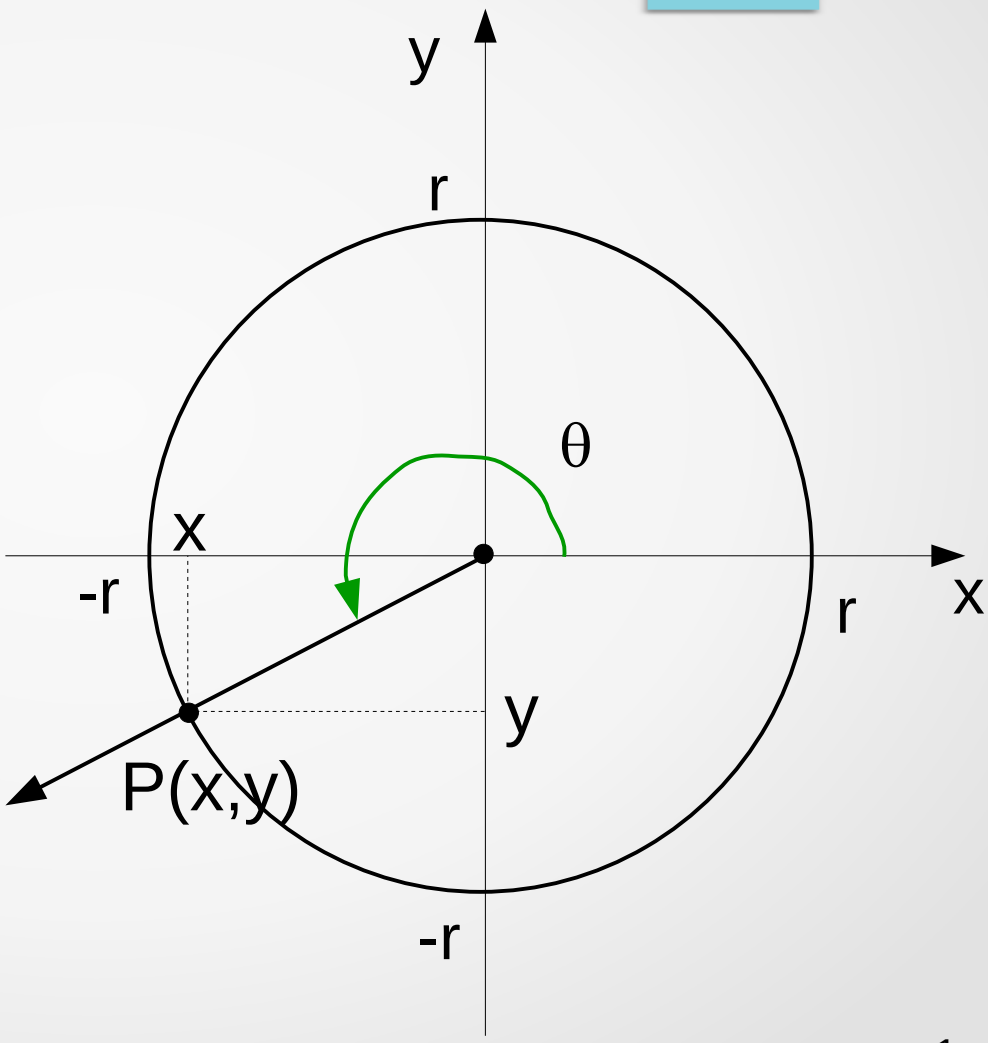
Trigonometric Functions of Any Angle

θ is any angle in standard position.

any circle of radius r

P(x,y) is any point on the terminal side of θ.

every point on the circle satisfies to
$x^2+y^2=r^2$



Trigonometric Functions of Any Angle

$$\sin\theta = \frac{y}{r}$$

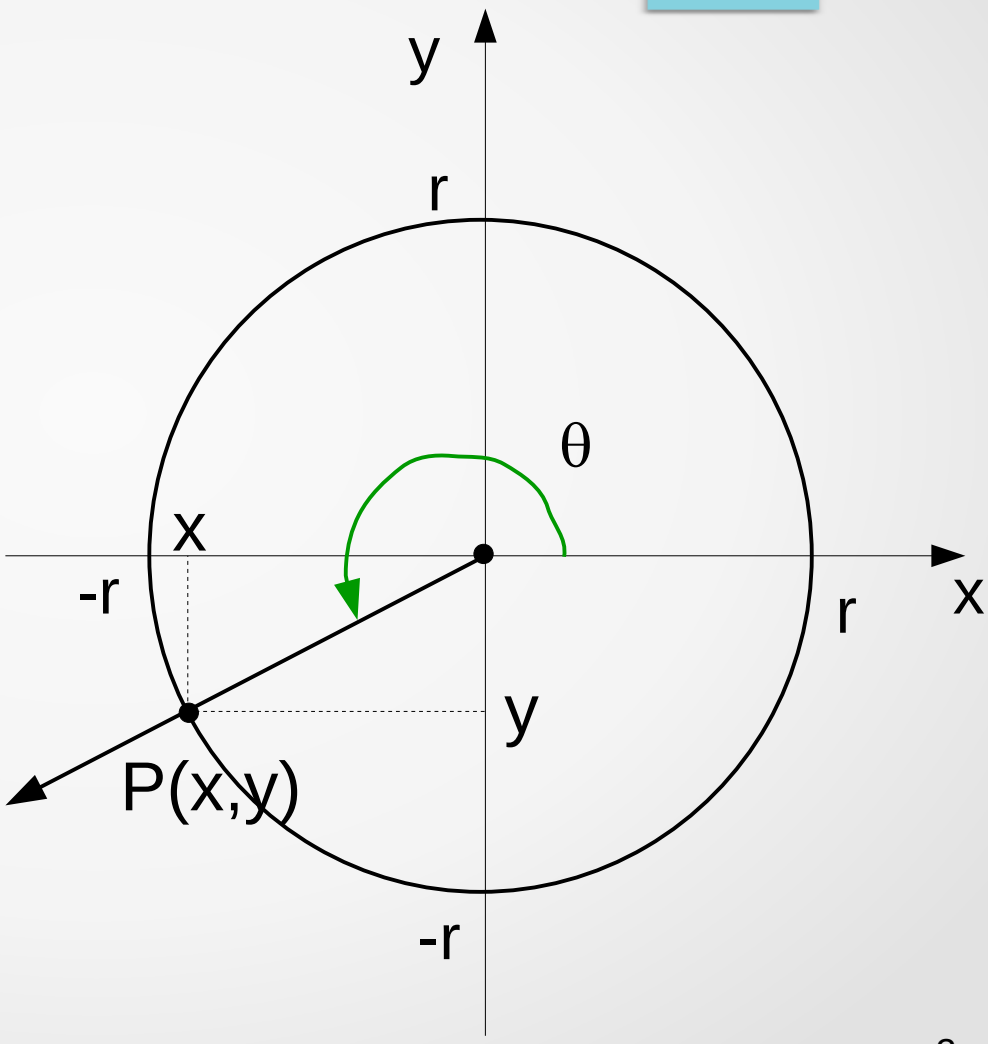
$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

$$\csc\theta = \frac{r}{y}$$

$$\sec\theta = \frac{r}{x}$$



Trigonometric Functions of Any Angle

$$\sin\theta = \frac{y}{r}$$

+	+
-	-

$$\cos\theta = \frac{x}{r}$$

-	+
-	+

$$\tan\theta = \frac{y}{x}$$

-	+
+	-

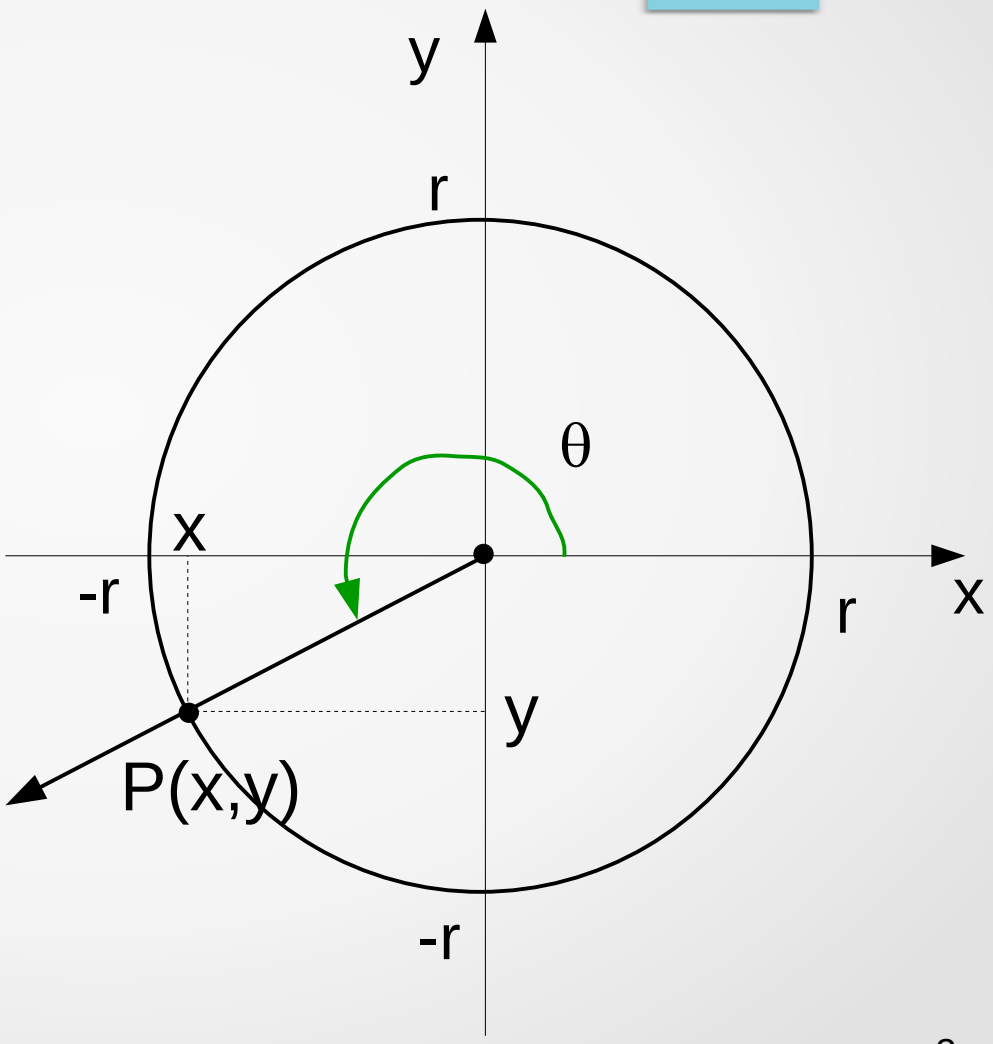
$$\cot\theta = \frac{x}{y}$$

$$\csc\theta = \frac{r}{y}$$

+	+
-	-

$$\sec\theta = \frac{r}{x}$$

-	+
-	+



Trigonometric Functions of Any Angle

Examples:

1) point (-12,5) lies on the terminal side of standard angle α. Find the exact values of *sin* α, *cos* α, *tan* α and *cot* α.

$$\sin\theta = \frac{y}{r}$$

+	+
-	-

$$\cos\theta = \frac{x}{r}$$

-	+
-	+

$$\tan\theta = \frac{y}{x}$$

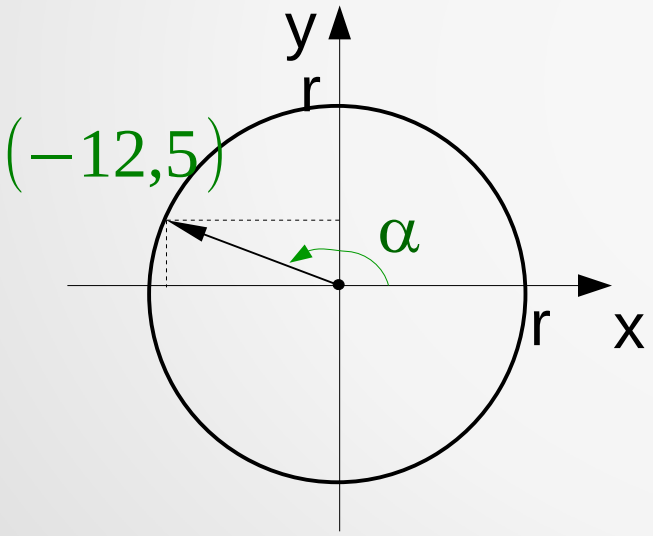
-	+
+	-

$$\cot\theta = \frac{x}{y}$$

Trigonometric Functions of Any Angle

Examples:

1) point (-12,5) lies on the terminal side of standard angle α. Find the exact values of *sin* α, *cos* α, *tan* α and *cot* α.



$$\sin\theta = \frac{y}{r}$$

+	+
-	-

$$\cos\theta = \frac{x}{r}$$

-	+
-	+

$$\tan\theta = \frac{y}{x}$$

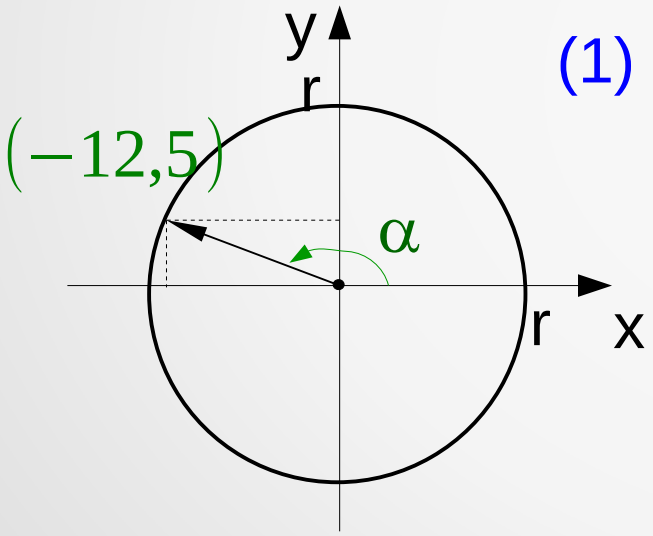
$$\cot\theta = \frac{x}{y}$$

-	+
+	-

Trigonometric Functions of Any Angle

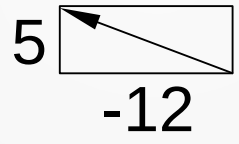
Examples:

1) point (-12,5) lies on the terminal side of standard angle α. Find the exact values of *sin* α, *cos* α, *tan* α and *cot* α.



Solution:

(1) find the radius:



$$\sin\theta = \frac{y}{r}$$

+	+
-	-

$$\cos\theta = \frac{x}{r}$$

-	+
-	+

$$\tan\theta = \frac{y}{x}$$

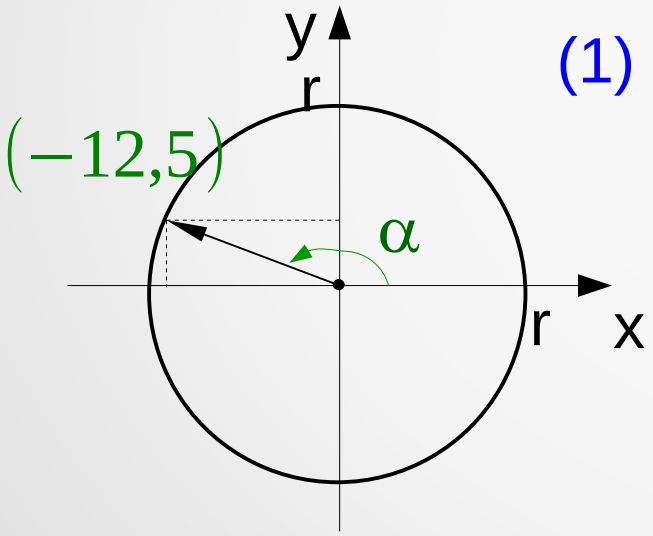
-	+
+	-

$$\cot\theta = \frac{x}{y}$$

Trigonometric Functions of Any Angle

Examples:

1) point (-12,5) lies on the terminal side of standard angle α. Find the exact values of *sin* α, *cos* α, *tan* α and *cot* α.



Solution:

(1) find the radius:

5

-12

$5^2+(-12)^2=r^2$

$25+144=r^2$

$r^2=169 \qquad r=13$

$\sin\theta=\frac{y}{r}$

+	+
-	-

$\cos\theta=\frac{x}{r}$

-	+
-	+

$\tan\theta=\frac{y}{x}$

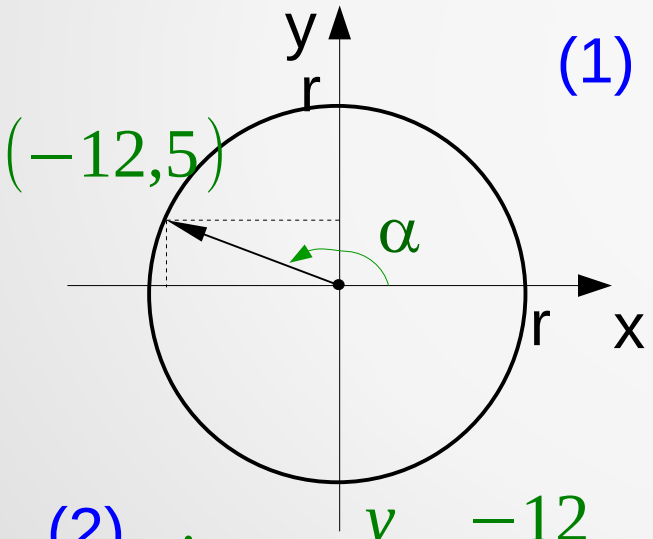
-	+
+	-

$\cot\theta=\frac{x}{y}$

Trigonometric Functions of Any Angle

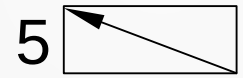
Examples:

1) point (-12,5) lies on the terminal side of standard angle α. Find the exact values of *sin* α, *cos* α, *tan* α and *cot* α.



Solution:

(1) find the radius:



$$5^2+(-12)^2=r^2$$

$$25+144=r^2$$

$$r^2=169 \qquad r=13$$

(2) $\sin\alpha=\frac{y}{r}=\frac{-12}{13}$

$\cos\alpha=\frac{x}{r}=\frac{5}{13}$

$\tan\alpha=\frac{y}{x}=-\frac{5}{12}$

$\cot\alpha=-\frac{12}{5}$

$\sin\theta=\frac{y}{r}$ $\cos\theta=\frac{x}{r}$ $\tan\theta=\frac{y}{x}$ $\cot\theta=\frac{x}{y}$

+	+
-	-

-	+
-	+

-	+
+	-

Trigonometric Functions of Any Angle

Examples:

2) Find $\tan\frac{3\pi}{2}, sec\,\pi, \cot\frac{\pi}{2}$

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

-	+
+	-

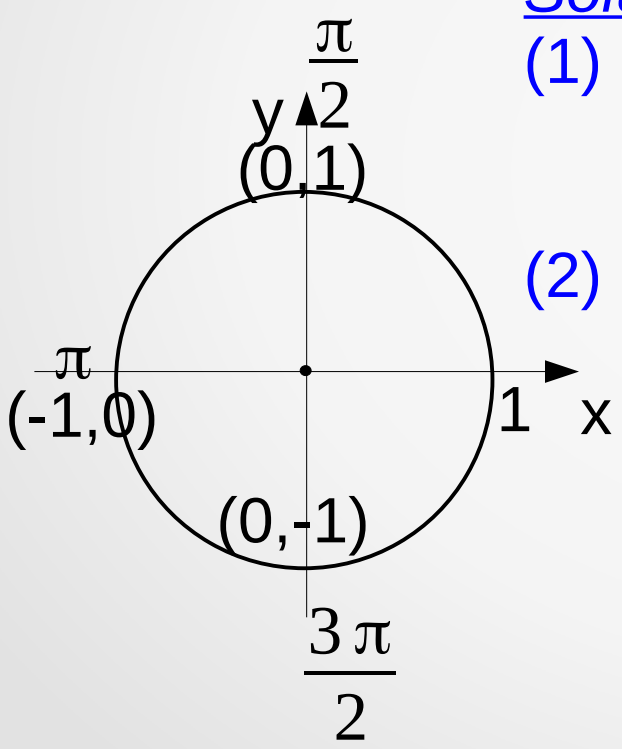
$sec\,\theta = \frac{r}{x}$

-	+
-	+

Trigonometric Functions of Any Angle

Examples:

2) Find $\tan\frac{3\pi}{2}, sec\,\pi, \cot\frac{\pi}{2}$



Solution:

(1) use the unit circle (radius = 1)

(2) $\tan\frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0} = undefined$

$sec\,\pi = \frac{r}{x} = \frac{1}{-1} = -1$

$\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

-	+
+	-

$sec\,\theta = \frac{r}{x}$

-	+
-	+

Trigonometric Functions of Any Angle

Examples:

3) Given $\cos\beta = -\frac{3}{5}$, β in quadrant II, find the exact value of each of the remaining trigonometric functions of β.

$\sin\theta = \frac{y}{r}$

+	+
-	-

$\cos\theta = \frac{x}{r}$

-	+
-	+

$\tan\theta = \frac{y}{x}$

-	+
+	-

$\cot\theta = \frac{x}{y}$

$csc\,\theta = \frac{r}{y}$

+	+
-	-

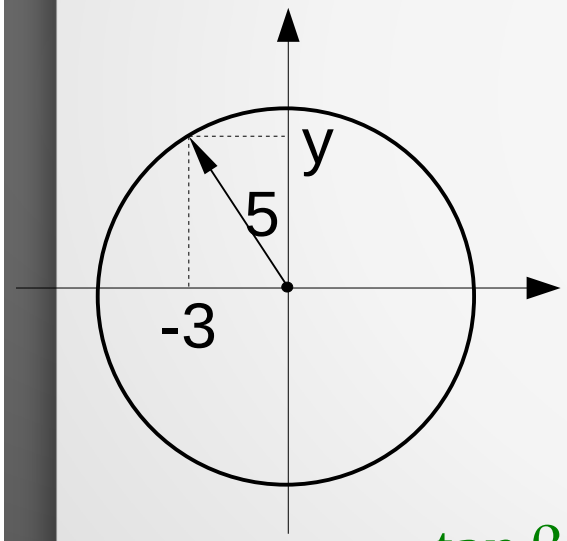
$sec\,\theta = \frac{r}{x}$

-	+
-	+

Trigonometric Functions of Any Angle

Examples:

3) Given $\cos\beta = -\frac{3}{5}$, β in quadrant II, find the exact value of each of the remaining trigonometric functions of β.



Solution:

(1) find the radius

$$5^2 = (-3)^2 + y^2$$

$$y^2 = 16 \qquad y = 4$$

(2) $\sin\beta = \frac{y}{r} = \frac{4}{5}$

$\tan\beta = \frac{y}{x} = -\frac{4}{3}$ $\quad \cot\beta = -\frac{3}{4}$

$\csc\beta = \frac{r}{y} = \frac{5}{4}$ $\quad \sec\beta = \frac{r}{x} = -\frac{5}{3}$

$\sin\theta = \frac{y}{r}$	+ + / - -
$\cos\theta = \frac{x}{r}$	- + / - +
$\tan\theta = \frac{y}{x}$	- + / + -
$\cot\theta = \frac{x}{y}$	
$\csc\theta = \frac{r}{y}$	+ + / - -
$\sec\theta = \frac{r}{x}$	- + / - +

Trigonometric Functions of Any Angle

Examples:

4) $\tan\theta = \frac{5}{12}$, $\cos\theta < 0$

find the exact value of each of the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r}$

+	+
-	-

$\cos\theta = \frac{x}{r}$

-	+
-	+

$\tan\theta = \frac{y}{x}$

-	+
+	-

$\cot\theta = \frac{x}{y}$

$csc\,\theta = \frac{r}{y}$

+	+
-	-

$sec\,\theta = \frac{r}{x}$

-	+
-	+

Trigonometric Functions of Any Angle

Examples:

4) $\tan\theta = \frac{5}{12}$, $\cos\theta < 0$

find the exact value of each of the remaining trigonometric functions of θ.

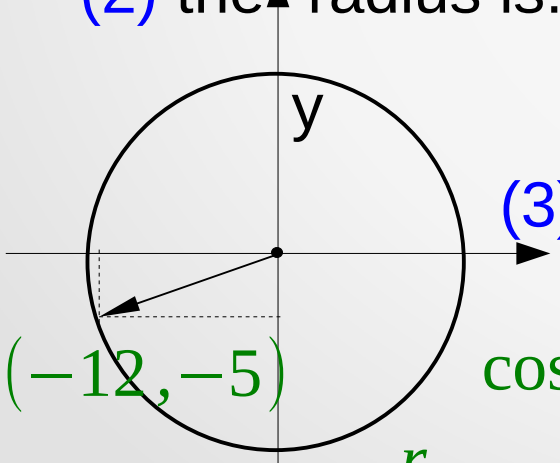
Solution:

(1) θ is in the III quadrant, point (-12,-5)

(2) the radius is:

$$(-5)^2+(-12)^2=r^2$$

$$r^2=169 \quad r=13$$



(3) $\sin\theta=\frac{y}{r}=\frac{-5}{13}$

$\cos\theta=\frac{x}{r}=\frac{-12}{13}$ $\quad \cot\theta=\frac{12}{5}$

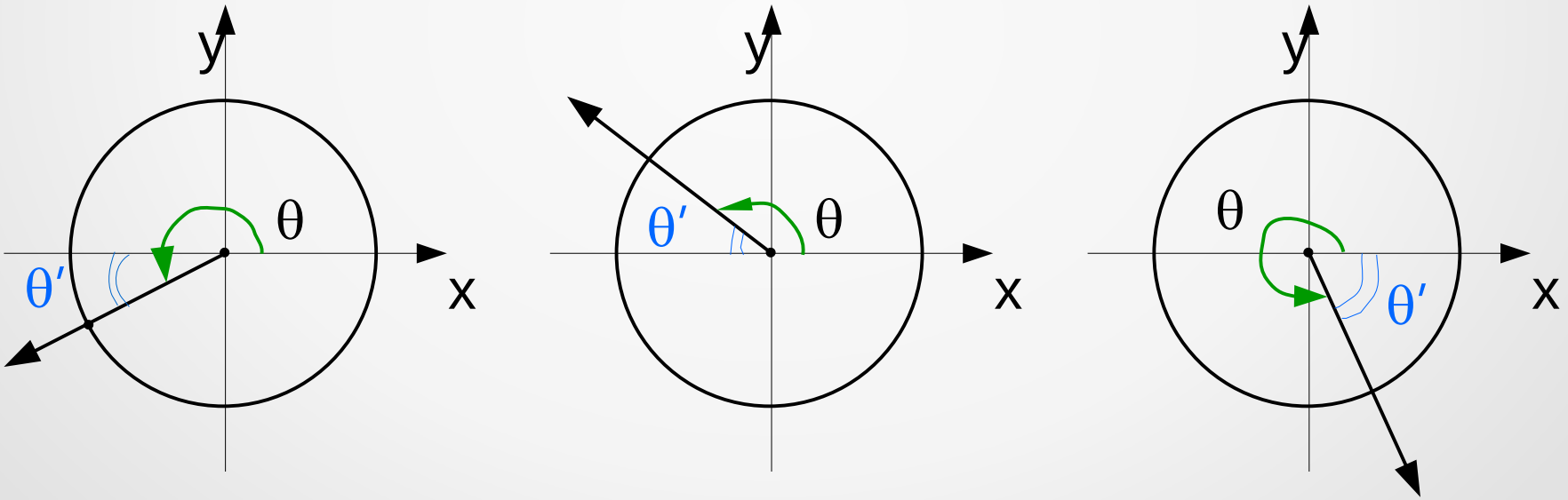
$\csc\theta=\frac{r}{y}=-\frac{13}{5}$ $\quad \sec\theta=\frac{r}{x}=-\frac{13}{12}$

$\sin\theta=\frac{y}{r}$	+ | + / - | -
$\cos\theta=\frac{x}{r}$	- | + / - | +
$\tan\theta=\frac{y}{x}$	- | + / + | -
$\cot\theta=\frac{x}{y}$	
$\csc\theta=\frac{r}{y}$	+ | + / - | -
$\sec\theta=\frac{r}{x}$	- | + / - | +

Trigonometric Functions of Any Angle

Reference Angles

If θ is a non-acute angle in standard position that lies in a quadrant, then its *reference angle* is a positive angle θ' formed by the terminal side of θ and x-axis.



Trigonometric Functions of Any Angle

Reference Angles

If θ is a non-acute angle in standard position that lies in a quadrant, then its *reference angle* is a positive angle θ' formed by the terminal side of θ and x-axis.

The values of trigonometric functions of a given angle θ are the same as the values of the trigonometric functions of the reference angle θ', except possibly for the sign.

Trigonometric Functions of Any Angle

Examples: use reference angles and trigonometric functions of special values table to evaluate

1) $\cos 135^{o}$

-	+
-	+

2) $\sin \frac{4\pi}{3}$

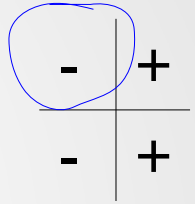
+	+
-	-

3) $\cot\left(-\frac{\pi}{4}\right)$

-	+
+	-

Trigonometric Functions of Any Angle

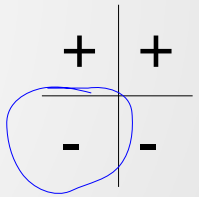
Examples: use reference angles and trigonometric functions of special values table to evaluate



1) $\cos 135^o$, θ is in the II quadrant

$\theta' = 180^o - 135^o = 45^o$,

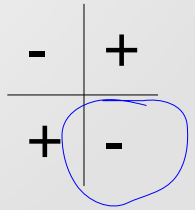
$$\cos 135^o = -\cos 45^o = -\frac{\sqrt{2}}{2}$$



2) $\sin \frac{4\pi}{3}$,θ is in the III quadrant

$$\theta' = \frac{4\pi}{3} - \pi = \frac{\pi}{3} \qquad \sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

3) $\cot\left(-\frac{\pi}{4}\right)$,θ is in the IV quadrant + cot is odd f.



$$\theta' = \frac{\pi}{4} \qquad \cot\left(-\frac{\pi}{4}\right) = -\cot\left(\frac{\pi}{4}\right) = -1$$

Trigonometric Functions of Any Angle

Examples: use reference angles and trigonometric functions of special values table to evaluate

4) $\sin \frac{14\pi}{3}$

+	+
-	-

5) $sec\, 240^{o}$

-	+
-	+

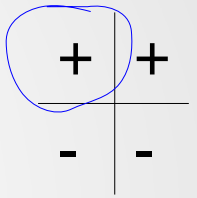
6) $\cos \frac{7\pi}{4}$

-	+
-	+

Trigonometric Functions of Any Angle

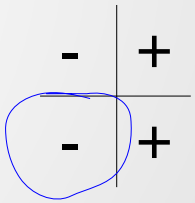
Examples: use reference angles and trigonometric functions of special values table to evaluate

4) $\sin\frac{14\pi}{3}$, θ is in the II quadrant



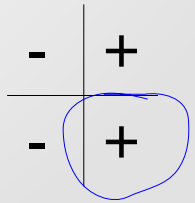
$\frac{14\pi}{3}=4\pi+\frac{2\pi}{3}$ $\quad \theta'=\pi\frac{2\pi}{3}=\frac{\pi}{3}$ $\quad \sin\frac{14\pi}{3}=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

5) $sec\,240^o$, θ is in the III quadrant



θ' = 240º – 180º = 60º, $sec\,240^o=-sec\,60^o=-2$

6) $\cos\frac{7\pi}{4}$, θ is in the IV quadrant



$\frac{7\pi}{4}=\pi+\frac{3\pi}{4}$ $\quad \theta'=2\pi-\frac{7\pi}{4}=\frac{\pi}{4}$ $\quad \cos\frac{7\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

Homework assignment

1) zyBooks: *review* Sections 5.4 and 5.2 (*Reference Angles*)

or

Textbook: *review* Section 4.4

2) We will have Quiz 16 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 16 (due date is in one week)