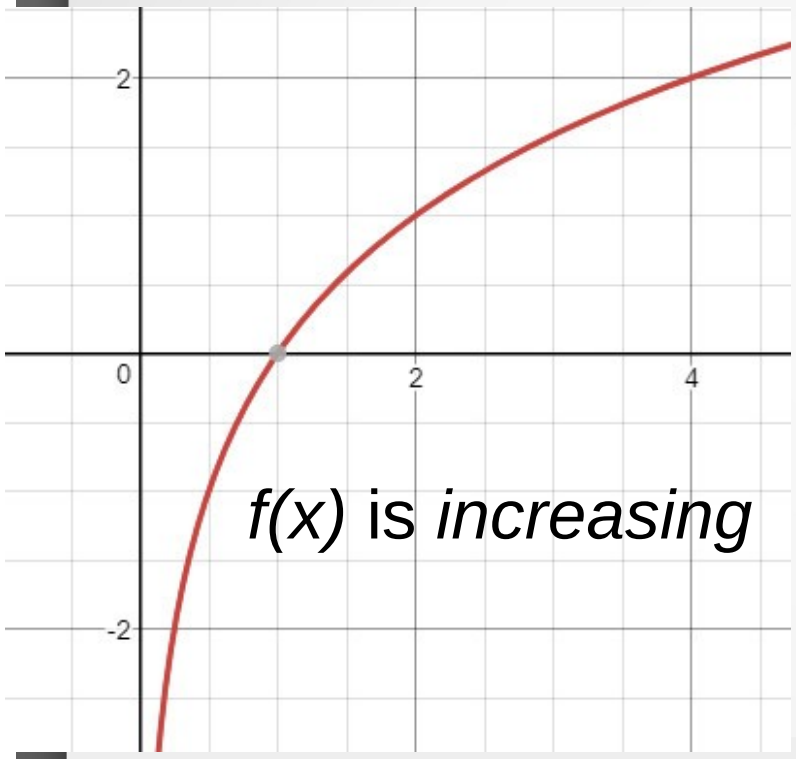

f(x) is *increasing*

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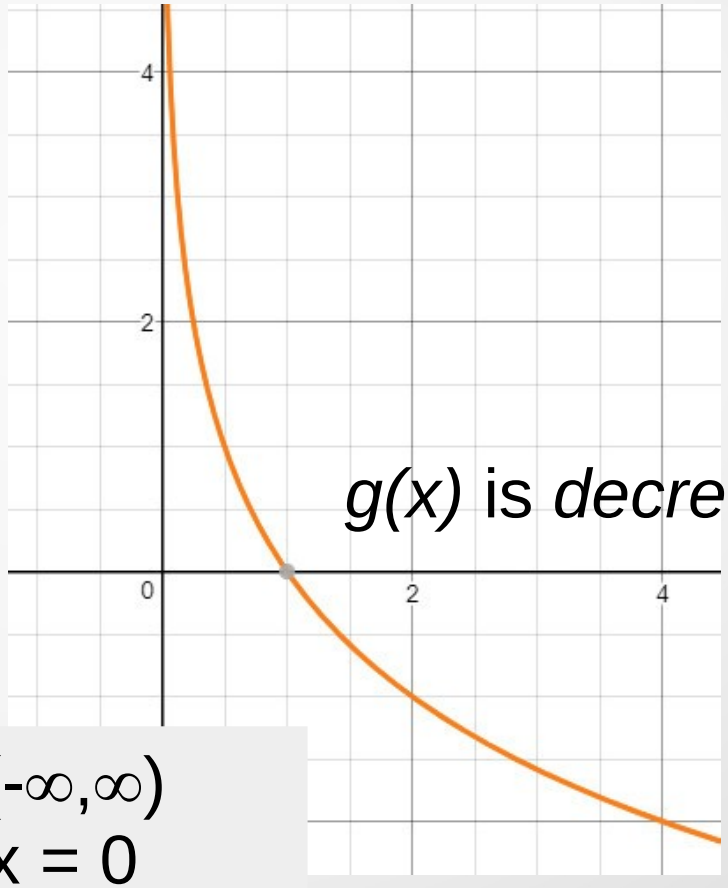


g(x) is *decreasing*

domain: (0,∞) *range*: (-∞,∞)
vertical asymptote: *vertical* x = 0
special point: (0,1)

Logarithmic Functions - transformations

Transformations of logarithmic functions

c is positive real number, $f(x) = \log_b x$

1) $g(x) = \log_b x + c$ shifts the graph of f(x) c units up
   $g(x) = \log_b x - c$ shifts the graph of f(x) c units down

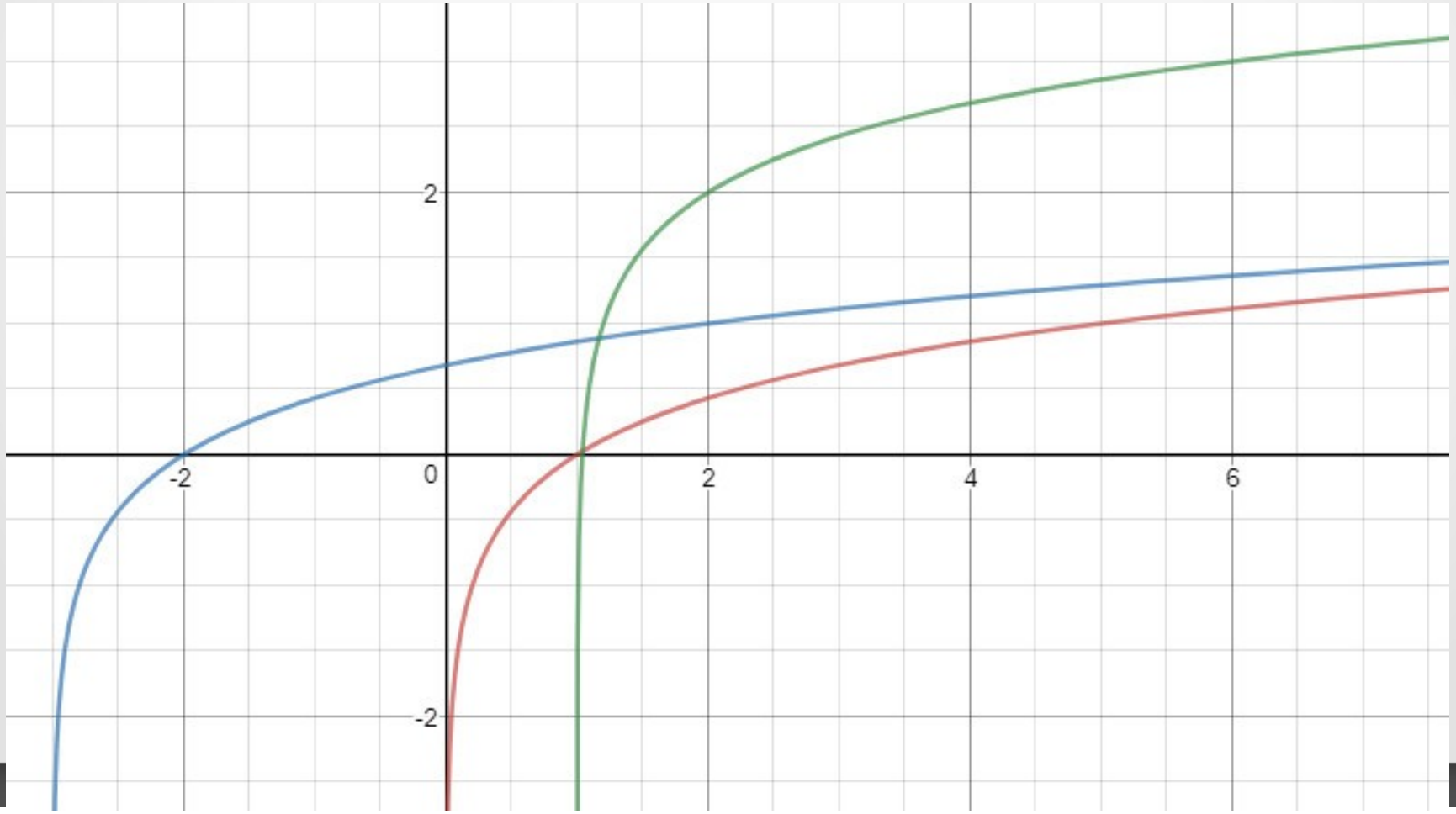
2) $g(x) = \log_b (x+c)$ shifts the graph of f(x) c units to the left
   $g(x) = \log_b (x-c)$ shifts the graph of f(x) c units to the right

3) $g(x) = -\log_b x$ reflection about x-axis
   $g(x) = \log_b (-x)$ reflection about y-axis

4) $g(x) = c\log_b x$ vertical stretch if c>1
   $g(x) = c\log_b x$ vertical shrinking if 0<c>1

Logarithmic Functions - transformations

Example: let's see the graphs of $f(x) = \log_5 x$, $g(x) = \log_5 (x+3)$, and $h(x) = \log_5 (x-1)+2$



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Finding the domain of a logarithmic function

consider a function $t(x) = 3\log_3 (5x-12)$

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Hence the domain is $\left(\frac{12}{5}, \infty\right)$

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Finding the vertical asymptote of a logarithmic function

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Where do you think the vertical asymptote is?

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$$x > \frac{12}{5}$$

Hence the domain is $\left(\frac{12}{5}, \infty\right)$

Where do you think the vertical asymptote is? $x = \frac{12}{5}$

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Finding the intercepts of a logarithmic function

What about x- and y-intercepts of the function $t(x) = 3\log_3(5x-12)$?

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$1 = 5x - 12$

$13 = 5x$

$x = \frac{13}{5}$ $\left(\frac{13}{5}, 0\right)$

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$x = \frac{13}{5}$

$\left(\frac{13}{5}, 0\right)$

In-class practice

Exercise 1: for the function $g(x)=\log_5(2x+9)-2$ state

(a) its *domain* and *range*

(b) its *vertical asymptote*

(c) its *x*- and *y-intercepts*, if possible

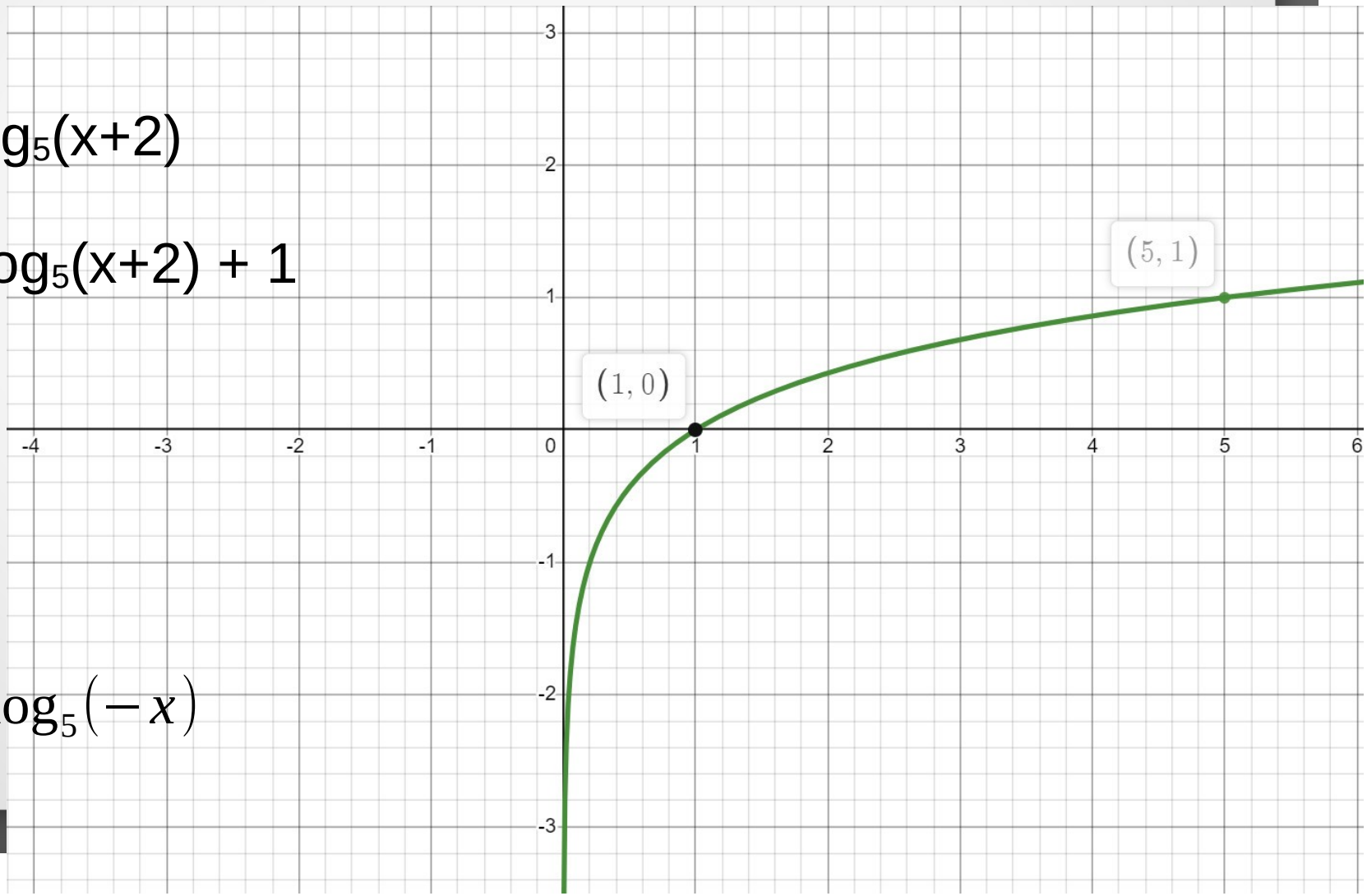
In-class practice

Exercise 2: given the graph of the function $f(x)=\log_5(x)$ graph

(a) $y = \log_5(x+2)$

(b) $y = -\log_5(x+2) + 1$

(c) $y=\frac{1}{5}\log_5(-x)$



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