Plan for today

Today we will discuss:

- trigonometric identities
- right triangle trigonometry

Trigonometric Functions: The Unit Circle

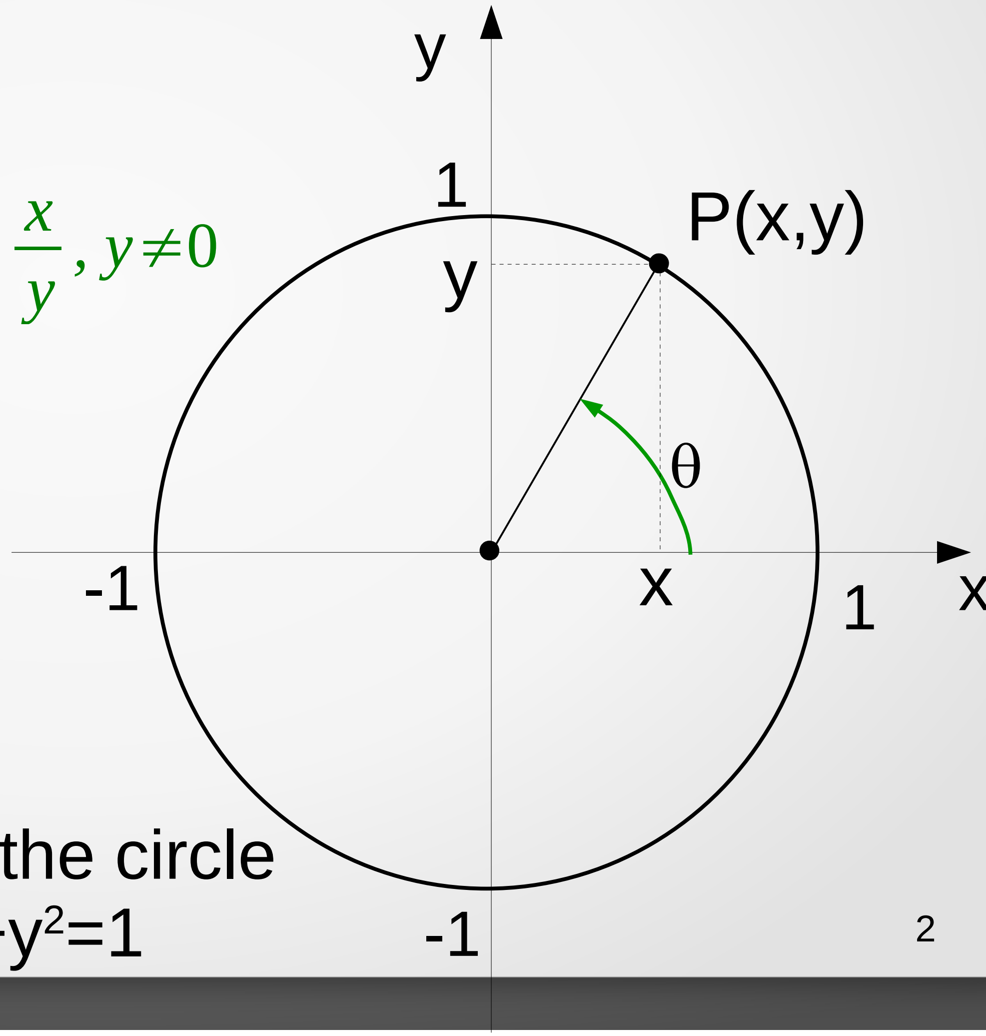
Recall the *unit circle* we used to define trigonometric functions:

$\sin\theta = y \quad \cos\theta = x$

$\tan\theta = \frac{y}{x}, x \neq 0 \qquad \cot\theta = \frac{x}{y}, y \neq 0$

$\csc\theta = \frac{1}{y}, y \neq 0$

$\sec\theta = \frac{1}{x}, x \neq 0$



every point on the circle satisfies to $x^2+y^2=1$

Trigonometric Functions: The Unit Circle

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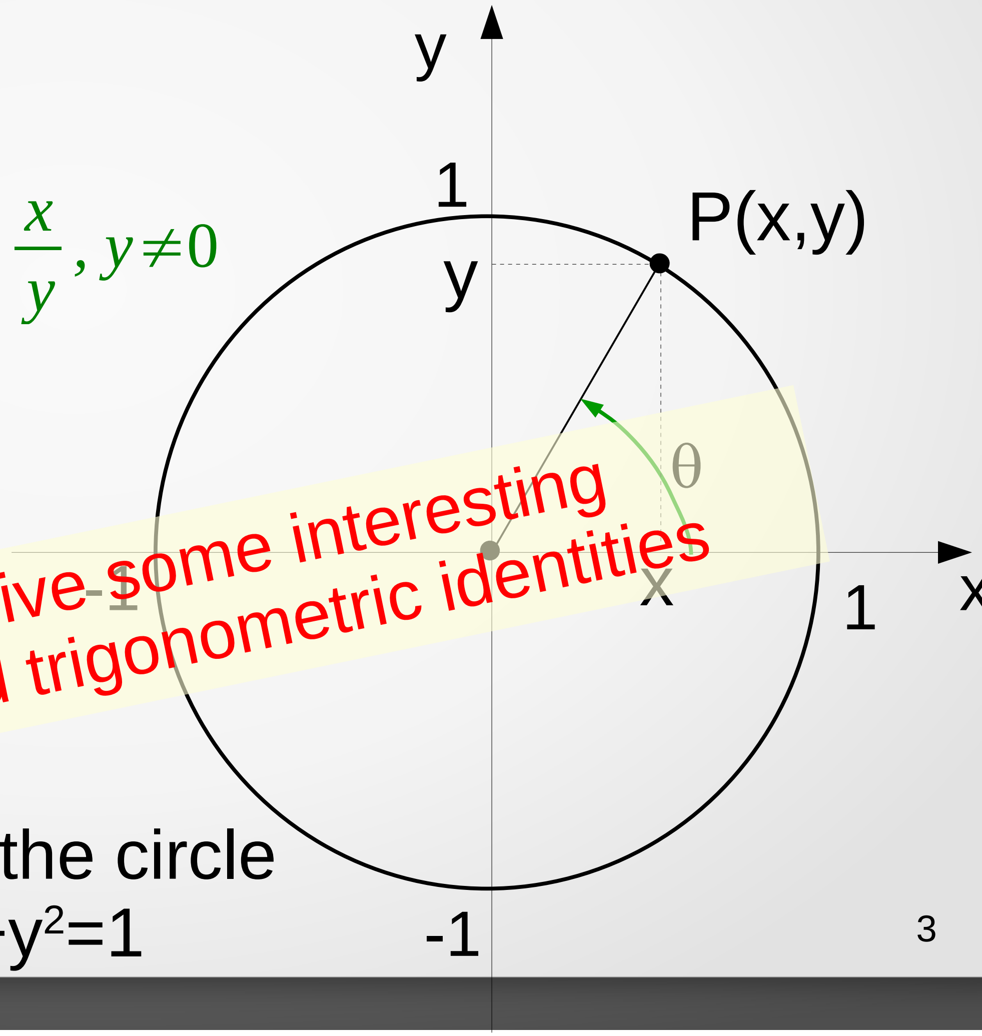


We can use them to derive some interesting identities that are called trigonometric identities

every point on the circle satisfies to $x^2+y^2=1$

Trigonometric Identities

Fundamental Identities

$$\sin t = \frac{1}{csc\, t} \qquad csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t} \qquad sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t} \qquad \cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1 \qquad 1 + \tan^2 t = sec^2 t \qquad 1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(1) given $\sin t = \frac{8}{17}, \cos t = \frac{15}{17}$

Find $\tan t, csc\, t, sec\, t$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

$$1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(1) given $\sin t = \frac{8}{17}, \cos t = \frac{15}{17}$

Find $\tan t, csc\, t, sec\, t$

$$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$$

$$csc\, t = \frac{1}{\sin t} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$$

$$sec\, t = \frac{1}{\cos t} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

$$1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(2) given $\sin t = \frac{1}{3}, \tan t = \frac{\sqrt{2}}{2}$

Find $csc\, t, \cot t$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

$$1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(2) given $\sin t = \frac{1}{3}, \tan t = \frac{\sqrt{2}}{2}$

Find $csc\, t, \cot t$

$$csc\, t = \frac{1}{\sin t} = \frac{1}{\frac{1}{3}} = 3$$

$$\cot t = \frac{1}{\tan t} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

$$1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(3) given $\sin t = \frac{6}{7}$ find $\cos t$, if $0 \le t < \frac{\pi}{2}$

$$\sin t = \frac{1}{\csc t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{\sec t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = \sec^2 t$$

$$1 + \cot^2 t = \csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(3) given $\sin t = \frac{6}{7}$ find $\cos t$, if $0 \le t < \frac{\pi}{2}$

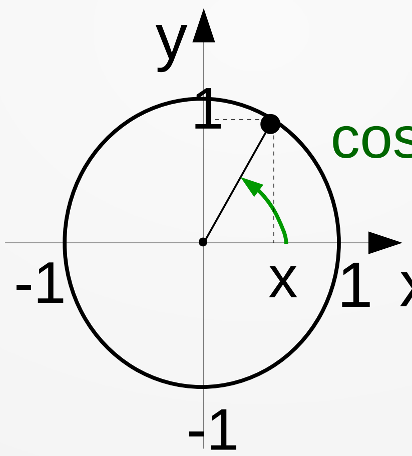
1) $\cos t > 0 \ for \ 0 \le t < \frac{\pi}{2}$

2) $\sin^2 t + \cos^2 t = 1$

$$\left(\frac{6}{7}\right)^2 + \cos^2 t = 1$$

$$\cos^2 t = 1 - \left(\frac{6}{7}\right)^2 = 1 - \frac{36}{49} = \frac{13}{49}$$

$$\cos t = \pm\sqrt{\frac{13}{49}} = \frac{\sqrt{13}}{7}$$



$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

$$1 + \cot^2 t = csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(4) without use of calculator find

$$csc^2\frac{\pi}{6}-\cot^2\frac{\pi}{6}$$

$$\sin t=\frac{1}{csc\,t}$$

$$csc\,t=\frac{1}{\sin t}$$

$$\cos t=\frac{1}{sec\,t}$$

$$sec\,t=\frac{1}{\cos t}$$

$$\tan t=\frac{\sin t}{\cos t}=\frac{1}{\cot t}$$

$$\cot t=\frac{\cos t}{\sin t}=\frac{1}{\tan t}$$

$$\sin^2 t+\cos^2 t=1$$

$$1+\tan^2 t=sec^2 t$$

$$1+\cot^2 t=csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(4) without use of calculator find

$$csc^2\frac{\pi}{6}-\cot^2\frac{\pi}{6}$$

$$1+\cot^2\frac{\pi}{6}=csc^2\frac{\pi}{6}$$

$$1=csc^2\frac{\pi}{6}-\cot^2\frac{\pi}{6}$$

$$csc^2\frac{\pi}{6}-\cot^2\frac{\pi}{6}=1$$

$$\sin t=\frac{1}{csc\, t}$$

$$csc\, t=\frac{1}{\sin t}$$

$$\cos t=\frac{1}{sec\, t}$$

$$sec\, t=\frac{1}{\cos t}$$

$$\tan t=\frac{\sin t}{\cos t}=\frac{1}{\cot t}$$

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$$\sin^2 t+\cos^2 t=1$$

$$1+\tan^2 t=sec^2 t$$

$$1+\cot^2 t=csc^2 t$$

Trigonometric Identities

Fundamental Identities

Examples:

(5) without use of calculator find

$$\cos 2.7\, sec\, 2.7$$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sin^2 t + \cos^2 t = 1$$

$$1 + \tan^2 t = sec^2 t$$

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Trigonometric Identities

Fundamental Identities

Examples:

(5) without use of calculator find

$$\cos 2.7\, sec\, 2.7$$

$$\cos 2.7\, sec\, 2.7 = \frac{1}{sec} 2.7 \times sec\, 2.7 = 1$$

$$\cos 2.7\, sec\, 2.7 = 1$$

$$\sin t = \frac{1}{csc\, t}$$

$$csc\, t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{sec\, t}$$

$$sec\, t = \frac{1}{\cos t}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$$

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$$\sin^2 t + \cos^2 t = 1$$

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Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

$$\sin\theta = y = \frac{y}{1} \qquad \cos\theta = x = \frac{x}{1}$$

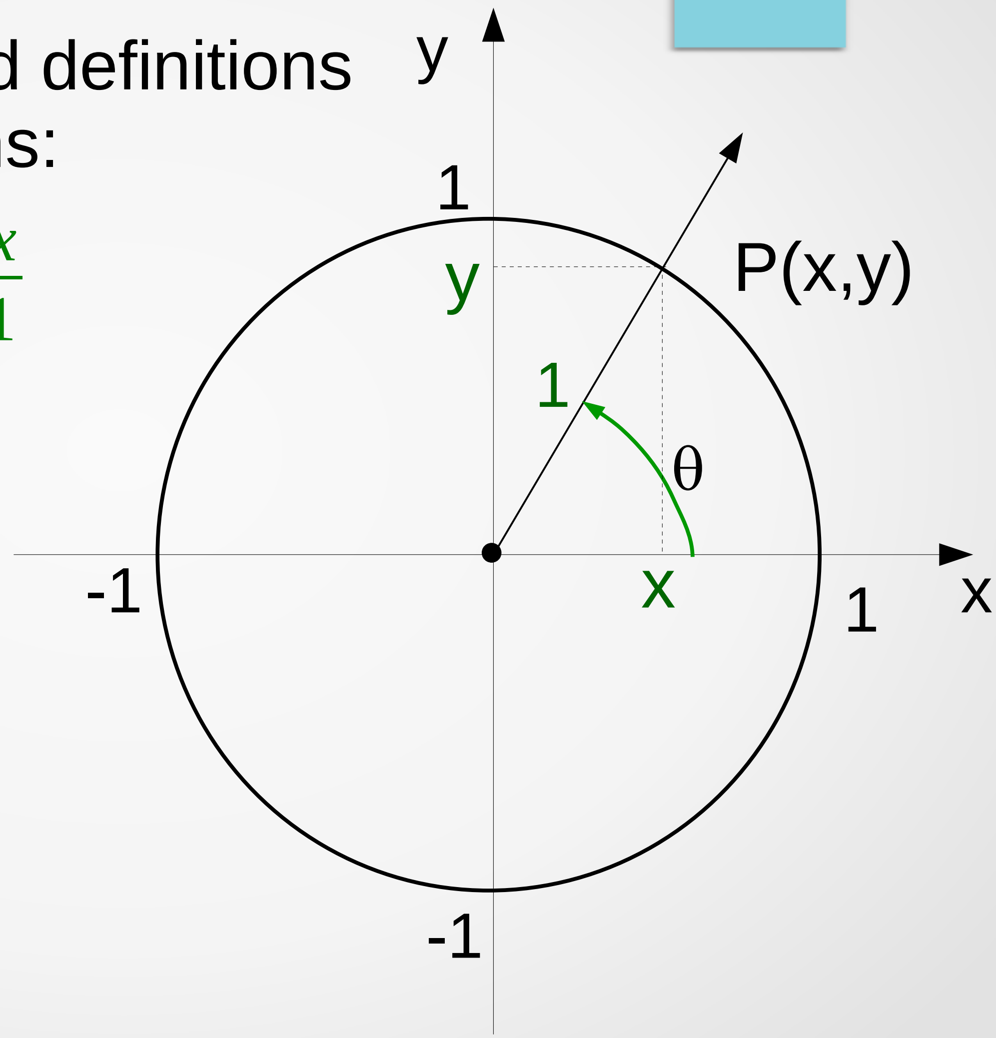
radius = 1

$$\tan\theta = \frac{y}{x}, x \neq 0$$

$$\cot\theta = \frac{x}{y}, y \neq 0$$

$$csc\,\theta = \frac{1}{y}, y \neq 0$$

$$sec\,\theta = \frac{1}{x}, x \neq 0$$



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radius = 1

$$\tan\theta = \frac{y}{x}, x \neq 0$$

$$\cot\theta = \frac{x}{y}, y \neq 0$$

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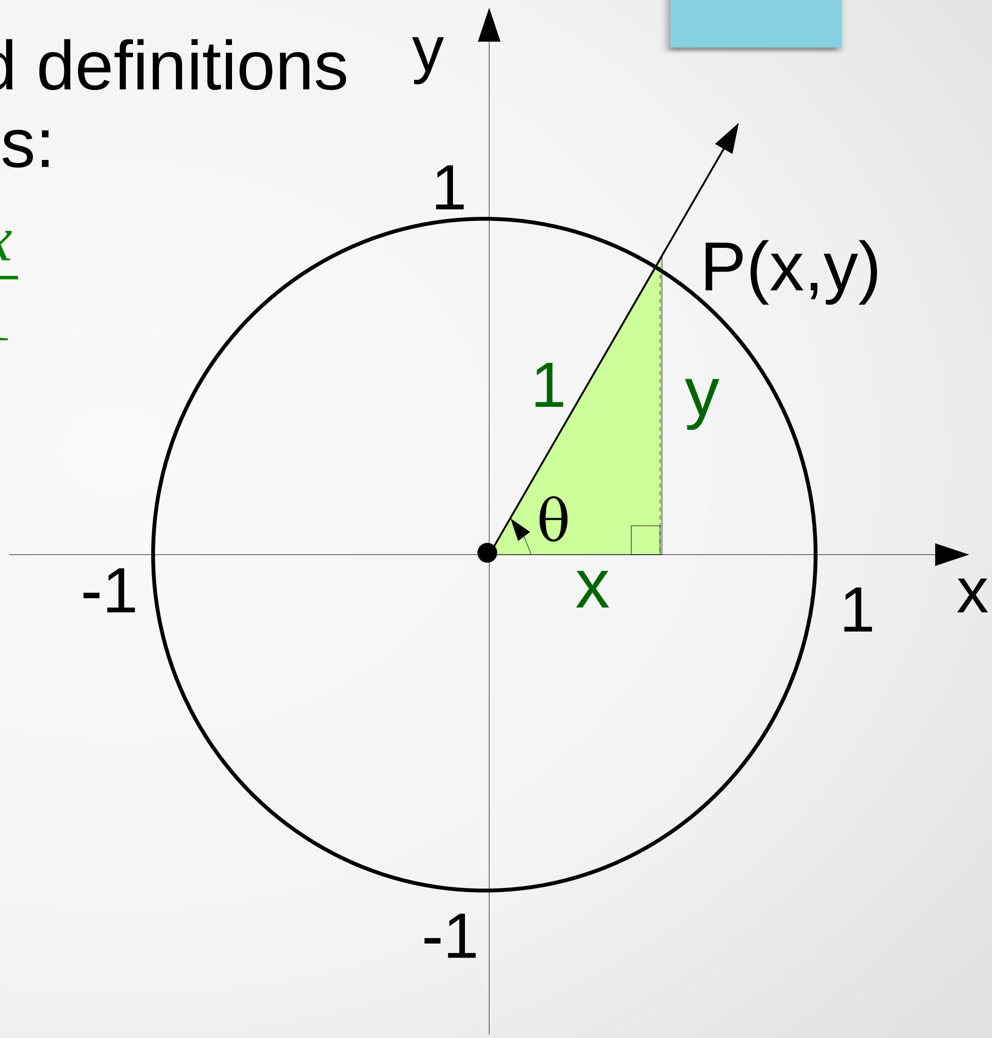


Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

$$\sin\theta = y = \frac{y}{1} \qquad \cos\theta = x = \frac{x}{1}$$

$$\tan\theta = \frac{y}{x}, x \neq 0$$

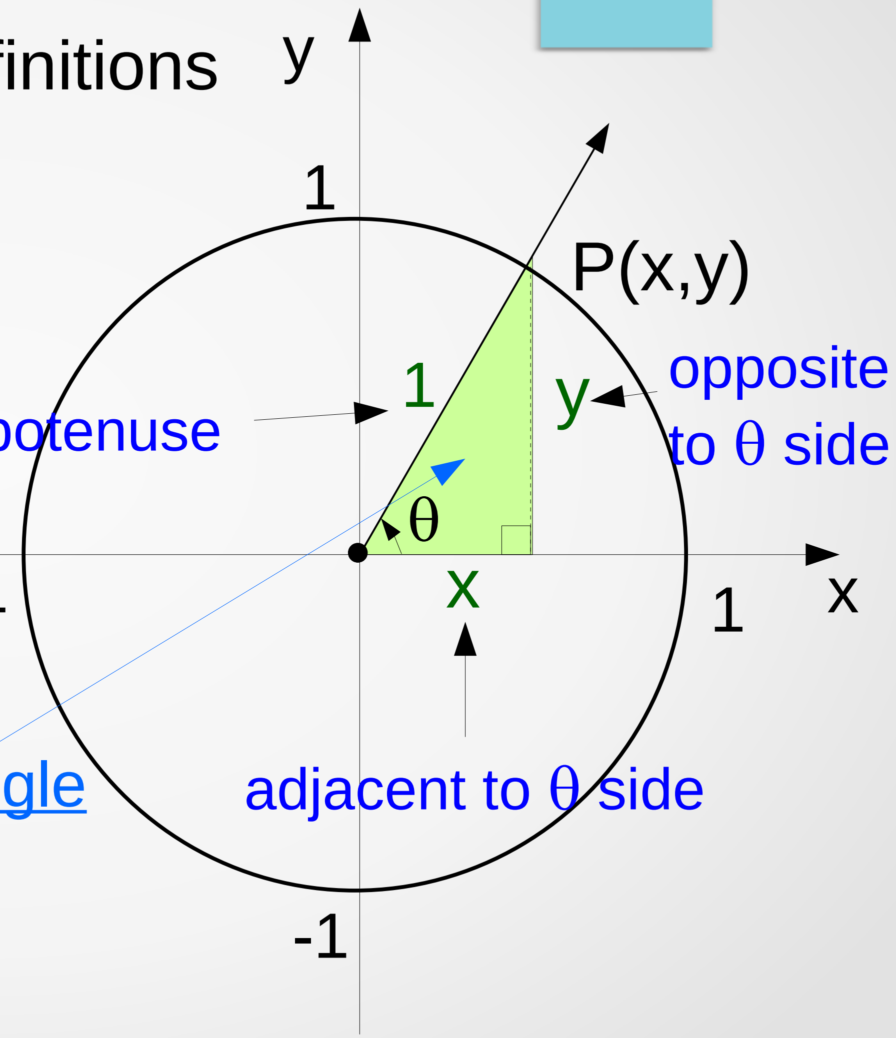
$$\cot\theta = \frac{x}{y}, y \neq 0$$

$$csc\,\theta = \frac{1}{y}, y \neq 0$$

$$sec\,\theta = \frac{1}{x}, x \neq 0$$

radius = 1 = hypotenuse

right triangle



Right Triangle Trigonometry

Recall the *unit circle* and definitions of trigonometric functions:

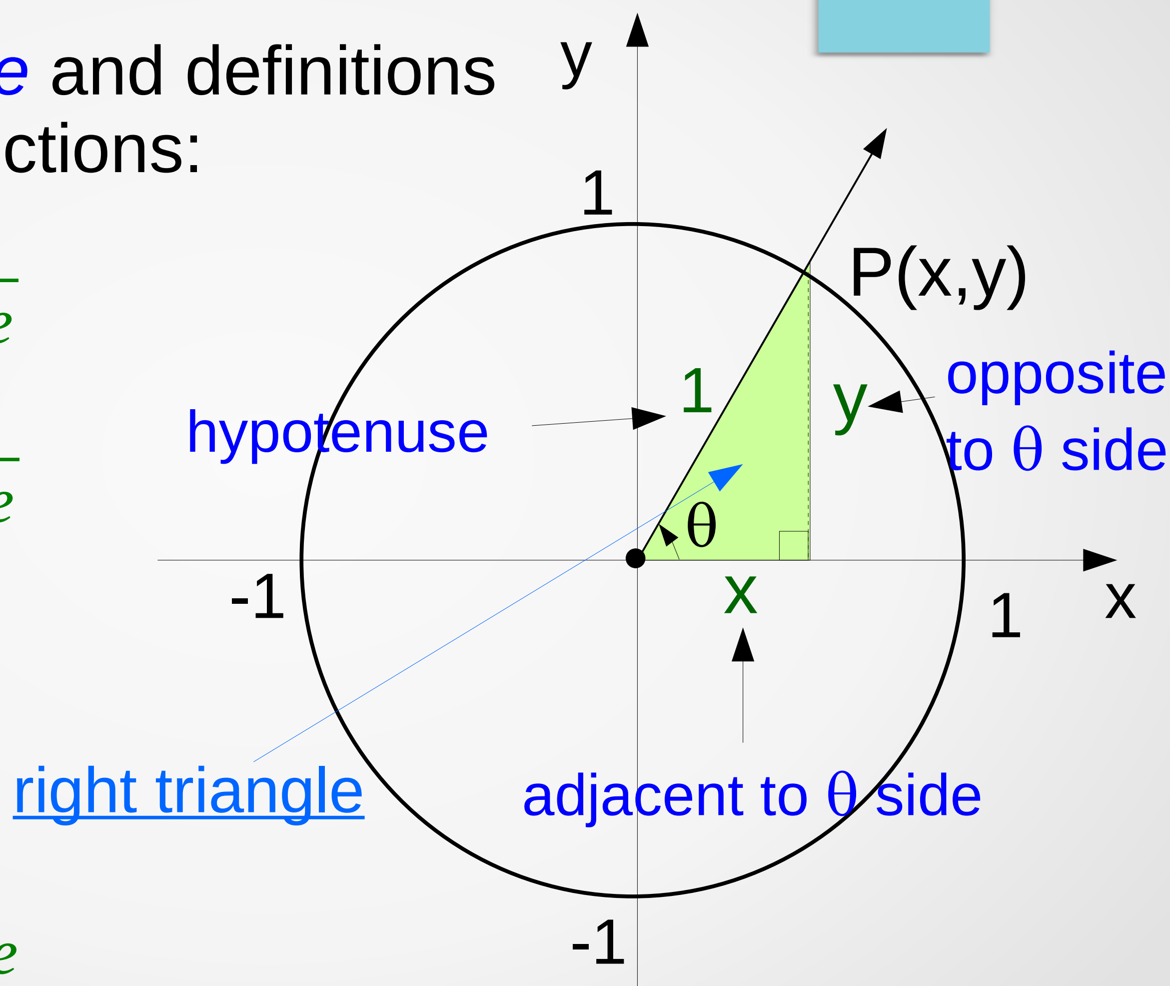
$$\sin\theta = \frac{y}{1} = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos\theta = \frac{x}{1} = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan\theta = \frac{y}{x} = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\cot\theta = \frac{x}{y} = \frac{\textit{adjacent}}{\textit{opposite}}$$

$$\csc\theta = \frac{1}{y} = \frac{\textit{hypotenuse}}{\textit{opposite}}$$

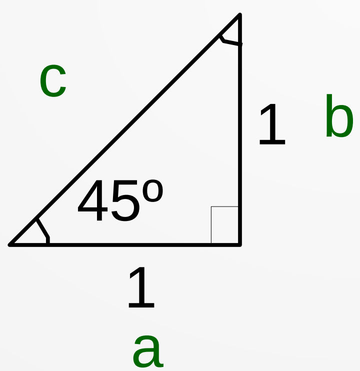


$$\sec\theta = \frac{1}{x} = \frac{\textit{hypotenuse}}{\textit{adjacent}}$$

Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:



Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

by *Pythagorean theorem*:

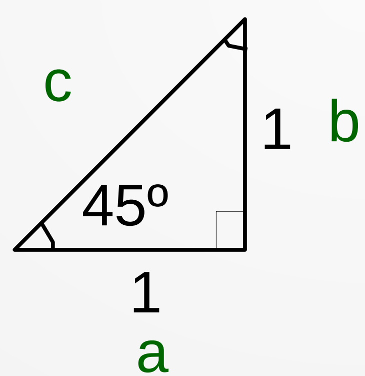
$a^2 + b^2 = c^2$

hence

$1^2 + 1^2 = c^2$

and finally

$c = \sqrt{2}$



Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

$\sin 45^o =$

by *Pythagorean theorem*:

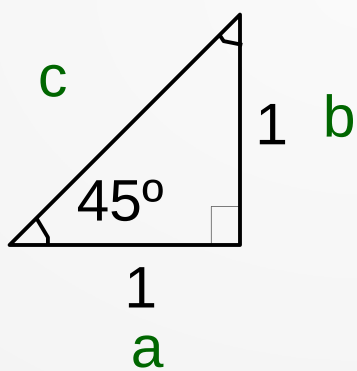
$a^2 + b^2 = c^2$

hence

$1^2 + 1^2 = c^2$

and finally

$c = \sqrt{2}$



$\cos 45^o =$

$\tan 45^o =$

$\cot 45^o =$

$csc\, 45^o =$

$sec\, 45^o =$

Trigonometric Functions

Function values for some special angles

Consider an *isosceles right triangle* with two equal sides of length 1:

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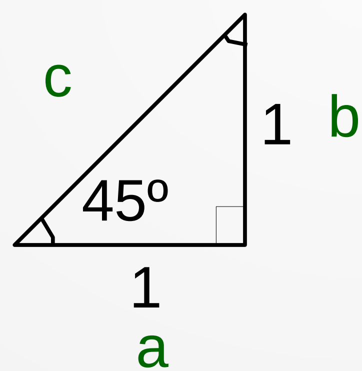
$a^2 + b^2 = c^2$

hence

$1^2 + 1^2 = c^2$

and finally

$c = \sqrt{2}$



$$\sin 45^o = \frac{opposite}{hypotenuse} = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^o = \frac{adjacent}{hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^o = \frac{opposite}{adjacent} = \frac{b}{a} = \frac{1}{1} = 1$$

$$\cot 45^o = \frac{adjacent}{opposite} = \frac{a}{b} = \frac{1}{1} = 1$$

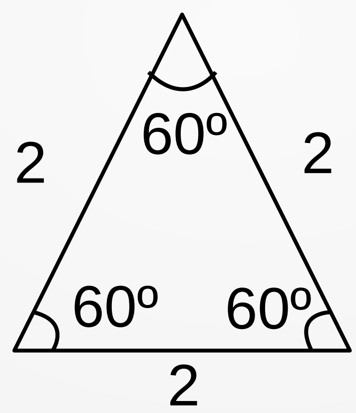
$$csc\, 45^o = \frac{hypotenuse}{opposite} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$sec\, 45^o = \frac{hypotenuse}{adjacent} = \sqrt{2}$$

Trigonometric Functions

Function values for some special angles

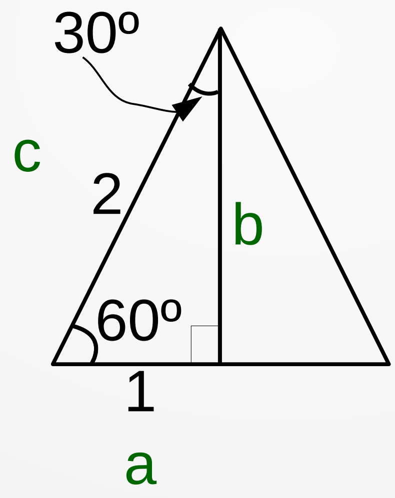
Consider an *equilateral triangle* with all equal sides of length 2:



Trigonometric Functions

Function values for some special angles

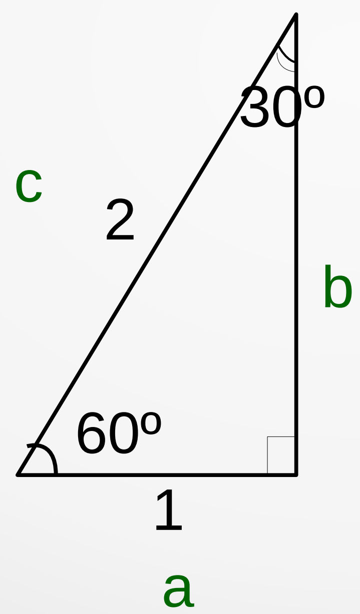
Consider an *equilateral triangle* with all equal sides of length 2:



Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:



Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

by *Pythagorean theorem*:

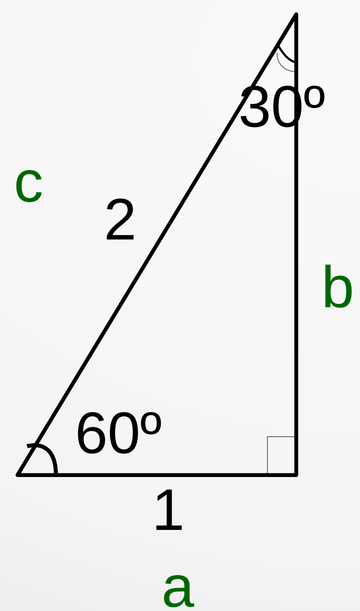
$a^2 + b^2 = c^2$

hence

$1^2 + b^2 = 2^2$

and finally

$b = \sqrt{3}$



Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

$\sin 30^o =$

by *Pythagorean theorem*:

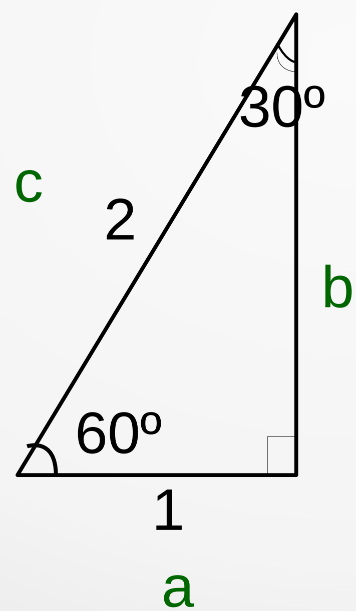
$a^2 + b^2 = c^2$

hence

$1^2 + b^2 = 2^2$

and finally

$b = \sqrt{3}$



$\sin 60^o =$

$\cos 30^o =$

$\cos 60^o =$

...

Trigonometric Functions

Function values for some special angles

Consider an *equilateral triangle* with all equal sides of length 2:

by *Pythagorean theorem*:

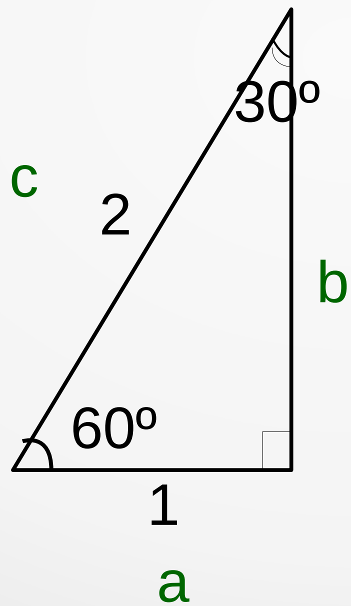
$a^2 + b^2 = c^2$

hence

$1^2 + b^2 = 2^2$

and finally

$b = \sqrt{3}$



$$\sin 30^o = \frac{opposite}{hypotenuse} = \frac{a}{c} = \frac{1}{2}$$

$$\sin 60^o = \frac{opposite}{hypotenuse} = \frac{b}{c} = \frac{\sqrt{3}}{2}$$

$$\cos 30^o = \frac{adjacent}{hypotenuse} = \frac{b}{c} = \frac{\sqrt{3}}{2}$$

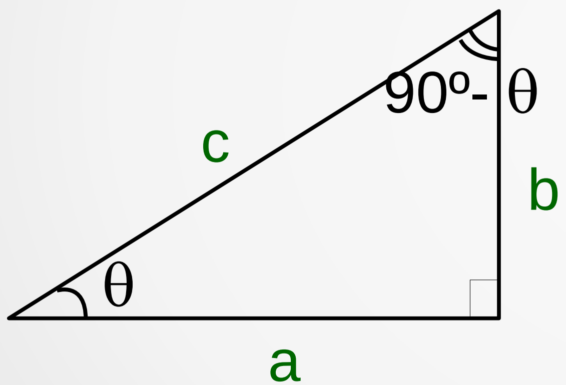
$$\cos 60^o = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \frac{1}{2}$$

...

Trigonometric Functions

Trigonometric Functions and Complements

Two positive angles are *complements* is their sum is 90^o or $\frac{\pi}{2}$



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c} = \cos(90^o - \theta)$$

$$\cos(90^o - \theta) = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c} = \sin(90^o - \theta)$$

$$\sin(90^o - \theta) = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

Similarly for tan, cot, sec, and csc

Trigonometric Functions

Trigonometric Functions and Complements

$$\sin\theta = \cos\left(90^o - \theta\right)$$

$$\cos\theta = \sin\left(90^o - \theta\right)$$

$$\tan\theta = \cot\left(90^o - \theta\right)$$

$$\cot\theta = \tan\left(90^o - \theta\right)$$

$$csc\,\theta = sec\left(90^o - \theta\right)$$

$$sec\,\theta = csc\left(90^o - \theta\right)$$

Trigonometric Functions

Trigonometric Functions and Complements

For example,

- If we are given sin 30º = ½, and asked to find cos 60º, then

cos 60º = cos (90º-30º) = sin 30º = ½

- If we know that tan 17º ≈ 0.3057 and asked to find cot 73º, then

cot 73º = cot (90-17º) = tan 17º ≈ 0.3057

$$\sin\theta = \cos(90^o - \theta)$$

$$\cos\theta = \sin(90^o - \theta)$$

$$\tan\theta = \cot(90^o - \theta)$$

$$\cot\theta = \tan(90^o - \theta)$$

$$csc\,\theta = sec(90^o - \theta)$$

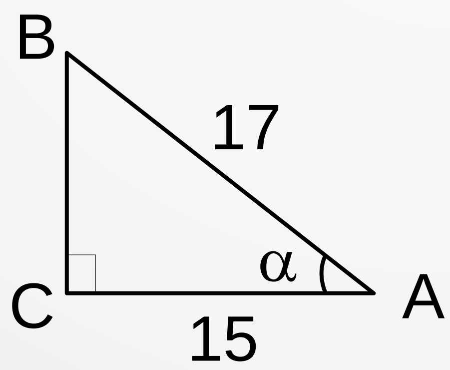
$$sec\,\theta = csc(90^o - \theta)$$

Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(1) Use *Pythagorean Theorem* to find the length of the missing side, then find the value of each of the six trigonometric functions.

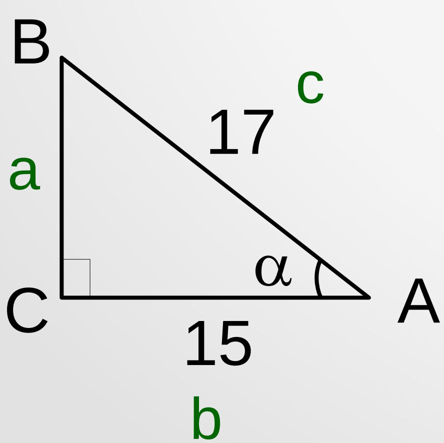


Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(1) Use *Pythagorean Theorem* to find the length of the missing side, then find the value of each of the six trigonometric functions.



$$a^2+b^2=c^2$$

$$a^2+15^2=17^2$$

$$a=\sqrt{289-225}=\sqrt{64}=8$$

$$\sin\alpha=\frac{opposite}{hypotenuse}=\frac{8}{17}$$

$$\cos\alpha=\frac{adjacent}{hypotenuse}=\frac{15}{17}$$

$$\tan\theta=\frac{opposite}{adjacent}=\frac{8}{15}$$

$$\cot\theta=\frac{adjacent}{opposite}=\frac{15}{8}$$

$$\csc\theta=\frac{hypotenuse}{opposite}=\frac{17}{8}$$

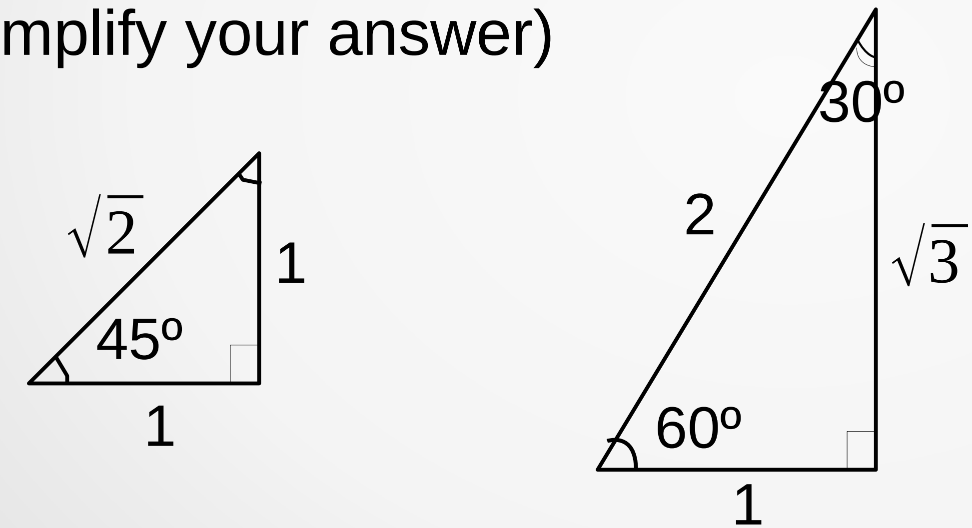
$$\sec\theta=\frac{hypotenuse}{adjacent}=\frac{17}{15}$$

Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(2) Use the given triangles to evaluate the given expressions (simplify your answer)



a) $\sin 30^o$ b) $\cot \frac{\pi}{4}$ c) $\cos \frac{\pi}{6} \, sec \frac{\pi}{6} - \cot \frac{\pi}{6}$

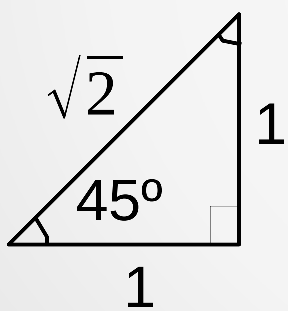
d) $2 \tan 60^0 + \cos 45^o \tan 30^o$

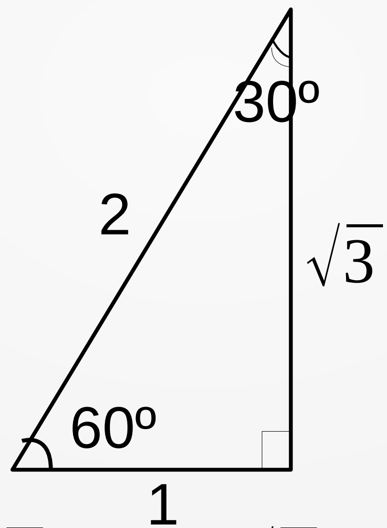
Trigonometric Functions

Trigonometric Functions and Complements

Examples:

(2) Use the given triangles to evaluate the given expressions (simplify your answer)





a) $\sin 30^o = \frac{1}{2}$

b) $\cot \frac{\pi}{4} = \frac{1}{1} = 1$

c) $\cos \frac{\pi}{6}\, sec \frac{\pi}{6} - \cot \frac{\pi}{6} = \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{1} = 1 - \sqrt{3}$

d) $2\tan 60^0 + \cos 45^o \tan 30^o = 2 \times \frac{\sqrt{3}}{1} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} = 2\sqrt{3} + \frac{1}{\sqrt{6}} = 2\sqrt{3} + \frac{\sqrt{6}}{6}$

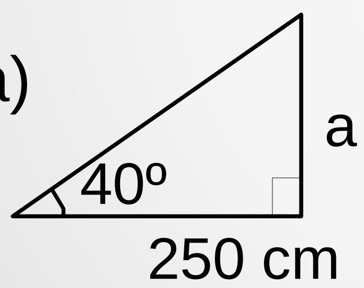
Trigonometric Functions

Trigonometric Functions and Complements

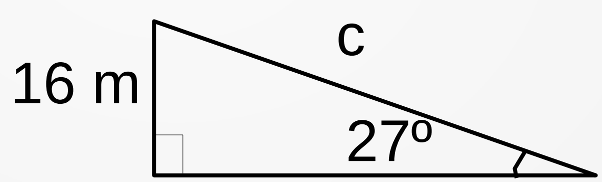
Examples:

(3) Find the length of the side marked with a lower case letter. Round the answer to the nearest whole number (use calculator)

a)



b)



Trigonometric Functions

Trigonometric Functions and Complements

Examples:

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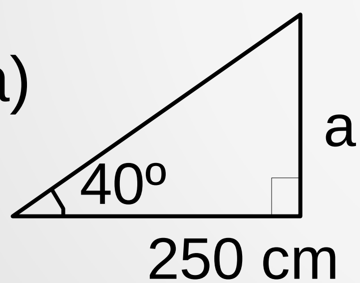
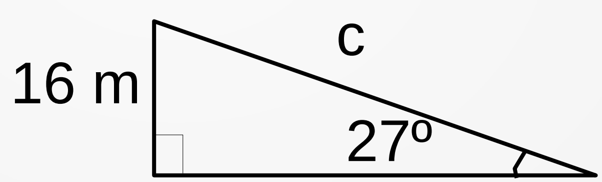
a) 

b) 

$$\tan 40^o = \frac{a}{250}$$

$$a = 250 \tan 40^o \approx 210\, cm$$

$$\sin 27^o = \frac{16}{c}$$

$$c = \frac{16}{\sin 27^o} \approx 35\, m$$

Homework assignment

1) zyBooks: *review* Sections 5.3 (skip the *Using reference angles to evaluate tangent, secant, cosecant, and cotangent*) and 5.4

or

Textbook: *review* Sections 4.2 and 4.3

2) We will have Quiz 15 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 15 (due date is in one week)