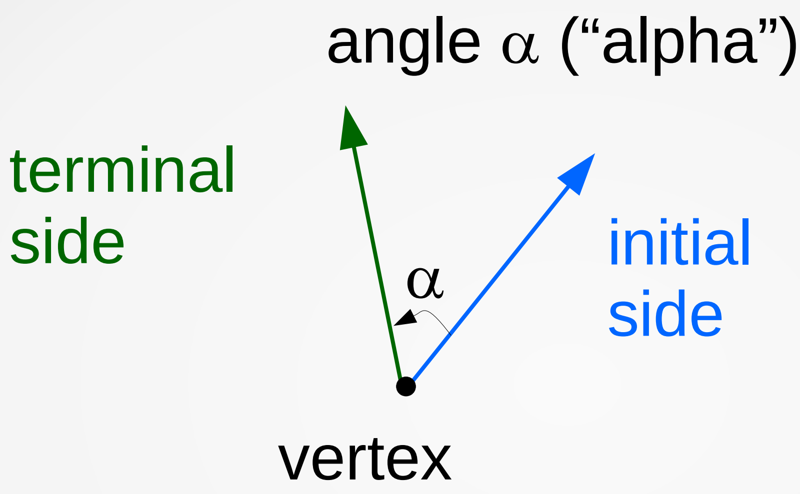
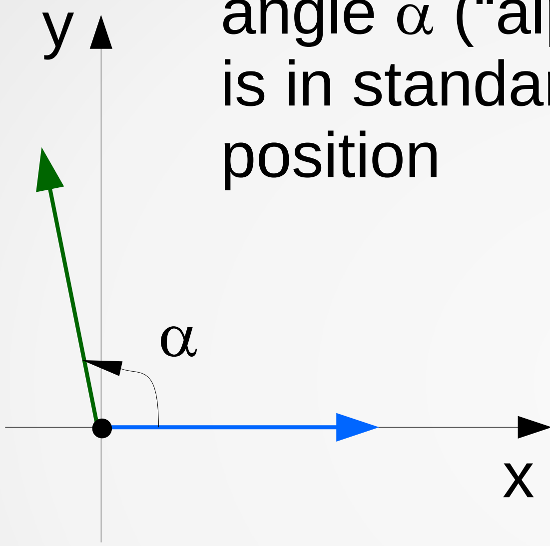
Topics to be covered today

- angles and radian measures
- trigonometric functions: the unit circle

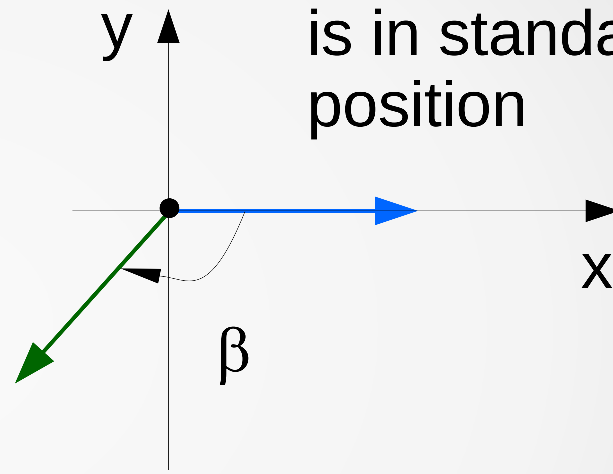
Angles and Radian Measures



Angles and Radian Measures



angle α (“alpha”) is in standard position



angle β (“beta”) is in standard position

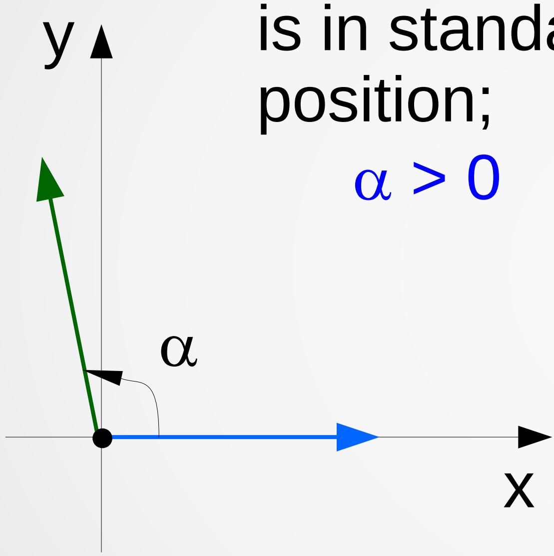
An *angle is in standard position* is when

- its vertex is at the origin of a rectangular coordinate system and
- its initial side lies along the positive x-axis

4.1 Angles and Radian Measures

angle α (“alpha”)
is in standard
position;

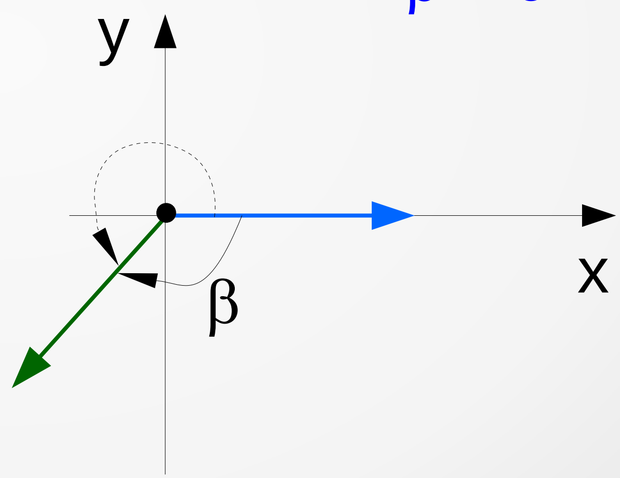
$\alpha > 0$



anticlockwise direction

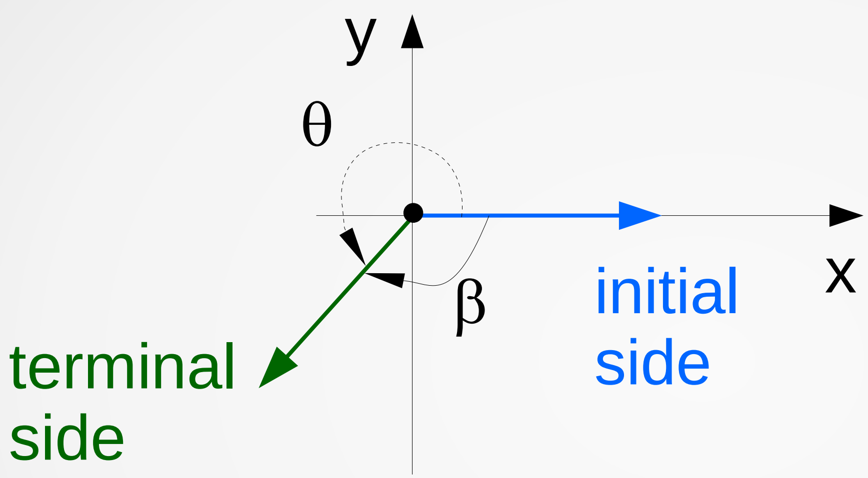
angle β (“beta”)
is in standard
position;

$\beta < 0$



clockwise direction

Angles and Radian Measures



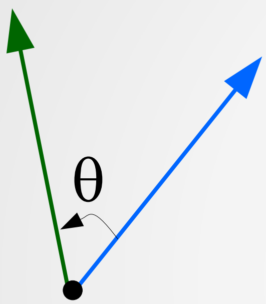
angles β ("beta") and θ ("theta") are *coterminal angles*

Two angles with the same initial and terminal sides but possibly different rotations are called *coterminal angles*.

We will see more of the later today.

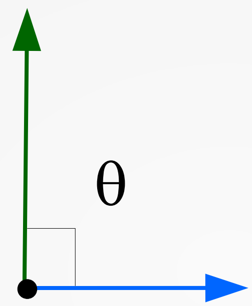
Angles and Radian Measures

Types of angles



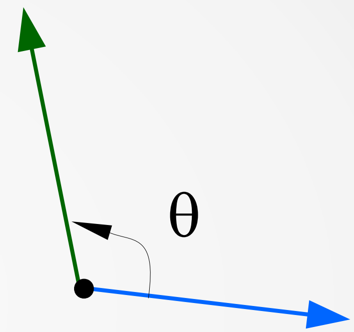
acute angle

$0 < \theta < 90^o$



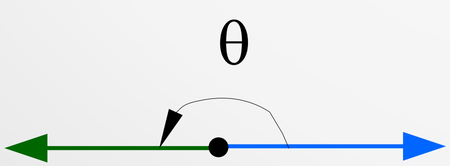
right angle

$\theta = 90^o$



obtuse angle

$90^o < \theta < 180^o$



straight angle

$\theta = 180^o$

full circle (one revolution) :

360^o

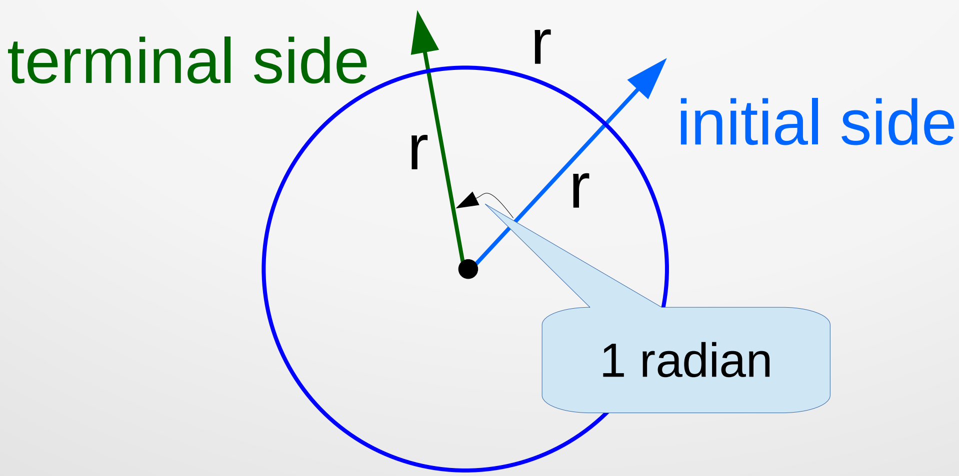
Angles and Radian Measures

Radians

- another way to measure angles

[Def] a *central angle* is an angle whose vertex is at the center of the circle.

[Def] *one radian* is the measure of the central angle of a circle that intercepts an *arc* equal in length to the radius of the circle.

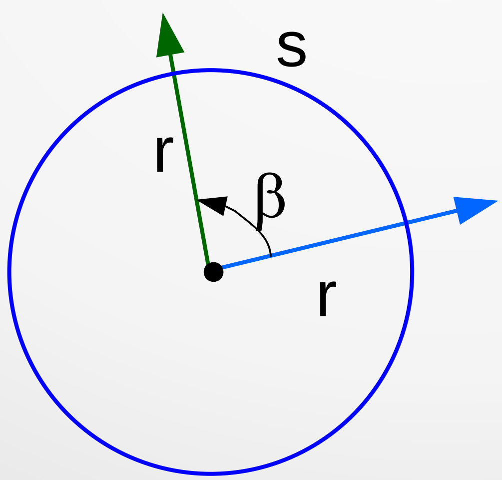


Angles and Radian Measures

Radians

The *radian measure* of any central angle

$$\beta = \frac{\text{length of the intercepted arc}}{\text{radius}} = \frac{s}{r} \quad \text{(rad)}$$



Angles and Radian Measures

Radians

The *radian measure* of any central angle

$$\beta = \frac{\text{length of the intercepted arc}}{\text{radius}} = \frac{s}{r} \quad \textit{(rad)}$$

Example: Find the radian measure of a *central angle*, if the radius of the circle is 5 and the length of the intercepted arc is 12.

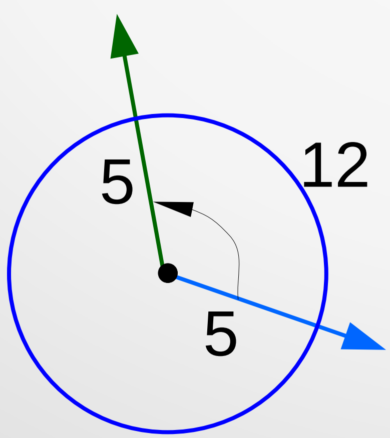
4.1 Angles and Radian Measures

Radians

The *radian measure* of any central angle

$$\beta = \frac{length\ of\ the\ intercepted\ arc}{radius} = \frac{s}{r} \quad \text{(rad)}$$

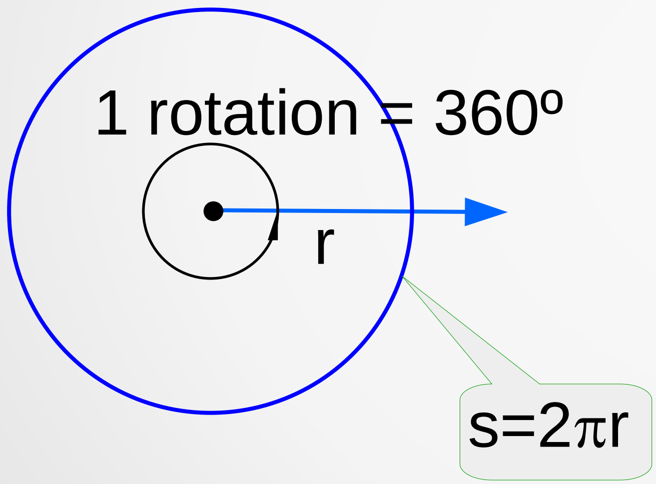
Example: Find the radian measure of a *central angle*, if the radius of the circle is 5 and the length of the intercepted arc is 12.



$$\beta = \frac{12}{5} = 2.4\,rad$$

Angles and Radian Measures

Relationship between radians and degrees



$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \ \textit{(rad)}$$

360º = 2π radians

or

180º = π radians

Angles and Radian Measures

Convertions between radians and degrees

$180^o = \pi$ radians
$x^o \quad = y$ radians

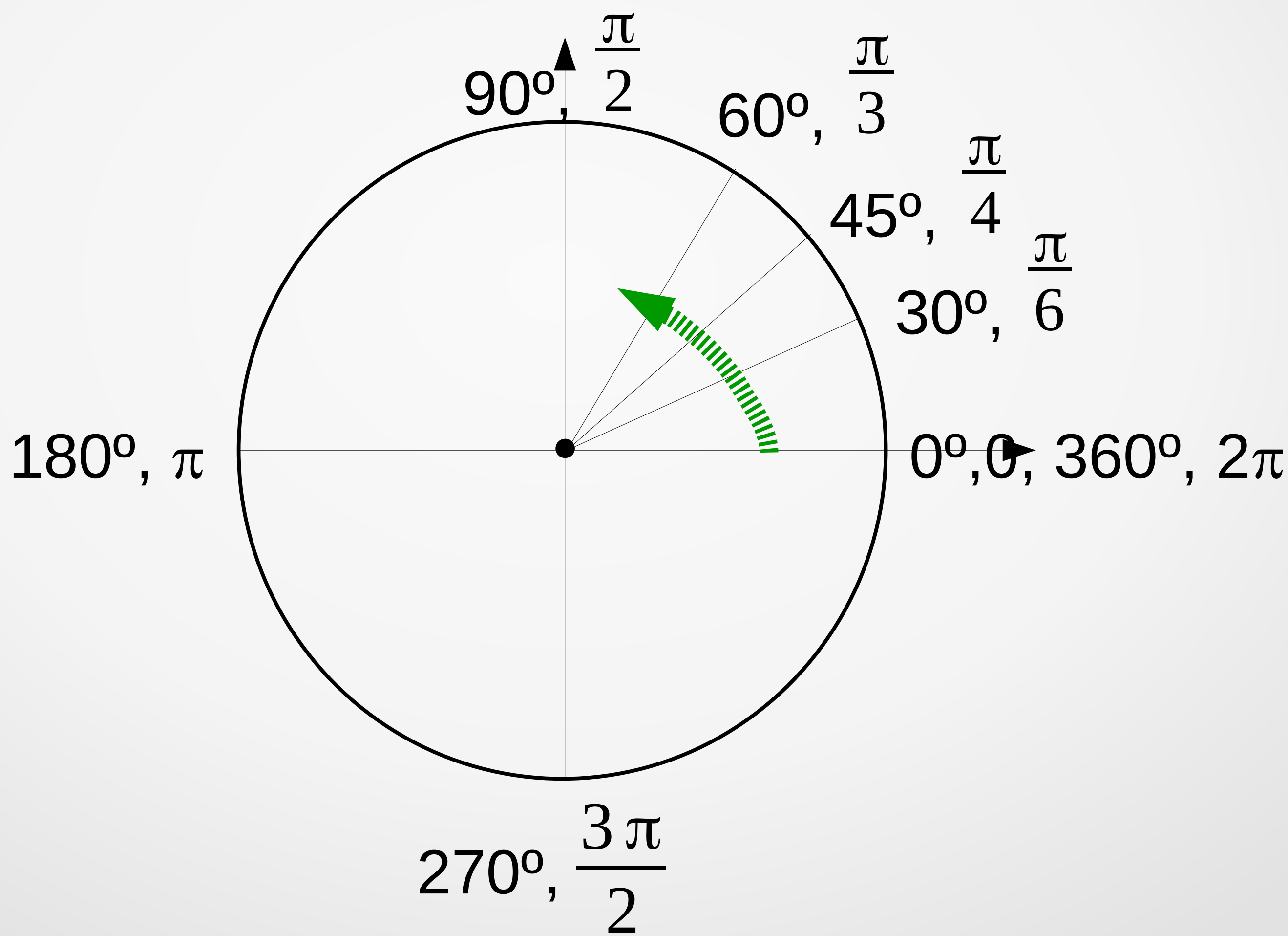
$$x = \frac{180^0 y}{\pi} \text{ degrees}$$

$$y = \frac{x \pi}{180^0} \text{ radians}$$

Note: when we measure angles in radians we prefer to keep π instead of approximating it.

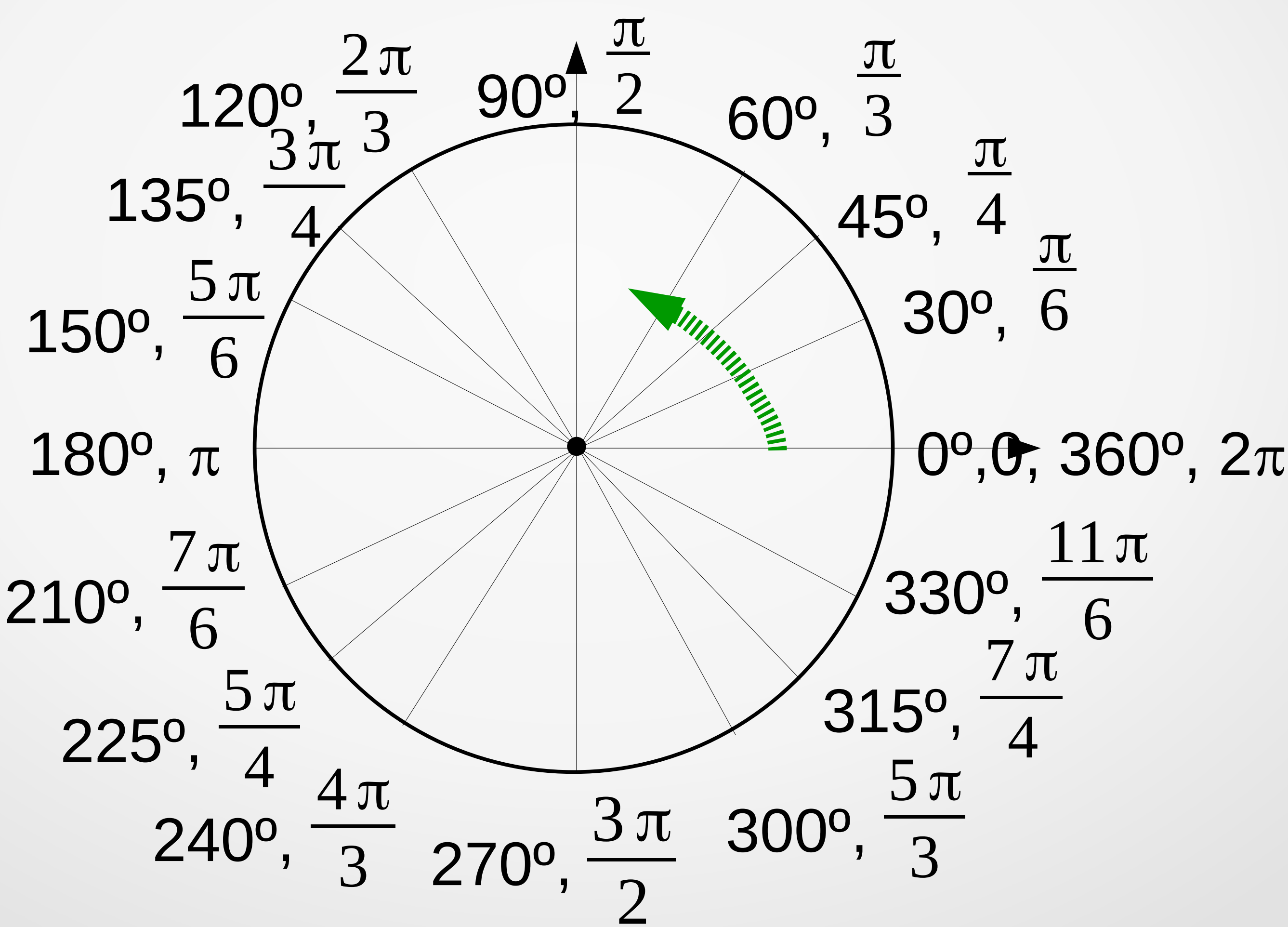
Angles and Radian Measures

Some common angles



Angles and Radian Measures

Some common angles



Angles and Radian Measures

Conversions between radians and degrees

$$x = \frac{180^0 y}{\pi} \quad \text{degrees}$$

$$y = \frac{x\pi}{180^0} \quad \text{radians}$$

Examples: convert from degrees to radians

(1) 45º

(2) 60º

(3) -240º

Angles and Radian Measures

Conversions between radians and degrees

$$x = \frac{180^0 y}{\pi} \text{ degrees}$$

$$y = \frac{x\pi}{180^0} \text{ radians}$$

Examples: convert from radians to degrees

(1) $\frac{3\pi}{2}$

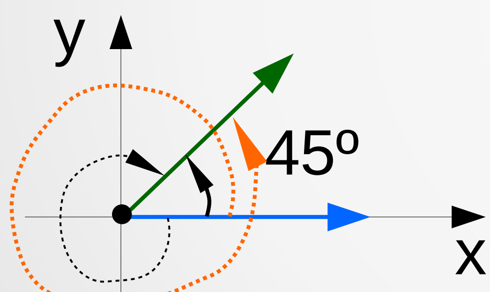
(2) $-\frac{\pi}{6}$

(3) $\frac{\pi}{17}$

Angles and Radian Measures

Coterminal angles

Two angles with the same initial and terminal sides but possibly different rotations are called *coterminal angles*.



$45^o - 360^o = -315^o$

$45^o + 360^o = 405^o$

$45^o + 2 \cdot 360^o = 765^o$

...

Consider angle $\theta = 45^o$

coterminal angles: $\theta \pm 360^o \cdot k,\ k \in Z^+$

$\theta \pm 2\pi k,\ k \in Z^+$

Angles and Radian Measures

Coterminal angles

coterminal angles: $\theta \pm 360^{o} \cdot k,\ k \in Z^{+}$

$\theta \pm 2\pi k,\ k \in Z^{+}$

Example: find a positive angle less than 360º (or 2π) what is *coterminal* with

(1) 395º

(2) -130º

(3) $\frac{15\pi}{4}$

Angles and Radian Measures

Coterminal angles

coterminal angles: $\theta \pm 360^{o} \cdot k,\ k \in Z^{+}$

$\theta \pm 2\pi k,\ k \in Z^{+}$

Example: find a positive angle less than 360º (or 2π) what is *coterminal* with

(1) 395º $\quad$ 395º-360º = 35º

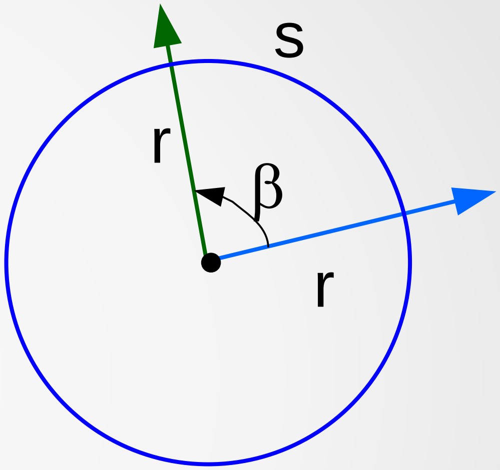
(2) -130º $\quad$ -130º + 360º = 230º

(3) $\frac{15\pi}{4}$ $\quad$ $\frac{15\pi}{4} > 2\pi$ $\quad$ hence $\frac{15\pi}{4} - 2\pi = \frac{7\pi}{4} < 2\pi$

$\frac{7\pi}{4}$

Angles and Radian Measures

Length of circular arc



We can use the formula

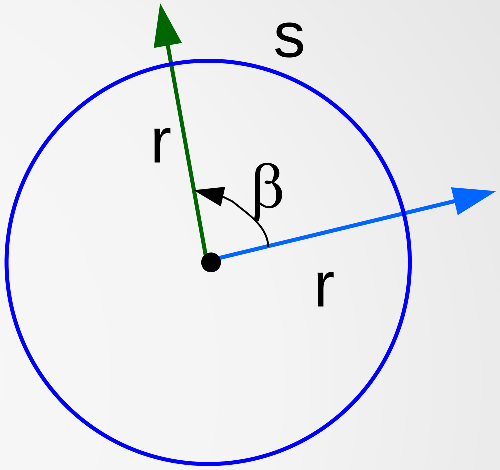
$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

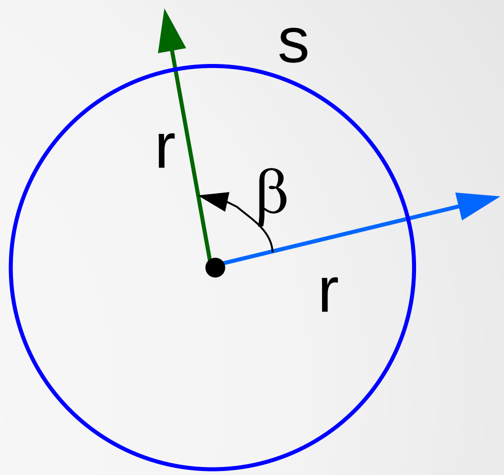
to find the length of the circular arc:

$$s = \beta r$$

Example: a circle has a radius 9". Find the length of a *circular arc* intercepted by a *central angle* of 120º

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

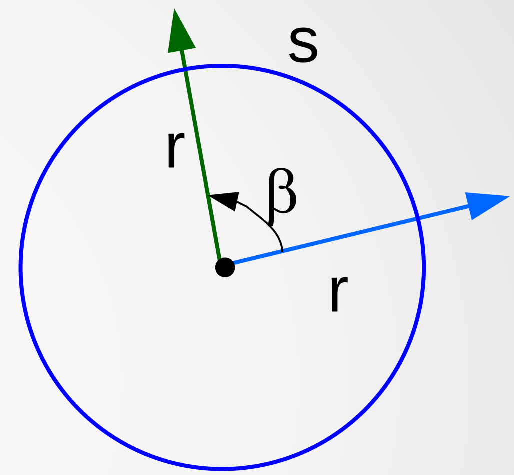
Example: a circle has a radius 9”. Find the length of a *circular arc* intercepted by a *central angle* of 120º

1) convert 120º to radians

$$y = \frac{x\pi}{180^0}$$

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

Example: a circle has a radius 9". Find the length of a *circular arc* intercepted by a *central angle* of 120º

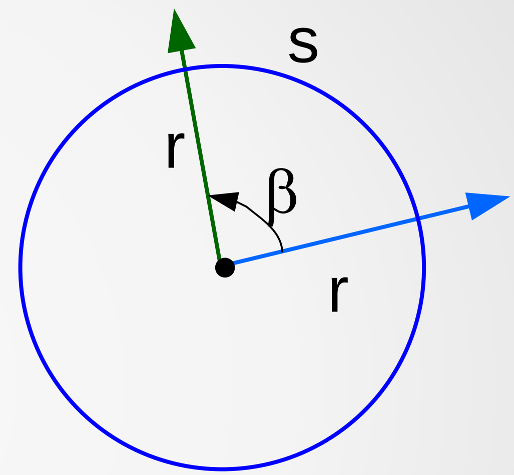
1) convert 120º to radians

$$\beta = \frac{120^0 \pi}{180^0} = \frac{2}{3}\pi$$

$$y = \frac{x\pi}{180^0}$$

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \ \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

Example: a circle has a radius 9”. Find the length of a *circular arc* intercepted by a *central angle* of 120º

$$y = \frac{x\pi}{180^0}$$

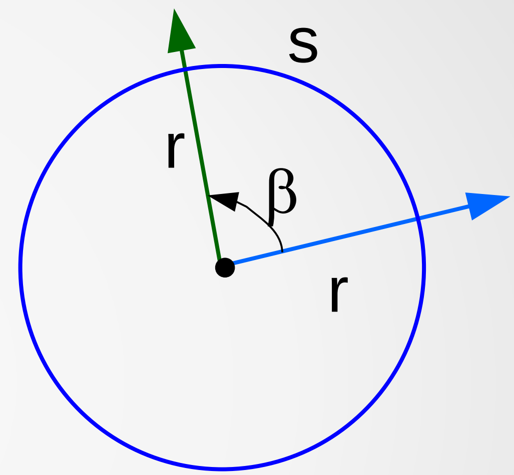
1) convert 120º to radians

$$\beta = \frac{120^0 \pi}{180^0} = \frac{2}{3}\pi$$

2) use the formula s = βr to find the length of the circular arc

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

Example: a circle has a radius 9”. Find the length of a *circular arc* intercepted by a *central angle* of 120º

$$y = \frac{x\pi}{180^0}$$

1) convert 120º to radians

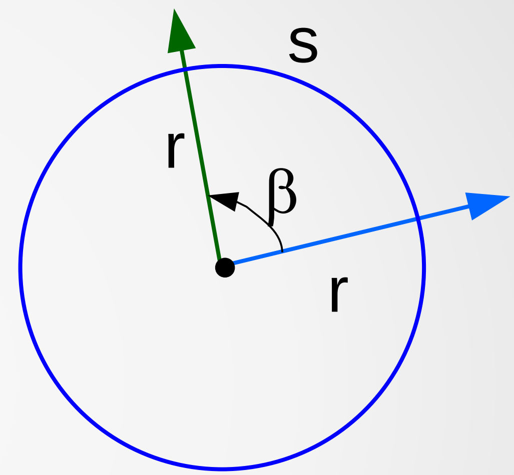
$$\beta = \frac{120^0\pi}{180^0} = \frac{2}{3}\pi$$

2) use the formula s = βr to find the length of the circular arc

$$s = \frac{2}{3}\pi \times 9 = 6\pi$$

Angles and Radian Measures

Length of circular arc



We can use the formula

$$\beta = \frac{s}{r} \quad \textit{(rad)}$$

to find the length of the circular arc:

$$s = \beta r$$

Example: a circle has a radius 9”. Find the length of a *circular arc* intercepted by a *central angle* of 120º

$$y = \frac{x\pi}{180^0}$$

1) convert 120º to radians

$$\beta = \frac{120^0 \pi}{180^0} = \frac{2}{3}\pi$$

2) use the formula s = βr to find the length of the circular arc

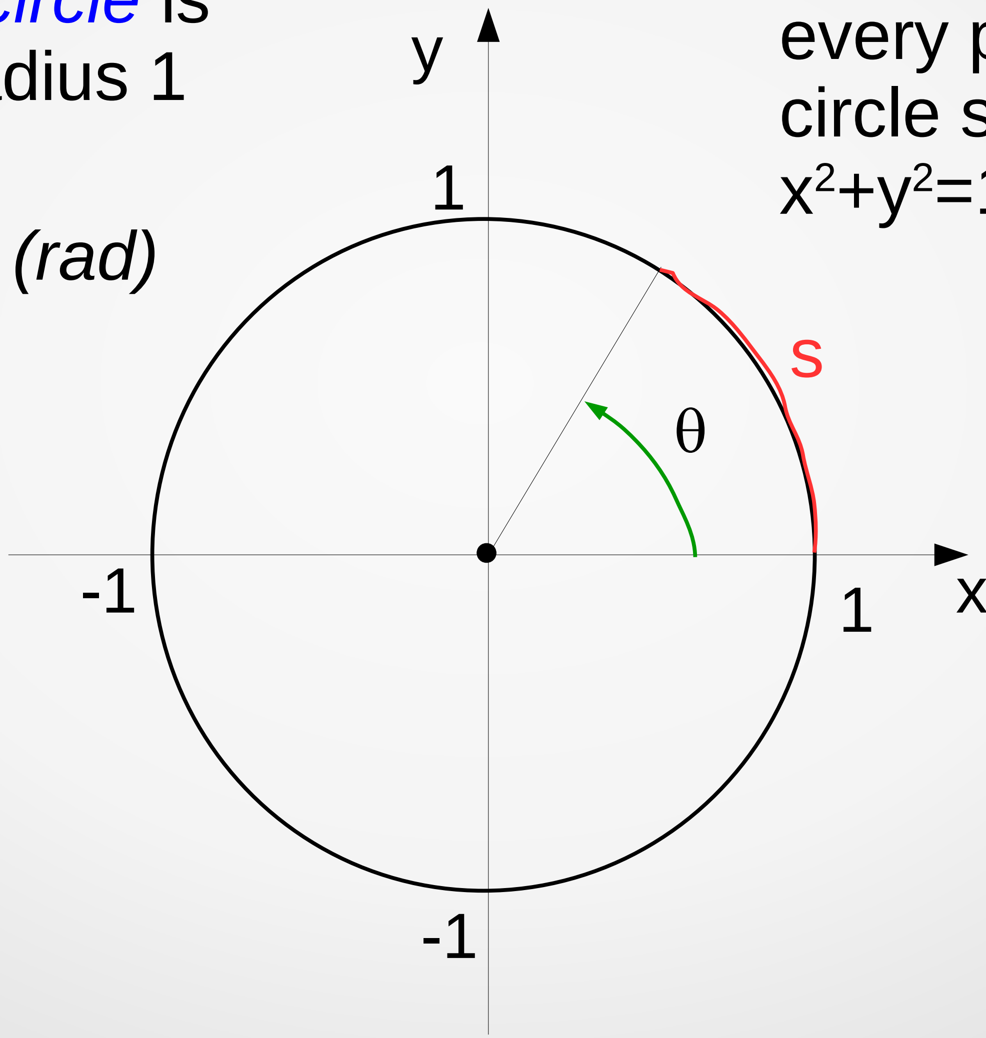
$$s = \frac{2}{3}\pi \times 9 = 6\pi$$

Answer: 6π inches ≈ 18.85 in

Trigonometric Functions: The Unit Circle

[Def] a *unit circle* is a circle of radius 1

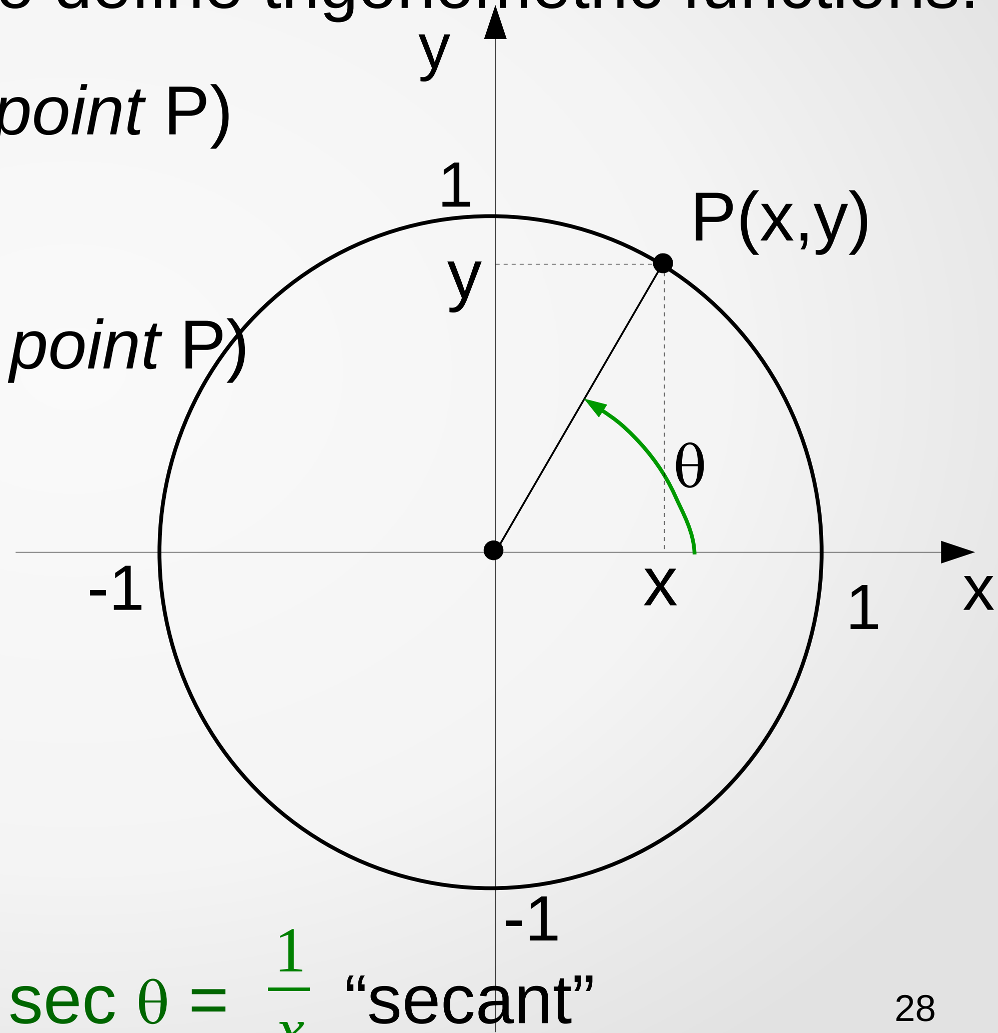
$$\theta = \frac{s}{r} = \frac{s}{1} = s \; (rad)$$



every point on the circle satisfies to $x^2+y^2=1$

Trigonometric Functions: The Unit Circle

We are using a *unit circle* to define trigonometric functions:



$\sin\theta = y$ (*y-coordinate of point* P)
"sine of theta"

$\cos\theta = x$ (*x-coordinate of point* P)
"cosine of theta"

$\tan\theta = \frac{y}{x}$
"tangent of theta"

$\cot\theta = \frac{x}{y}$
"cotangent of theta"

$\csc\theta = \frac{1}{y}$ "cosecant"

$\sec\theta = \frac{1}{x}$ "secant"

Trigonometric Functions: The Unit Circle

We are using a *unit circle* to define trigonometric functions:

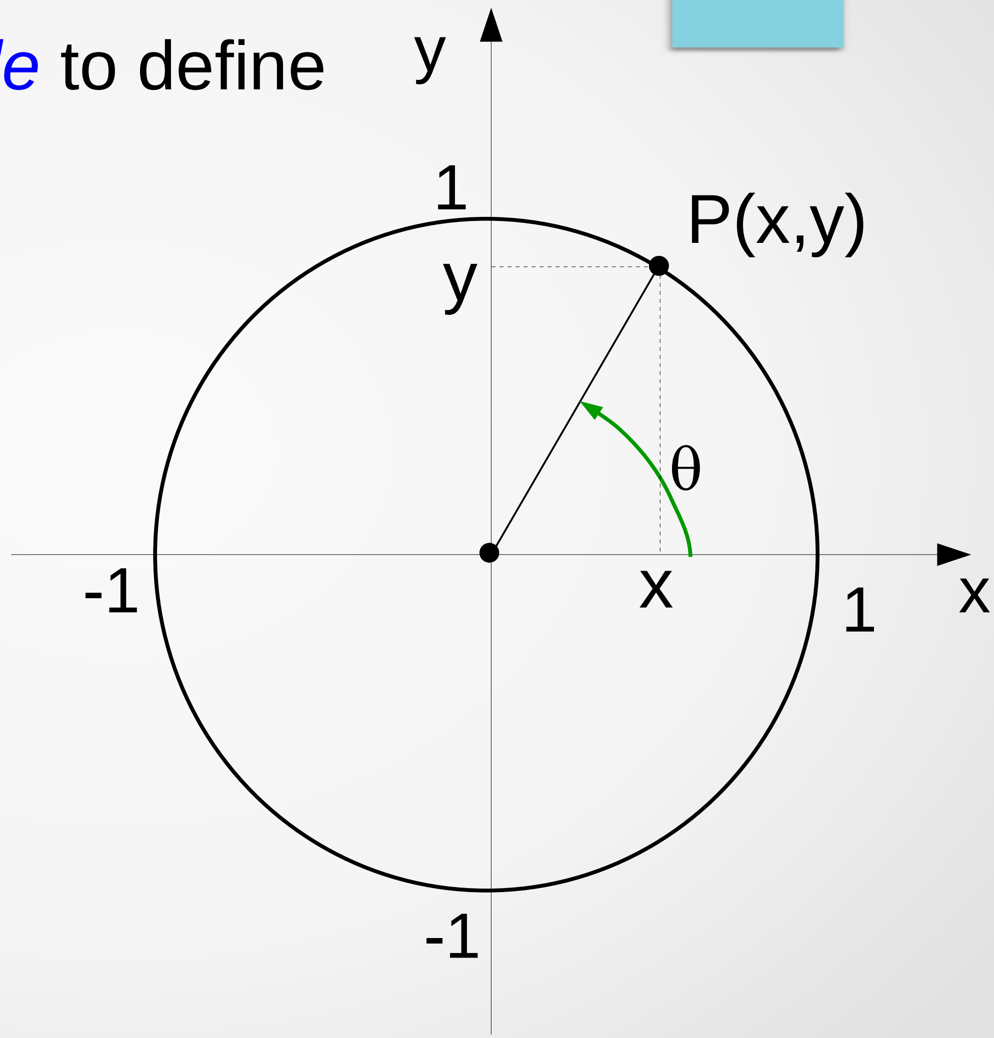
$\sin\theta = y$
$\cos\theta = x$

$\tan\theta = \frac{y}{x}, x \neq 0$

$\cot\theta = \frac{x}{y}, y \neq 0$

$\csc\theta = \frac{1}{y}, y \neq 0$

$\sec\theta = \frac{1}{x}, x \neq 0$

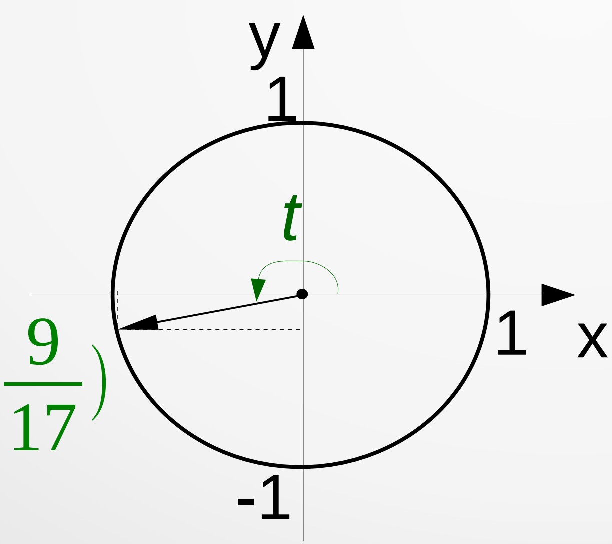


Sometimes there functions are called *circular functions*.

Trigonometric Functions: The Unit Circle

Example 1: for the point $P(-\frac{15}{17}, -\frac{9}{17})$, find the values of the six trigonometric functions at angle *t*, measured in radians

$P(-\frac{15}{17}, -\frac{9}{17})$



using a *unit circle* to define trigonometric functions:

$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x}, x \neq 0$

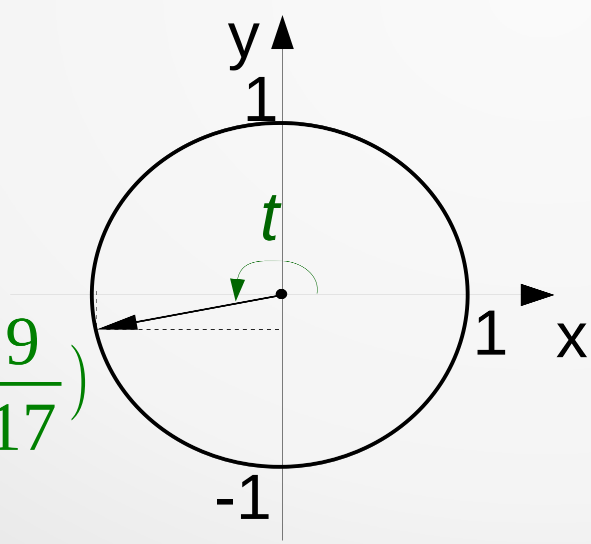
$\cot\theta = \frac{x}{y}, y \neq 0$

$\csc\theta = \frac{1}{y}, y \neq 0$

$\sec\theta = \frac{1}{x}, x \neq 0$

Trigonometric Functions: The Unit Circle

Example 1: for the point $P\left(-\frac{15}{17}, -\frac{9}{17}\right)$, find the values of the six trigonometric functions at angle *t*, measured in radians



$P\left(-\frac{15}{17}, -\frac{9}{17}\right)$

using a *unit circle* to define trigonometric functions:

$\sin\theta = y = -\frac{9}{17}$

$\cos\theta = x = -\frac{15}{17}$

$\tan\theta = \frac{y}{x} = \frac{3}{5}$

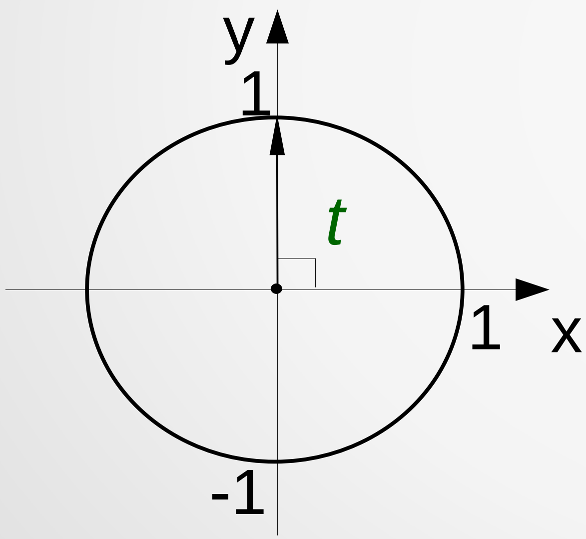
$\cot\theta = \frac{x}{y} = \frac{5}{3}$

$\csc\theta = \frac{1}{y} = -\frac{17}{9}$

$\sec\theta = \frac{1}{x} = -\frac{17}{15}$

Trigonometric Functions: The Unit Circle

Example 2: find the values of the six trigonometric functions at angle $t = \frac{\pi}{2}$



using a *unit circle* to define trigonometric functions:

$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x}, x \neq 0$

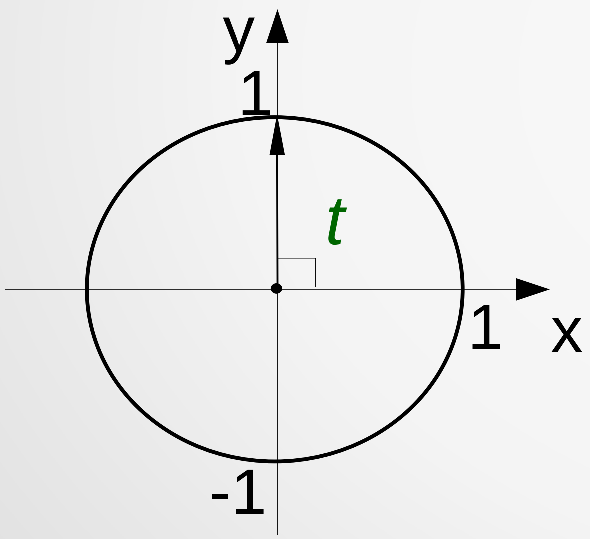
$\cot\theta = \frac{x}{y}, y \neq 0$

$\csc\theta = \frac{1}{y}, y \neq 0$

$\sec\theta = \frac{1}{x}, x \neq 0$

Trigonometric Functions: The Unit Circle

Example 2: find the values of the six trigonometric functions at angle $t = \frac{\pi}{2}$



using a *unit circle* to define trigonometric functions:

$\sin\theta = y = 1$

$\cos\theta = x = 0$

$\tan\theta = \frac{y}{x} = undefined$

$\cot\theta = \frac{x}{y} = 0$

$\csc\theta = \frac{1}{y} = 1$

$\sec\theta = \frac{1}{x} = undefined$

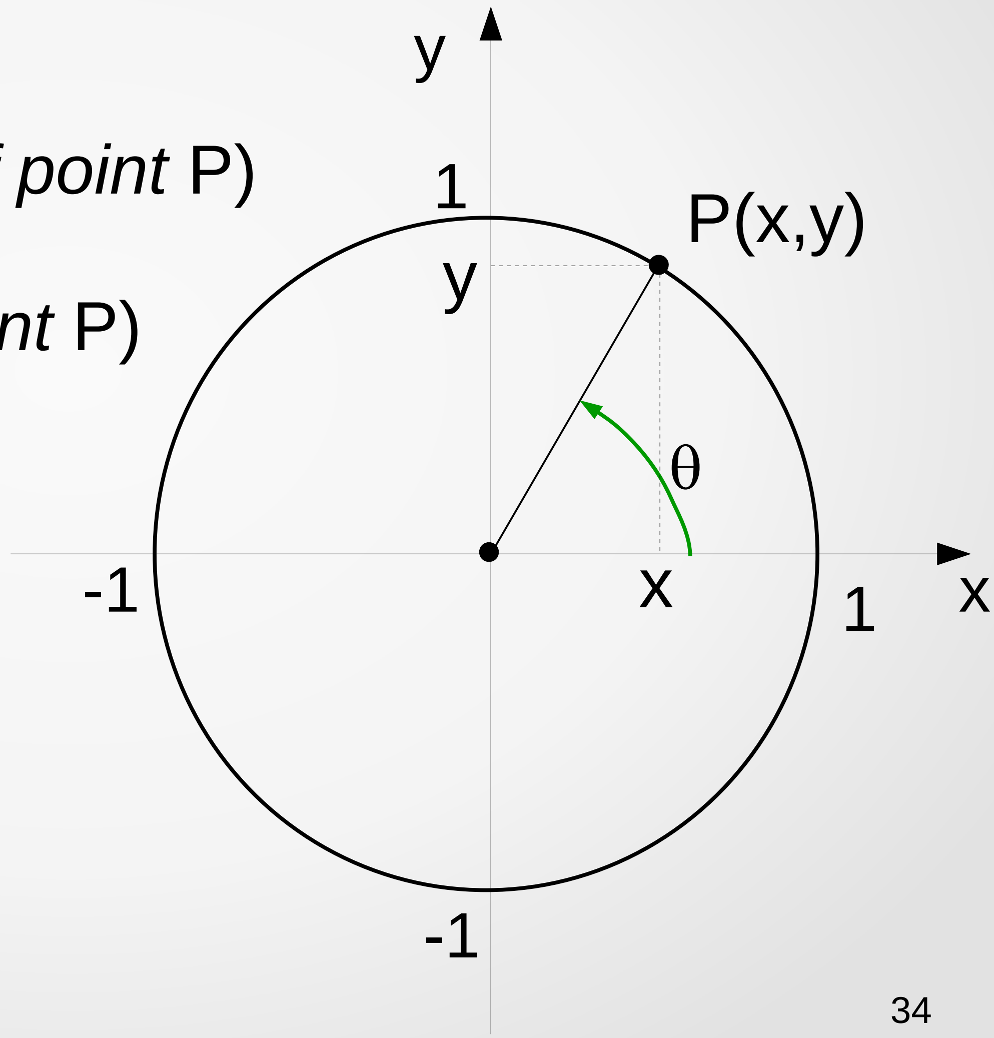
Trigonometric Functions: The Unit Circle

Domain and range of sin, cos functions

sin θ = y   (*y-coordinate of point* P)

cos θ = x   (*x-coord. of point* P)

domain: (-∞,∞)
range: [-1,1]



Trigonometric Functions: The Unit Circle

Even and odd trigonometric functions

$\sin\theta = y$ sin($-\theta$) =

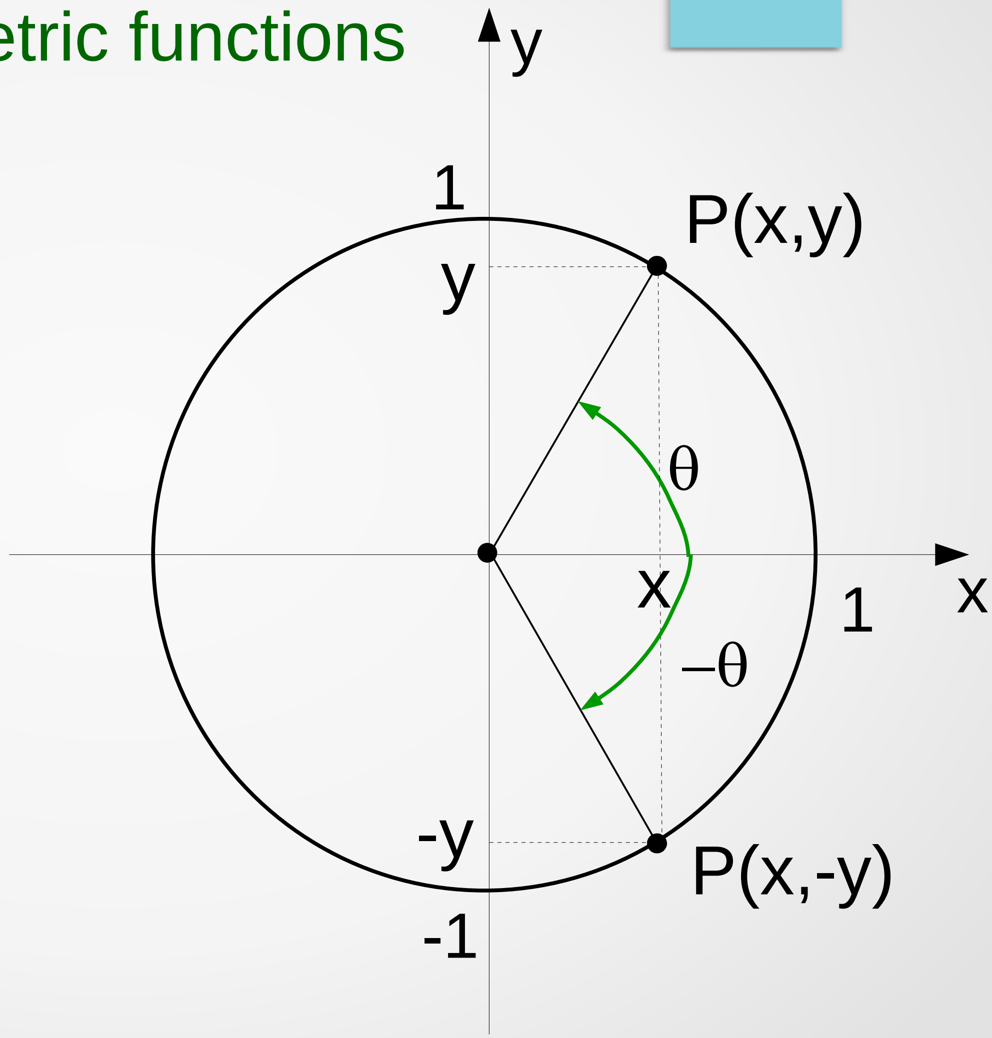
$\cos\theta = x$ cos($-\theta$) =

$\tan\theta = \dfrac{y}{x}$ tan($-\theta$) =

$\cot\theta = \dfrac{x}{y}$ cot($-\theta$) =

$\csc\theta = \dfrac{1}{y}$ csc($-\theta$)=

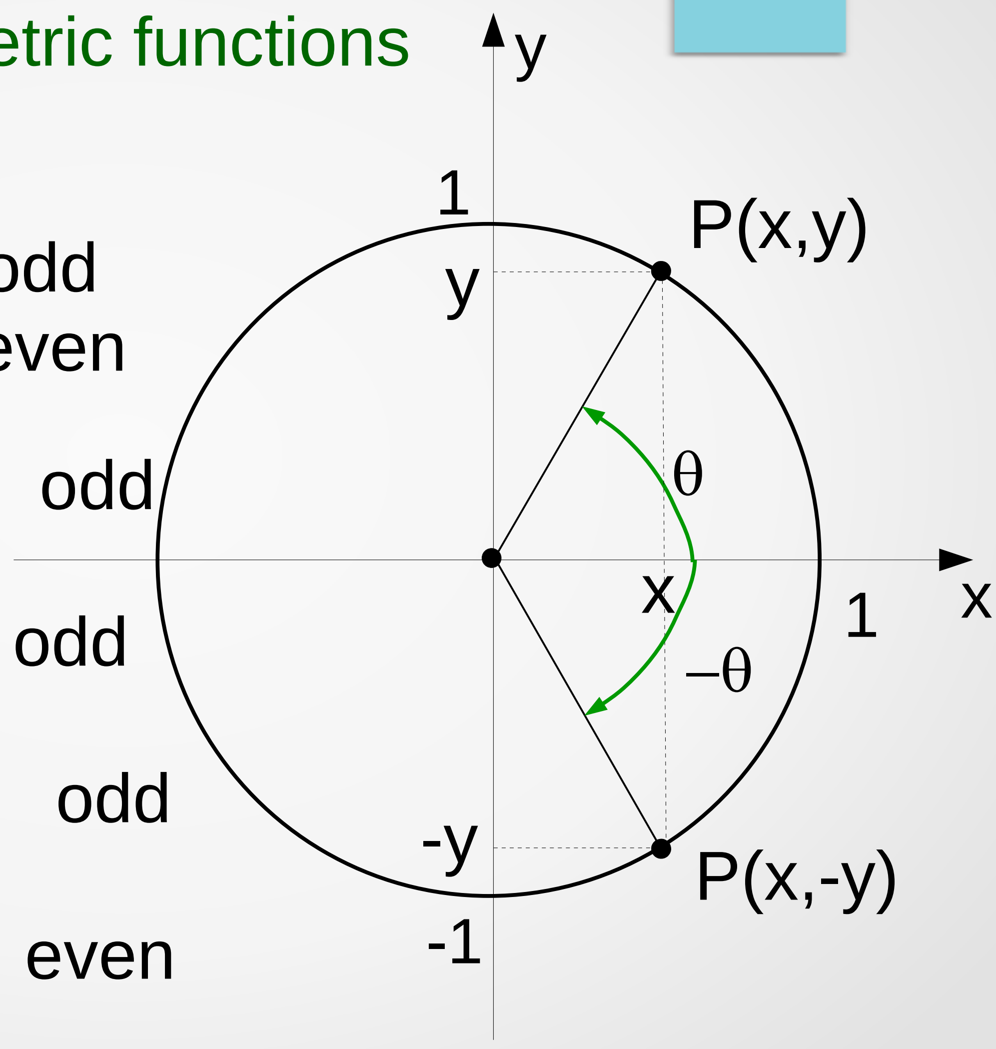
$\sec\theta = \dfrac{1}{x}$ sec($-\theta$) =



Trigonometric Functions: The Unit Circle

Even and odd trigonometric functions

$\sin\theta = y$	$\sin(-\theta) = -y$	odd
$\cos\theta = x$	$\cos(-\theta) = x$	even
$\tan\theta = \frac{y}{x}$	$\tan(-\theta) = \frac{-y}{x}$	odd
$\cot\theta = \frac{x}{y}$	$\cot(-\theta) = \frac{x}{-y}$	odd
$\csc\theta = \frac{1}{y}$	$\csc(-\theta) = \frac{1}{-y}$	odd
$\sec\theta = \frac{1}{x}$	$\sec(-\theta) = \frac{1}{x}$	even



Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

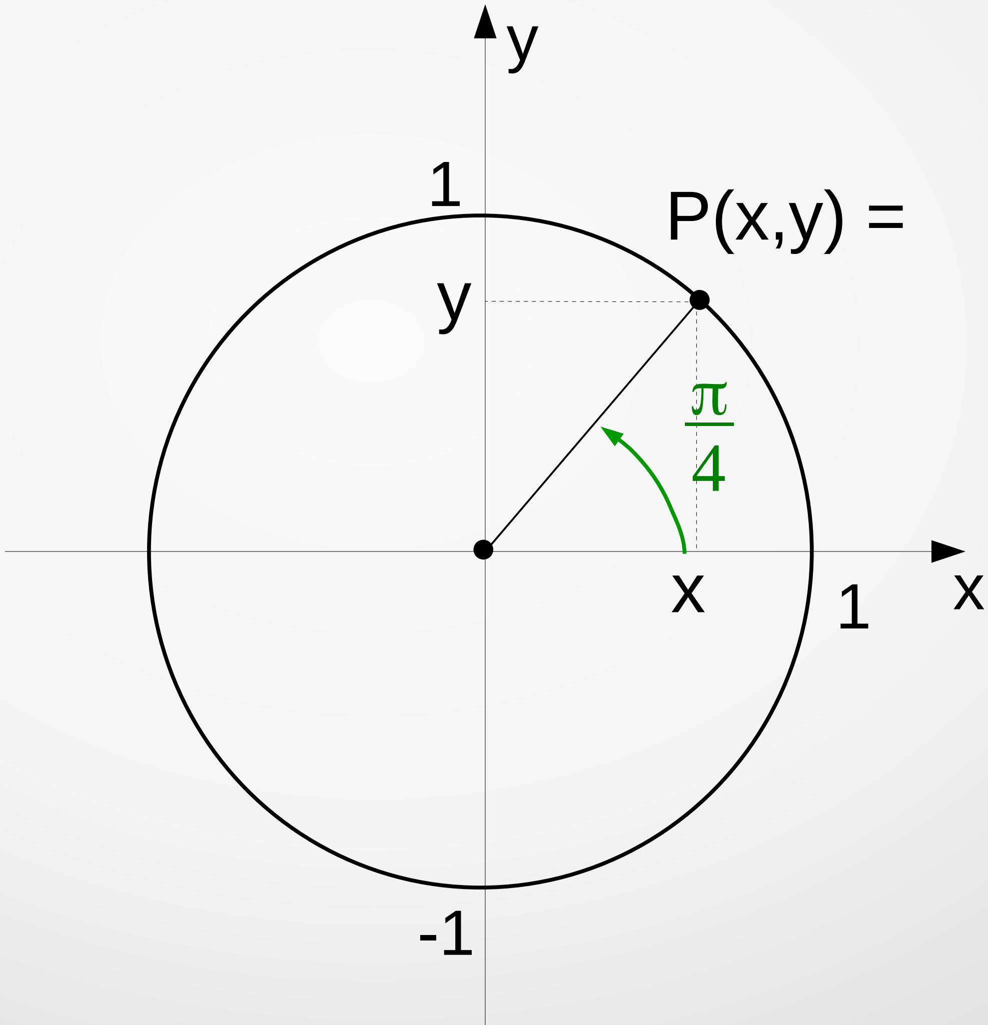
$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$



Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

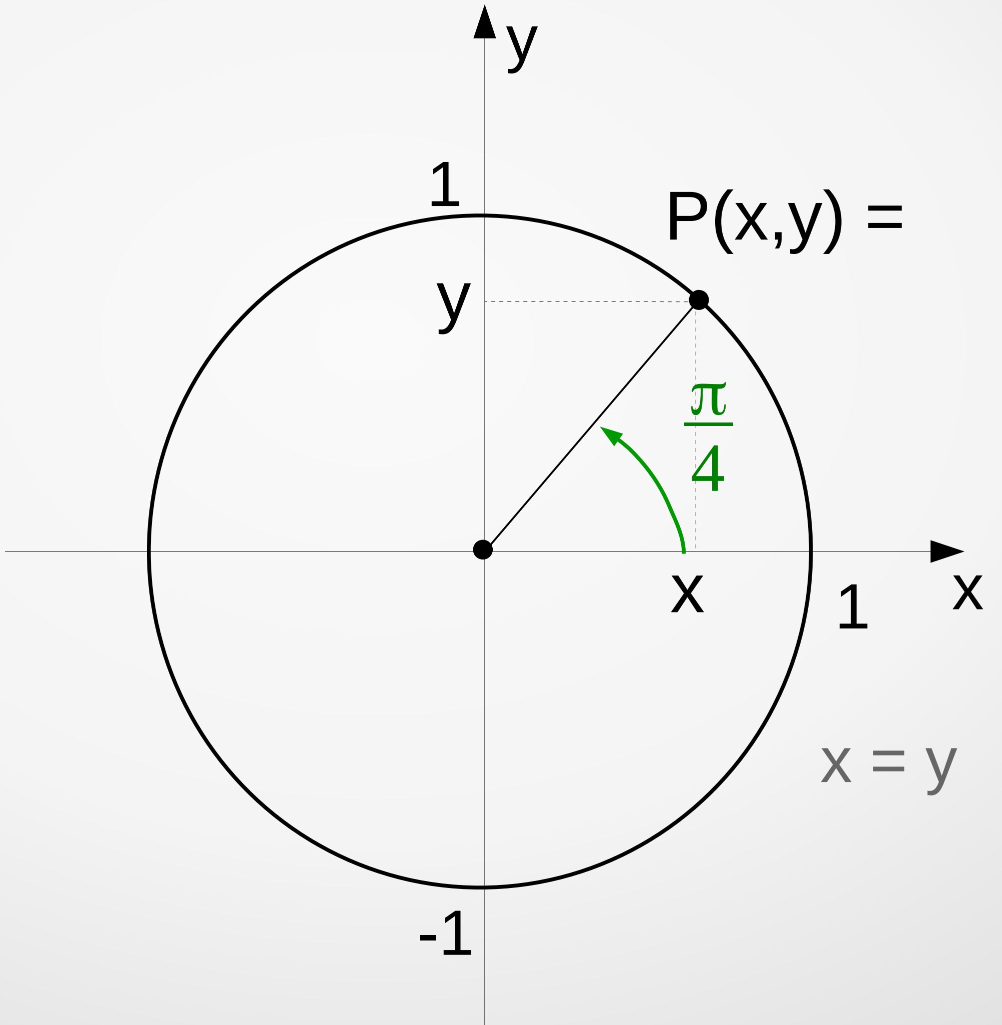
$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$



Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

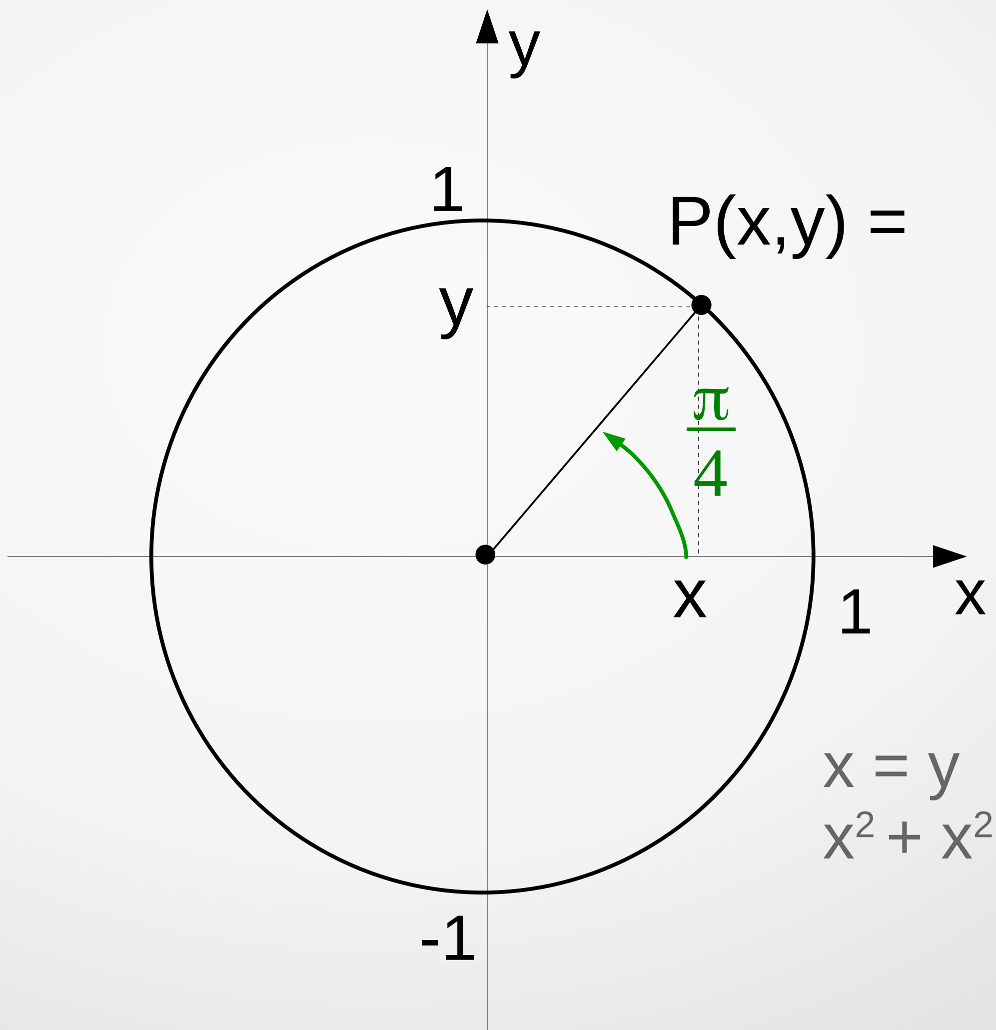
$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$



$x = y$

$x^2 + x^2 = 1$ …

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

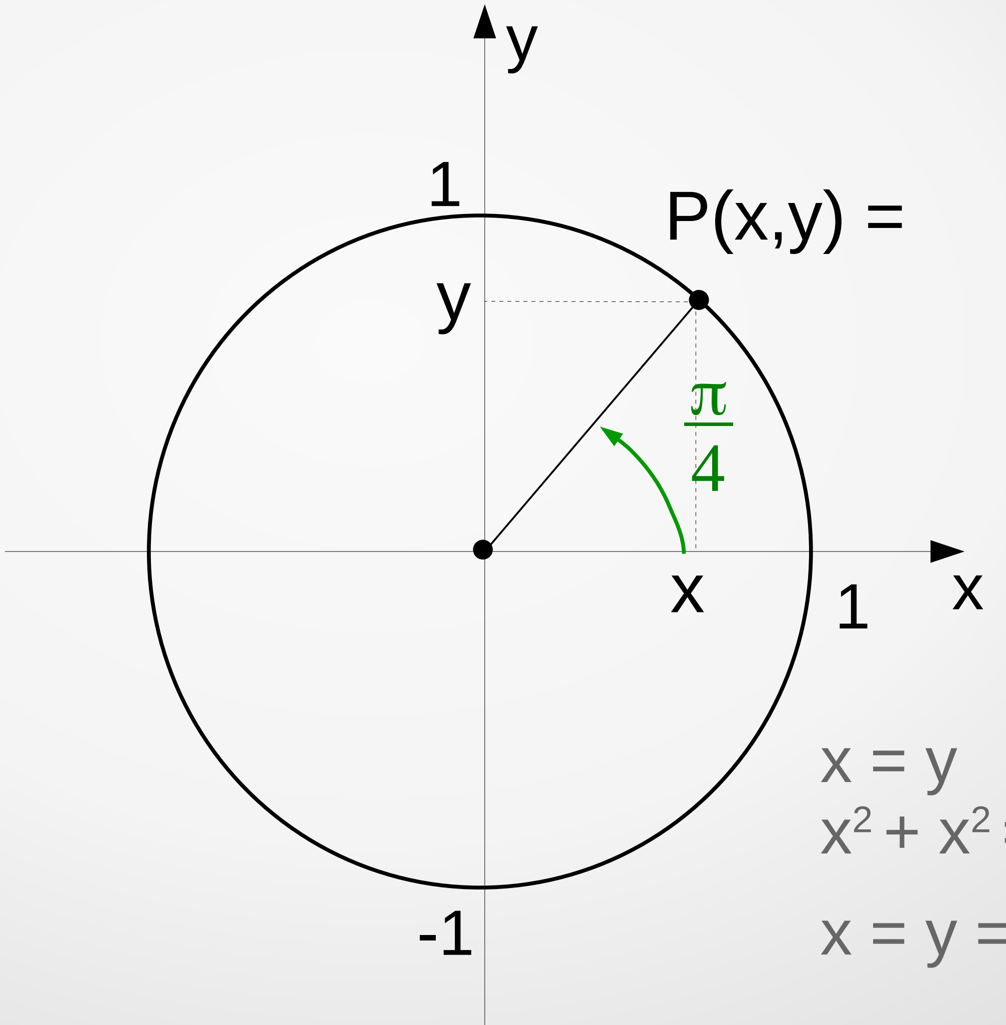
$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$



$x = y$

$x^2 + x^2 = 1 \ldots$

$x = y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)$

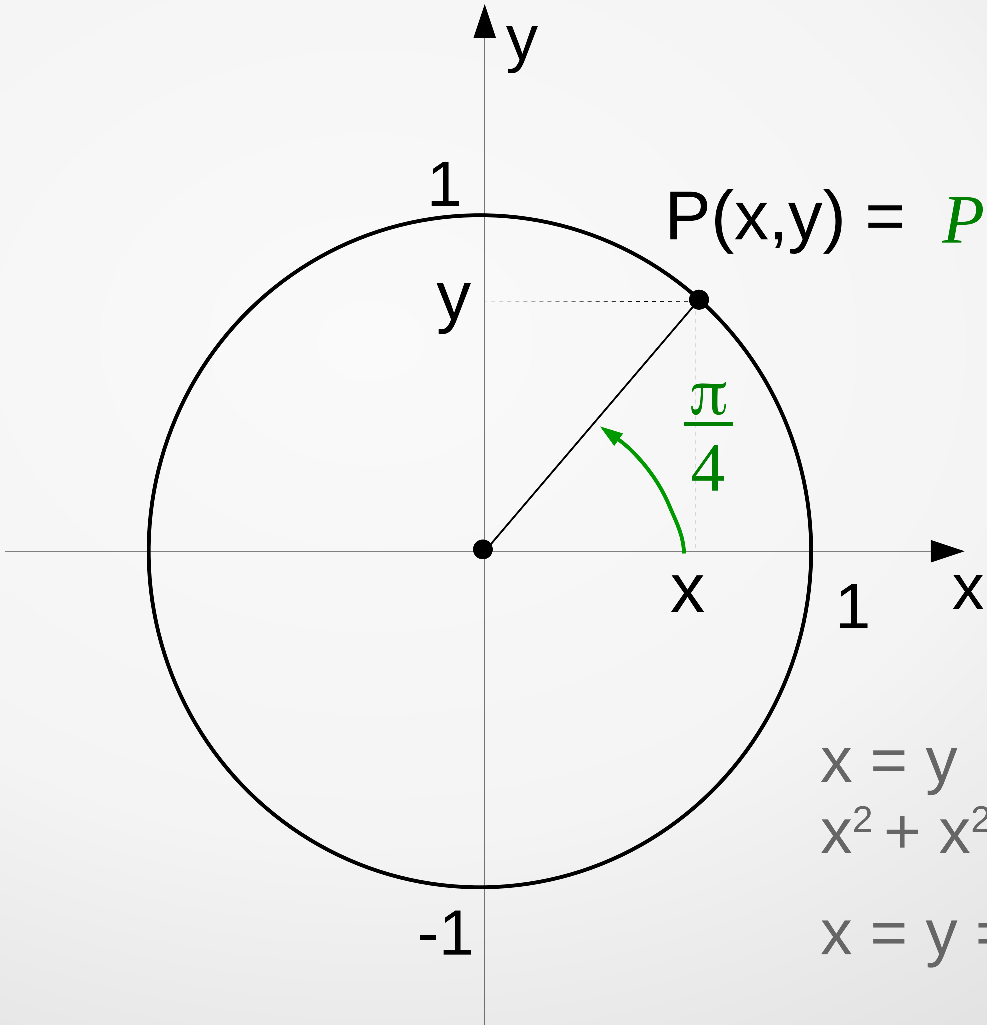
$\cos\left(\frac{\pi}{4}\right)$

$\tan\left(\frac{\pi}{4}\right)$

$\cot\left(\frac{\pi}{4}\right)$

$csc\left(\frac{\pi}{4}\right)$

$sec\left(\frac{\pi}{4}\right)$



P(x,y) = $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$x = y$

$x^2 + x^2 = 1 \ldots$

$x = y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Let's find values of trigonometric functions at $\frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

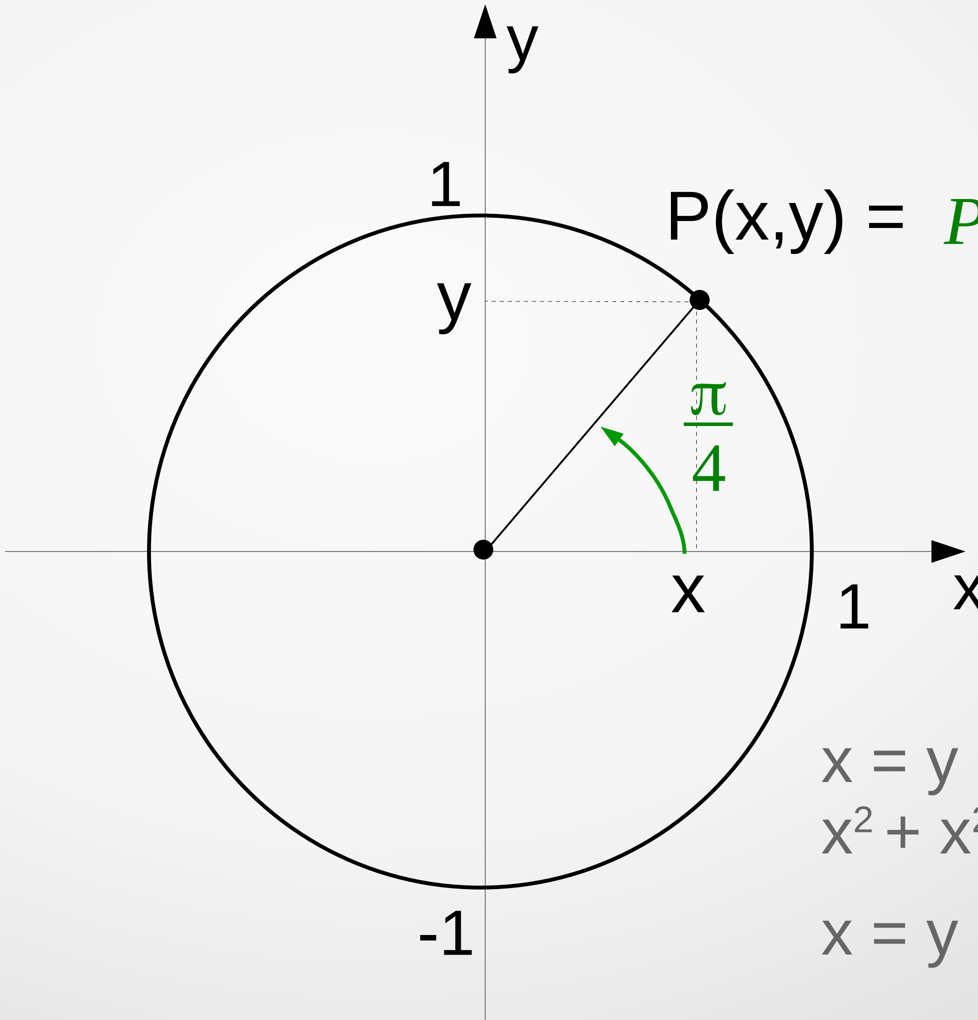
$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$\tan\left(\frac{\pi}{4}\right)=1$

$\cot\left(\frac{\pi}{4}\right)=1$

$csc\left(\frac{\pi}{4}\right)=\sqrt{2}$

$sec\left(\frac{\pi}{4}\right)=\sqrt{2}$



$P(x,y) = P\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$x = y$

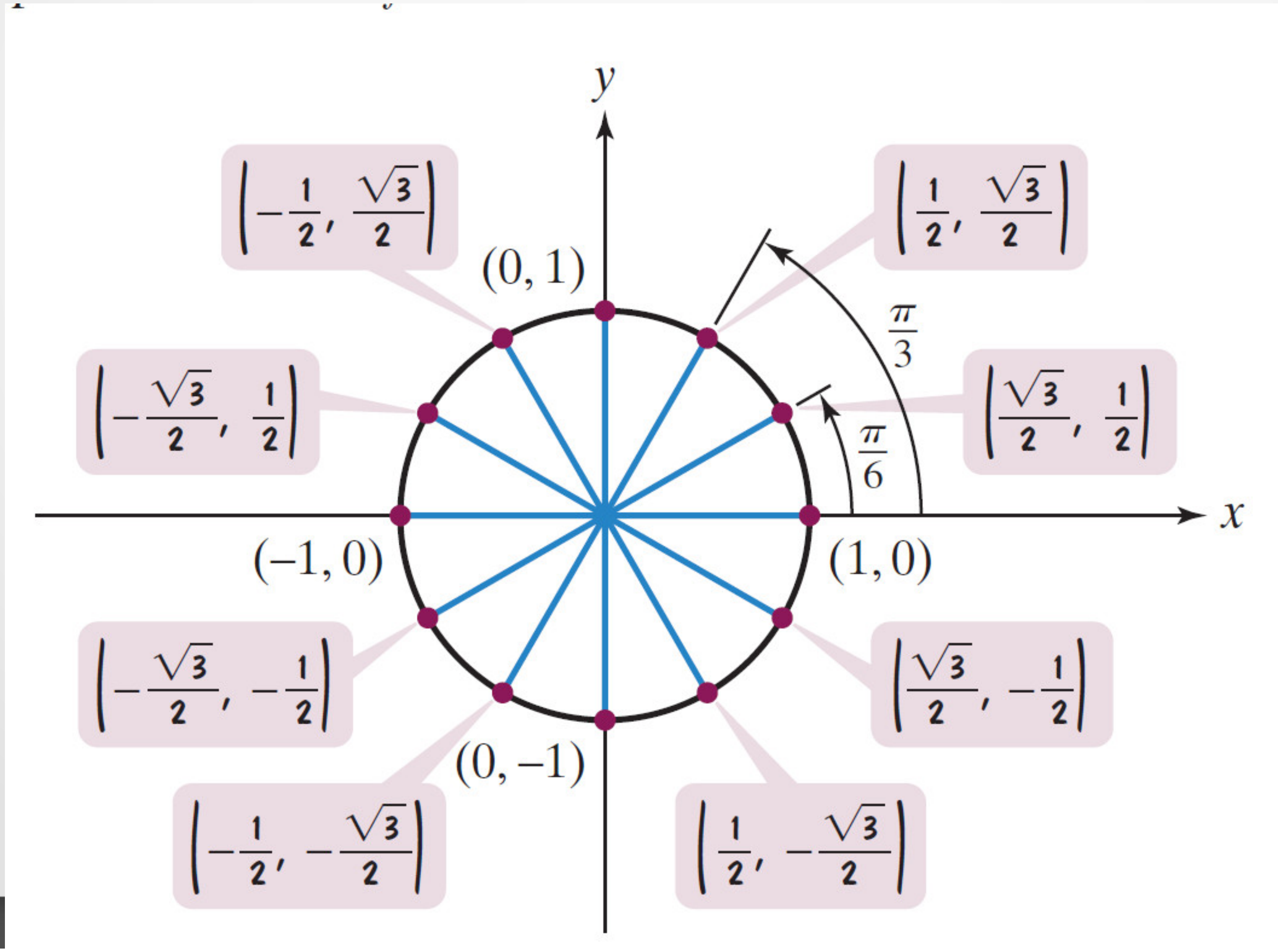
$x^2 + x^2 = 1 \ldots$

$x = y = \frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

Trigonometric Functions: The Unit Circle

Trigonometric Functions at some other angles

can be found using this picture of the unit circle



Trigonometric Functions: The Unit Circle

Trigonometric Functions at some other angles

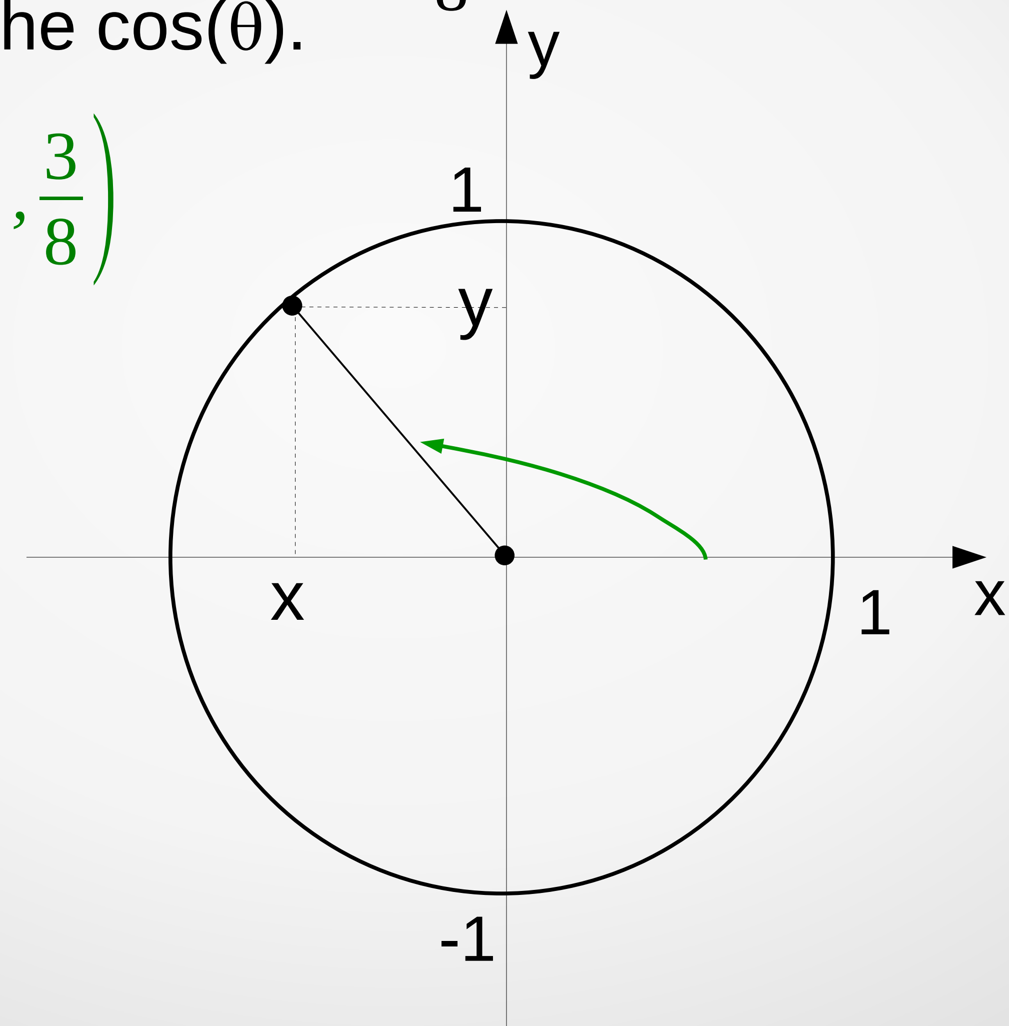
θ	**0°**	$\frac{\pi}{6}$ (30°) $\frac{5\pi}{6}$ (150°)	$\frac{7\pi}{6}$ (210°) $\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°) $\frac{3\pi}{4}$ (135°)	$\frac{5\pi}{4}$ (225°) $\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°) $\frac{2\pi}{3}$ (120°)	$\frac{4\pi}{3}$ (240°) $\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Sinθ	0	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	1	0	-1
θ	**0°**	$\frac{\pi}{6}$ (30°) $\frac{11\pi}{6}$ (330°)	$\frac{5\pi}{6}$ (150°) $\frac{7\pi}{6}$ (210°)	$\frac{\pi}{4}$ (45°) $\frac{7\pi}{4}$ (315°)	$\frac{3\pi}{4}$ (135°) $\frac{5\pi}{4}$ (225°)	$\frac{\pi}{3}$ (60°) $\frac{5\pi}{3}$ (300°)	$\frac{2\pi}{3}$ (120°) $\frac{4\pi}{3}$ (240°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{-1}{2}$	0	-1	0
θ	**0°**	$\frac{\pi}{6}$ (30°) $\frac{7\pi}{6}$ (210°)	$\frac{5\pi}{6}$ (150°) $\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°) $\frac{5\pi}{4}$ (225°)	$\frac{3\pi}{4}$ (135°) $\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°) $\frac{4\pi}{3}$ (240°)	$\frac{2\pi}{3}$ (120°) $\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Tanθ	0	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	1	-1	$\sqrt{3}$	$-\sqrt{3}$	*	0	*
Cotθ	*	$\sqrt{3}$	$-\sqrt{3}$	1	-1	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	*	0
θ	**0°**	**30°, 150°**	**210°, 330°**	**45°, 135°**	**225°, 315°**	**60°, 120°**	**240°, 300°**	**90°**	**180°**	**270°**
Cosecθ	*	2	-2	$\sqrt{2}$	$-\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{-2}{\sqrt{3}}$	1	*	-1
θ	**0°**	**30°, 330°**	**150°, 240°**	**45°, 315°**	**135°, 225°**	**60°, 300°**	**120°, 240°**	**90°**	**180°**	**270°**
Secθ	1	$\frac{2}{\sqrt{3}}$	$\frac{-2}{\sqrt{3}}$	$\sqrt{2}$	$-\sqrt{2}$	2	-2	*	-1	*

available on our web-page in the Notices

Trigonometric Functions: The Unit Circle

Example: Given that $\sin(\theta) = \frac{3}{8}$ and θ is in the second quadrant, find the $\cos(\theta)$.

P(x,y) = $P\left(?, \frac{3}{8}\right)$



Homework assignment

1) zyBooks: *review* Sections 5.1 (up to *Finding the area of a sector of a circle*, excluding), 5.2 (up to *Reference Angle, excluding*), and 5.3

or

Textbook: *review* Sections 4.1 and 4.2

2) We will have Quiz 14 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 14 (due date is in one week)