Exponential and logarithmic functions

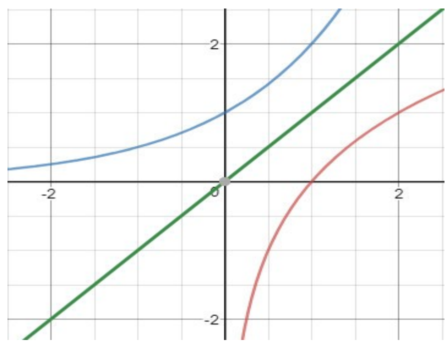
[Def] The *exponential function* with the base b is defined by **f(x) = b^x** or **y = b^x**, $b > 0, b \neq 1, x \in R$

Examples: $y=2^x, g(x)=3^{2x-5}, f(x)=\left(\frac{1}{3}\right)^{x+1}$

[Def] The inverse function of the exponential function with base b is called *logarithmic function* with base b: **f(x) = $\log_b x$** or **y = $\log_b x$**, $x > 0, b > 0, b \neq 1$

1) f(x) = 2^x

x	2^x
0	1
1	2
2	4
-1	½
-2	¼



2) g(x) = $\log_2 x$

x	$\log_2 x$
1	0
2	1
4	2
½	-1
¼	-2

Their graphs are reflections about the y=x line

both functions are one-to-one

Domain: (-∞,∞)
Range: (0,∞)
asymptote: horizontal, y = 0
special point: (0,1)
if b > 1, f(x) is increasing
if 0 < b < 1, f(x) is decreasing

Domain: (0,∞)
Range: (-∞,∞)
asymptote: vertical, x = 0
special point: (1,0)
if b > 1, g(x) is increasing
if 0 < b < 1, g(x) is decreasing

Changing from logarithmic form to exponential form:

$y=\log_b x$ is equivalent $b^y=x$

Examples: $2 = \log_3 9$ because $3^2 = 9$

$a^t = k$ is equivalent to $\log_a k = t$

Common logarithms: with the base 10: $\log_{10} 100{,}000 = \log 100{,}000 = 6$
Natural logarithms: with base e: $\log_e e^7 = \ln e^7 = 7$

Properties of logarithms:

(1) $\log_b b = 1$ *because* $b^1 = b$

(2) $\log_b 1 = 0$ *because* $b^0 = 1$

(3) $\log_b b^x = x$ *because* $b^x = b^x$

(4) $b^{\log_b x}=x$ *because* $\log_b x=\log_b x$

Common logarithms: $f(x) = \log_{10} x = \log x$
$\log 1 = 0$ $\log 10 = 1$ $\log 10^x = x$ $10^{\log x} = x$
Natural logarithms: $f(x) = \log_e x = \ln x$
$\ln 1 = 0$ $\ln e = 1$ $\ln e^x = x$ $e^{\ln x} = x$