

Exponential Functions

Learning Objectives:

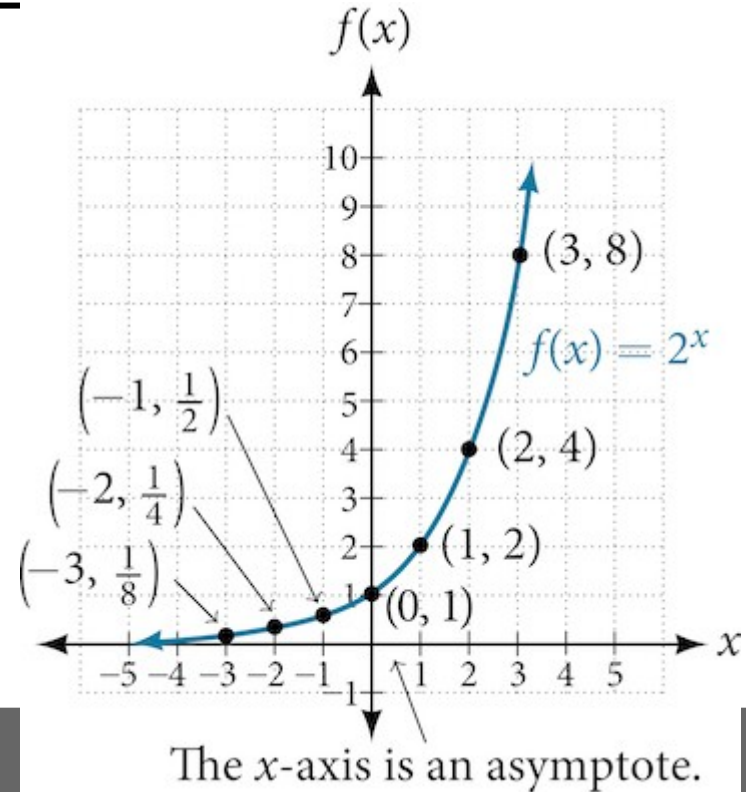
In this section, we will:

- Graphed exponential functions
- Graphed exponential functions using transformations.

Exponential Functions

We sketched the graph of $f(x) = 2^x$

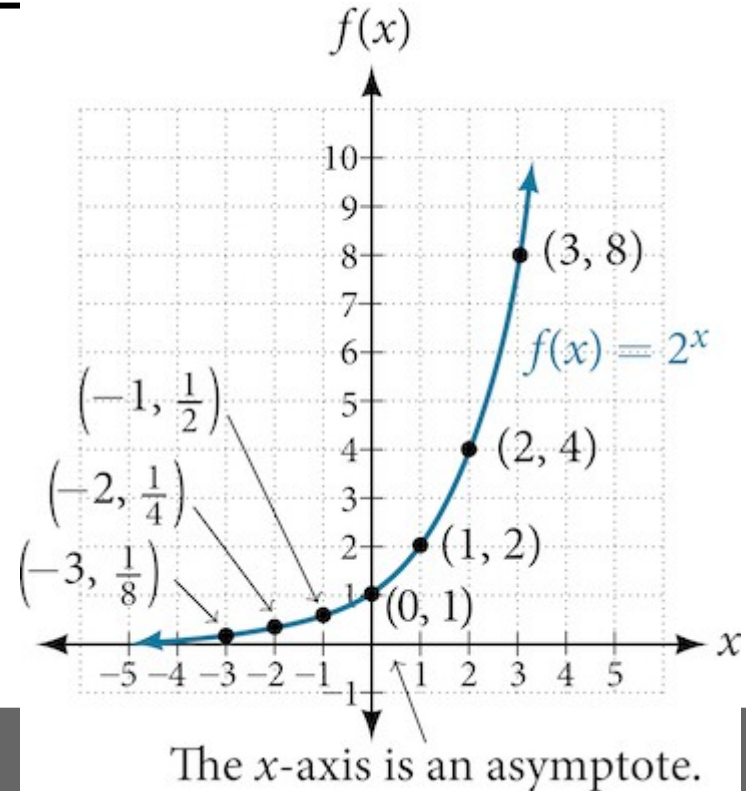
x	2^x
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
-1	$2^{-1} = 0.5$
-2	$2^{-2} = 0.25$
-3	$2^{-3} = 0.125$



Exponential Functions

We sketched the graph of $f(x) = 2^{-x}$

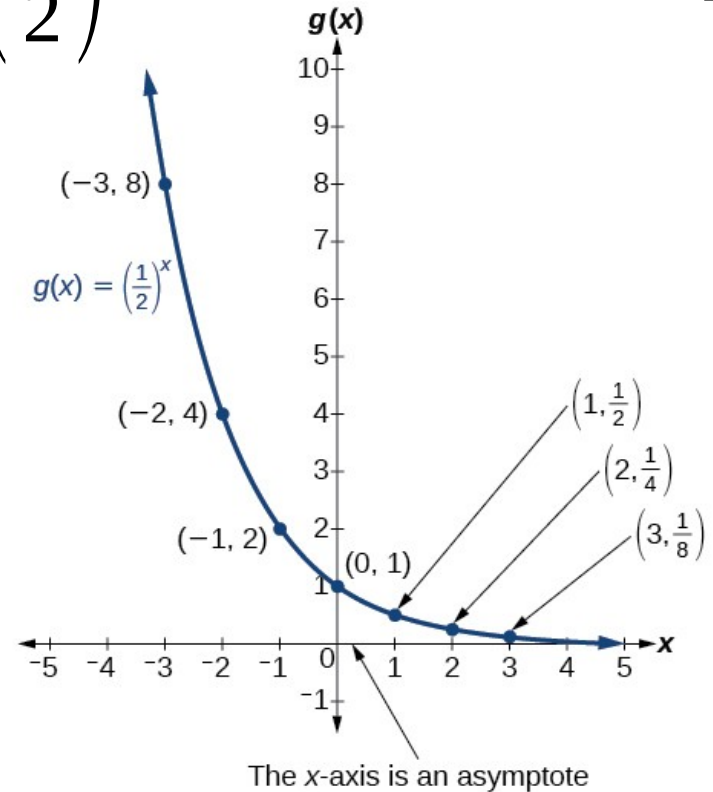
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4	$2^4 = 16$
-1	$2^{-1} = 0.5$
-2	$2^{-2} = 0.25$
-3	$2^{-3} = 0.125$



Exponential Functions

We sketched the graph of $f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$

x	2^x
0	$2^0 = 1$
1	$2^{-1} = \frac{1}{2} = 0.5$
2	$2^{-2} = \frac{1}{4} = 0.25$
3	$2^{-3} = \frac{1}{8} = 0.125$
-1	$2^{-(-1)} = 2$
-2	$2^{-(-2)} = 2^2 = 4$
-3	$2^{-(-3)} = 2^3 = 8$

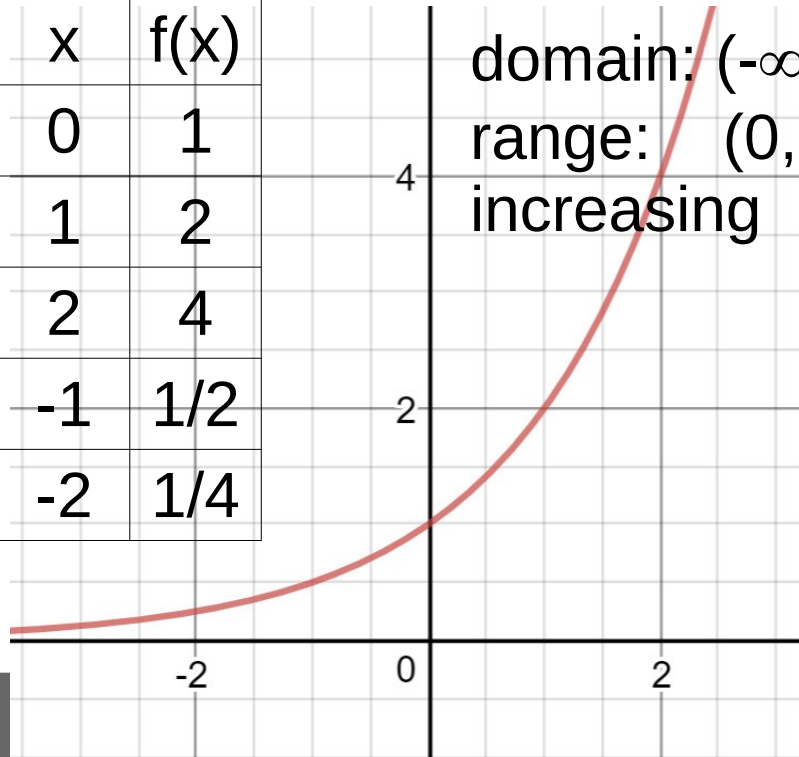


Exponential Functions

$$f(x) = 2^x$$

x	f(x)
0	1
1	2
2	4
-1	1/2
-2	1/4

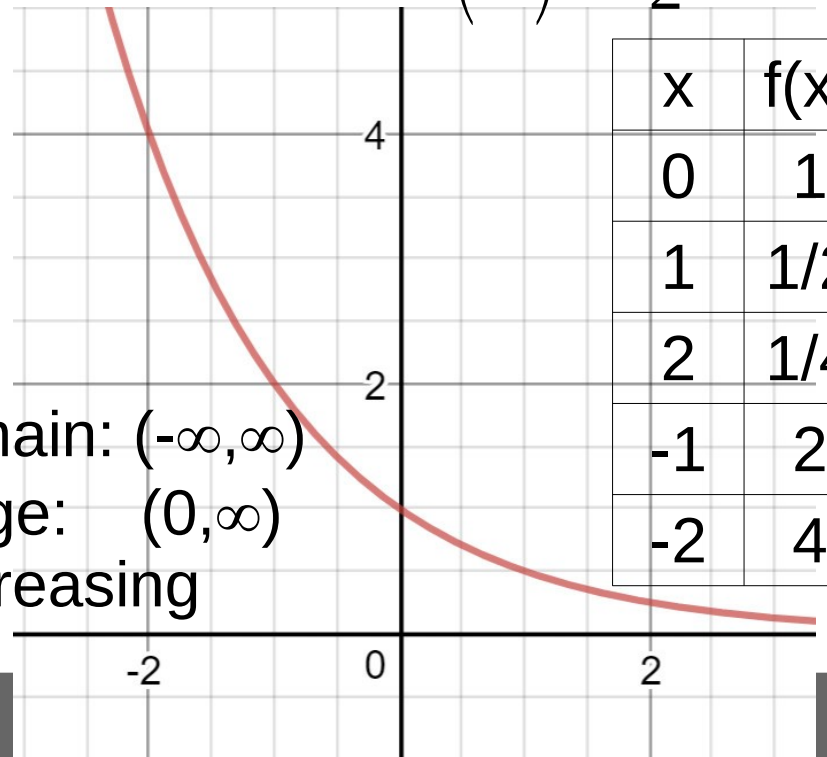
domain: $(-\infty, \infty)$
range: $(0, \infty)$
increasing



$$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x = \frac{1}{2^x}$$

x	f(x)
0	1
1	1/2
2	1/4
-1	2
-2	4

domain: $(-\infty, \infty)$
range: $(0, \infty)$
decreasing



Exponential Functions

Characteristics of graphs of functions of the form $f(x) = b^x$

- 1) *domain:* $(-\infty, \infty)$, *range:* $(0, \infty)$
- 2) *horizontal asymptote:* $y = 0$
- 3) graph passes through the point $(0, 1)$
- 4) function $f(x)$ is *one-to-one*, i.e. has an *inverse*
- 5) if $b > 1$, $f(x)$ is *increasing*
If $0 < b < 1$, the function is *decreasing*

Exponential Functions

Let's sketch graphs of the given exponential functions, using the graph of $f(x) = 2^x$.

(a) $g(x) = 2^x - 4$

(b) $h(x) = 2^{x-3}$

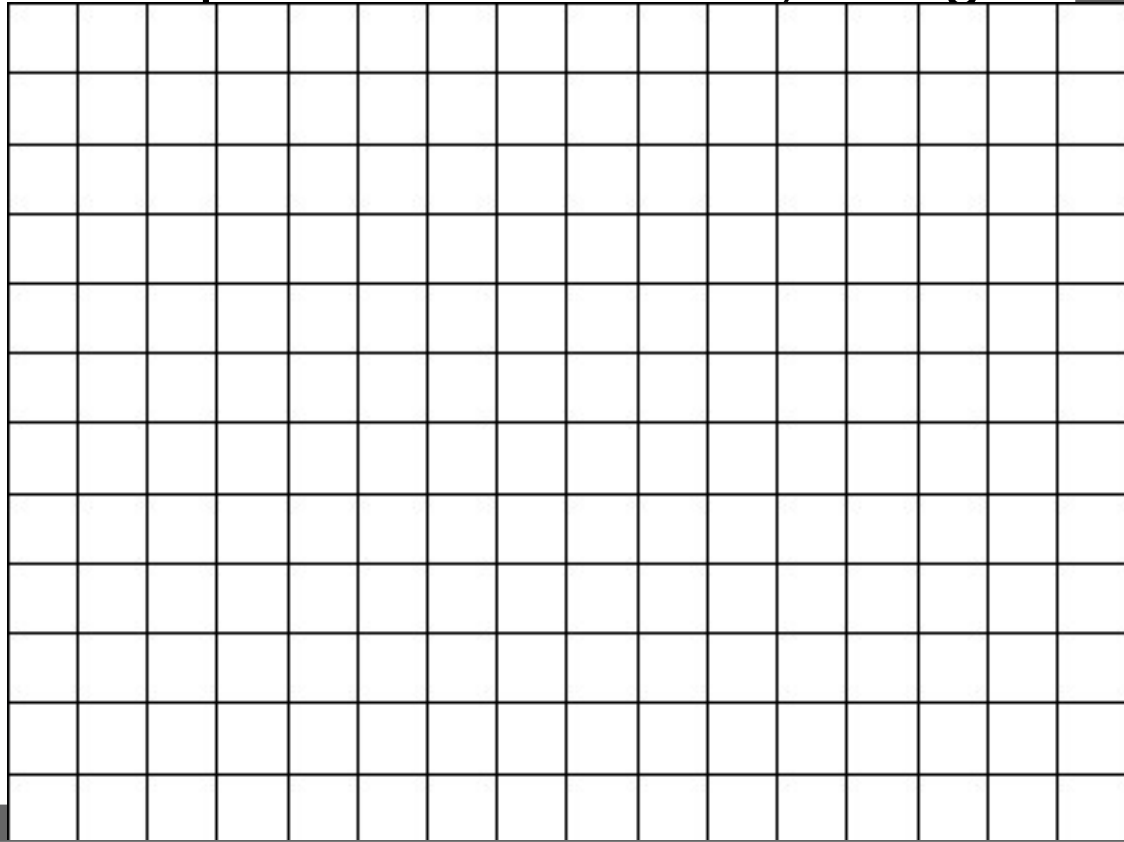
(c) $t(x) = 2^{x-3} - 4$

(d) $m(x) = 3(2)^x$

(e) $k(x) = -3(2)^x$

(f) $n(x) = -3(2)^{-x}$

(g) $r(x) = \frac{1}{3} x^{x+1}$



Translations of Exponential Functions

A translation of an exponential function has the form

$$f(x) = ab^{x+c} + d$$

where the parent function, $y = b^x$, $b > 1$, is

- shifted horizontally c units to the left.
- stretched vertically by a factor of $|a|$ if $|a| > 1$.
- compressed vertically by a factor of $|a|$ if $0 < |a| < 1$.
- shifted vertically d units.
- reflected about the x -axis when $a < 0$.

Note: the order of the shifts, transformations, and reflections follow the order of operations.

Also, check Table 6 in the textbook

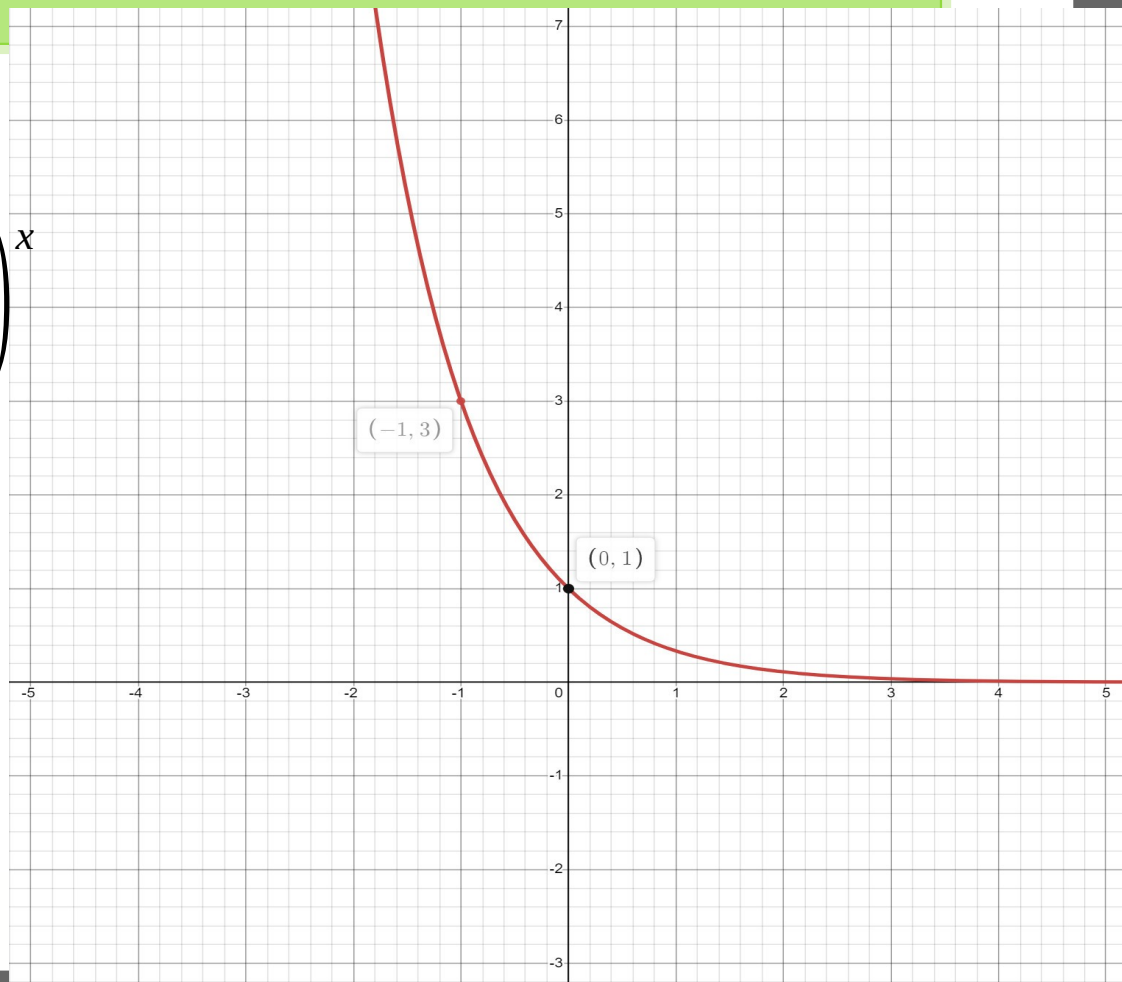
In-class practice

sketch graphs of the given exponential functions, using the graph of $g(x) = \left(\frac{1}{3}\right)^x$

(a) $h(x) = \left(\frac{1}{3}\right)^{(x+3)}$

(b) $g(x) = \left(\frac{1}{3}\right)^x - 2$

(c) $g(x) = 2\left(\frac{1}{3}\right)^{(-x)}$



Exponential Functions

Today we:

- Graphed exponential functions
- Graphed exponential functions using transformations.