Rational Functions

Learning Objectives

In this section, you will:

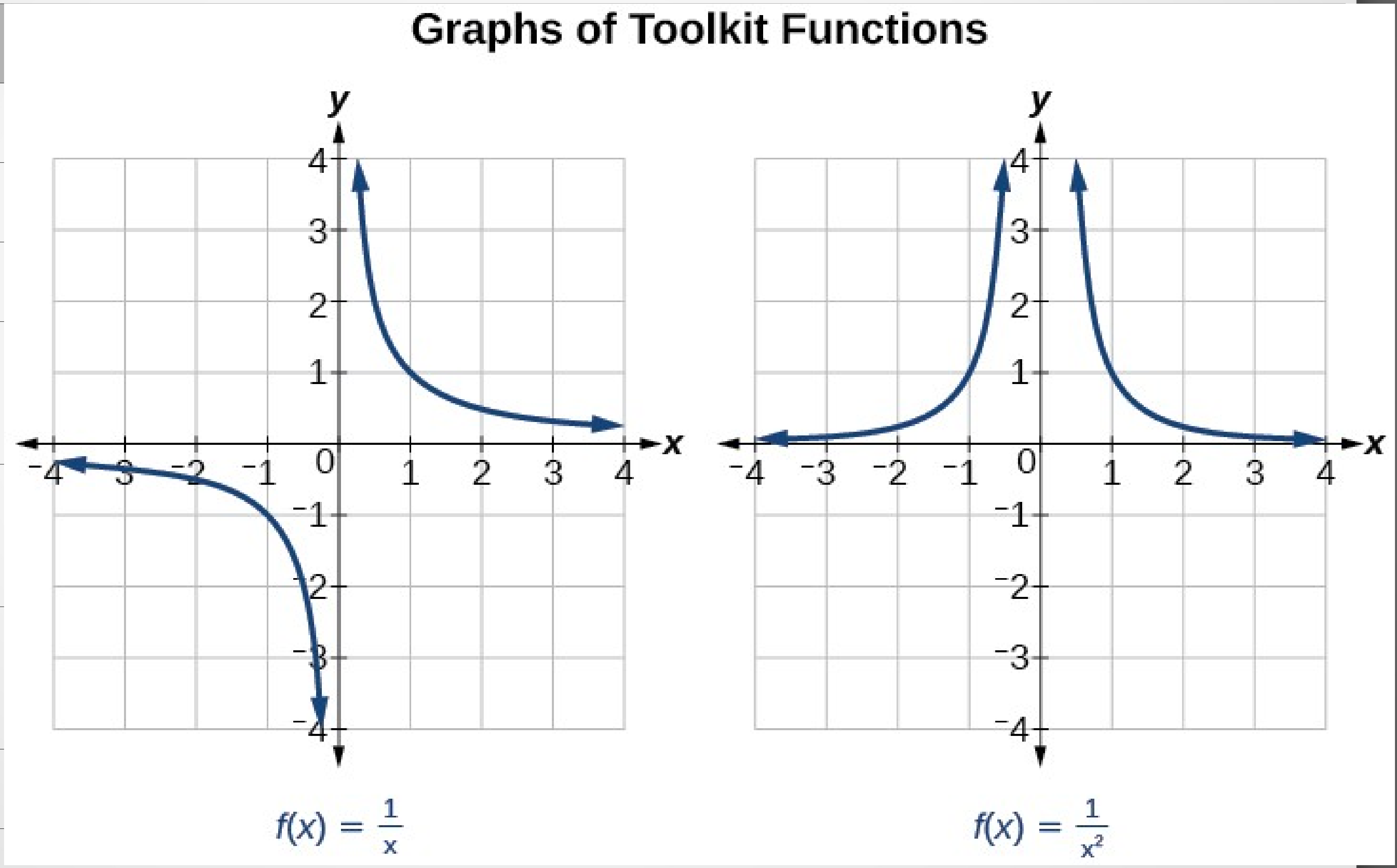
- Use arrow notation to show that x or $f(x)$ is approaching a particular value.
- Find the domains of rational functions.
- Identify vertical and horizontal asymptotes.
- Graph rational functions.

Rational Functions: arrow notation

Symbol	Meaning
$x \to a^-$	x approaches a from the left ($x<a$ but close to a)
$x \to a^+$	x approaches a from the right ($x>a$ but close to a)
$x \to \infty$	x approaches infinity (x increases without bound)
$x \to -\infty$	x approaches negative infinity (x decreases without bound)
$f(x) \to \infty$	the output approaches infinity (the output increases without bound)
$f(x) \to -\infty$	the output approaches negative infinity (the output decreases without bound)
$f(x) \to a$	the output approaches a

Rational Functions: arrow notation

Symbol
$x \rightarrow a^-$
$x \rightarrow a^+$
$x \rightarrow \infty$
$x \rightarrow -\infty$
$f(x) \rightarrow \infty$
$f(x) \rightarrow -\infty$
$f(x) \rightarrow a$



Rational Functions: the end behavior

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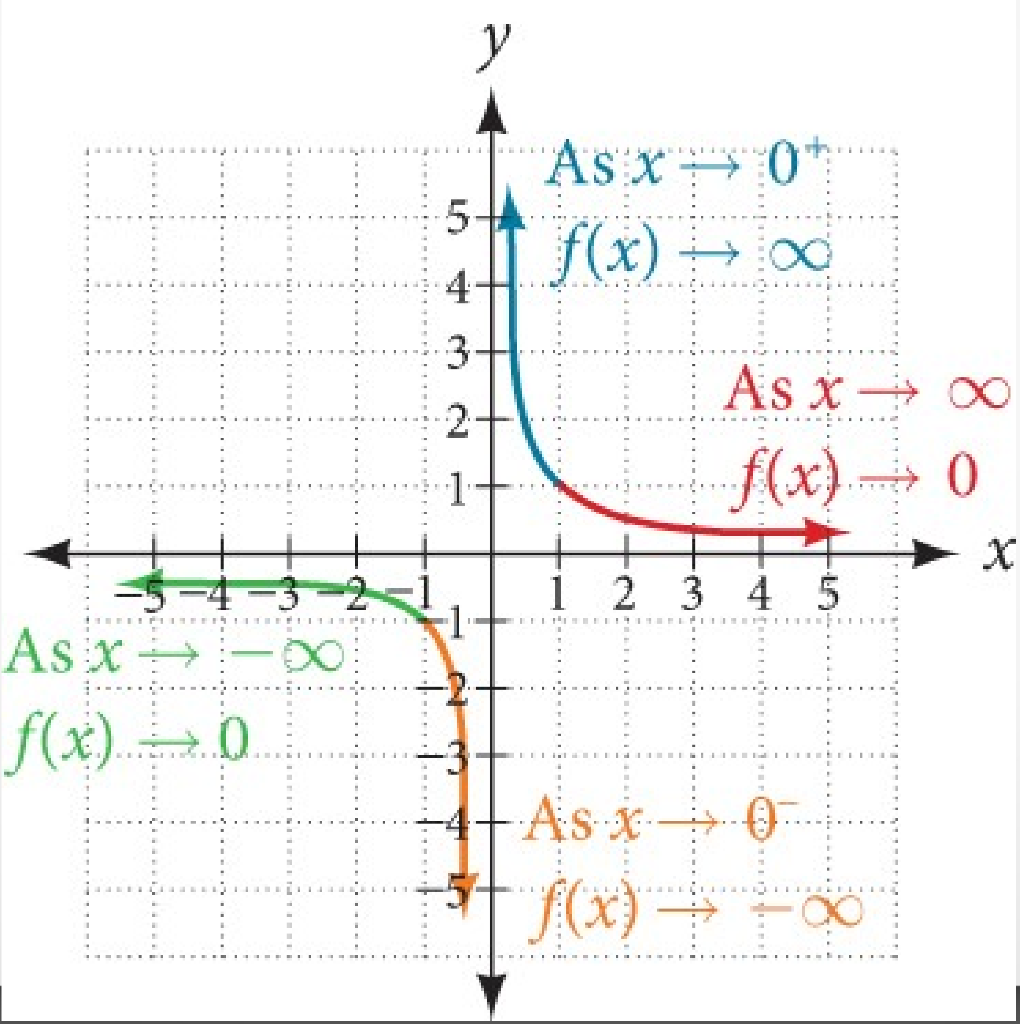


Rational Functions: vertical asymptote

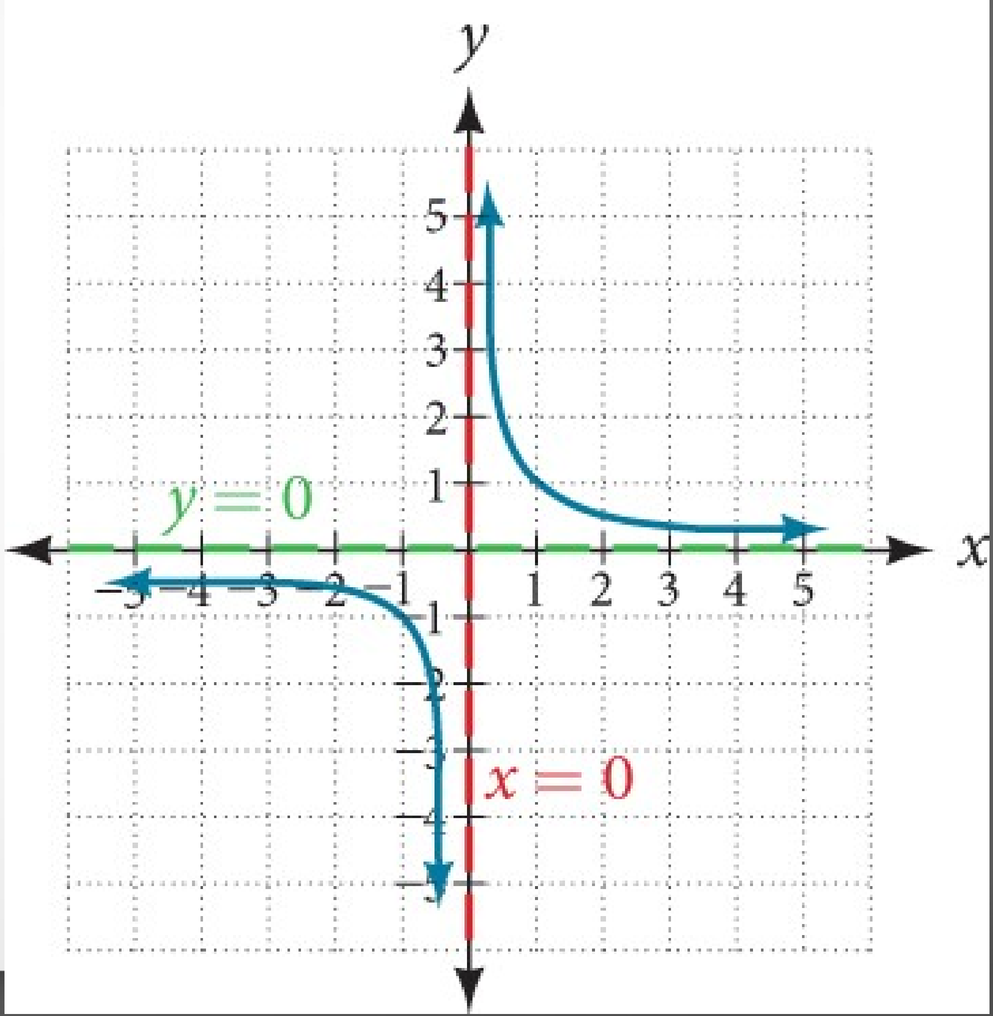
Vertical asymptote

A *vertical asymptote* of a graph is a vertical line $x = a$ where the graph tends toward positive or negative infinity as the inputs approach a.

We write

As $x \to a, \ f(x) \to \infty$, or

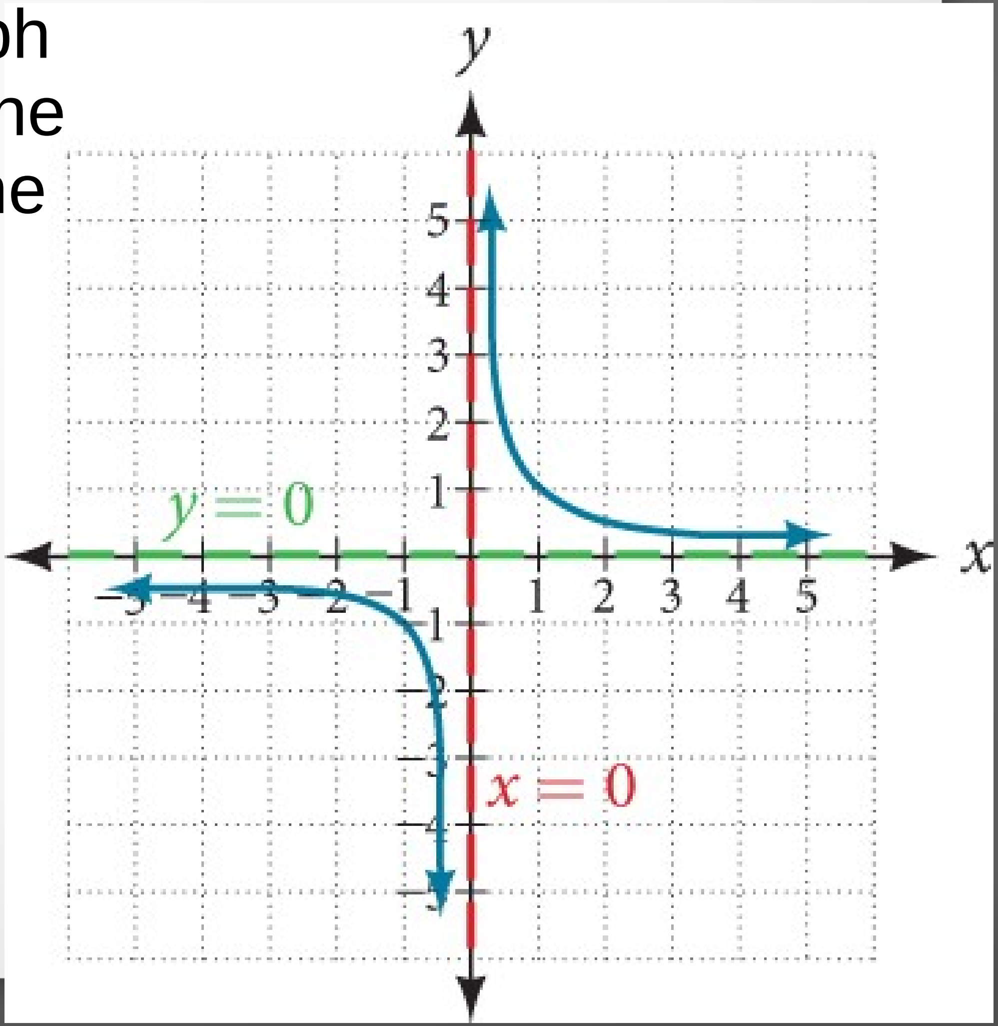
as $x \to a, \ f(x) \to -\infty$.



Rational Functions: horizontal asymptote

Horizontal asymptote

A *horizontal asymptote* of a graph is a horizontal line $y=b$ where the graph approaches the line as the inputs increase or decrease without bound.

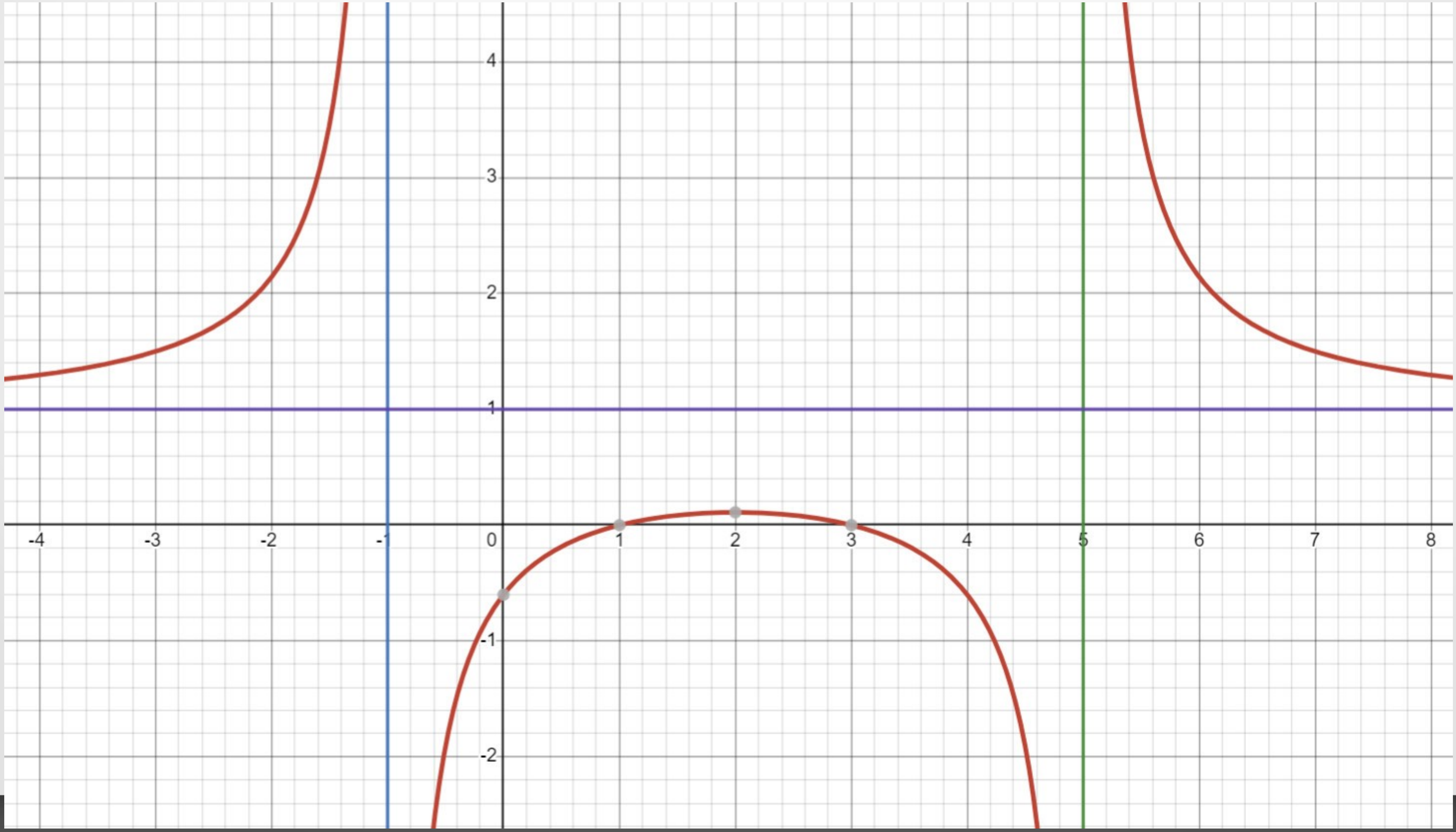


We write

As $x\to\infty$ or $x\to-\infty$, $f(x)\to b$.

In-class practice

Consider the following graph of the function.
State its *end-behavior*, *vertical* and *horizontal asymptotes*.



Rational Functions

Rational Functions as the Quotient

A rational function is a function that can be written as the quotient of two polynomial functions, $P(x)$ and $Q(x)$:

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_p x^p+a_{p-1}x^{p-1}+\ldots+a_1 x+a_0}{b_q x^q+b_{q-1}x^{q-1}+\ldots+b_1 x+b_0},\ Q(x)\neq 0$$

Examples:

$$k(x)=\frac{x^2-3x+5}{x-5}$$

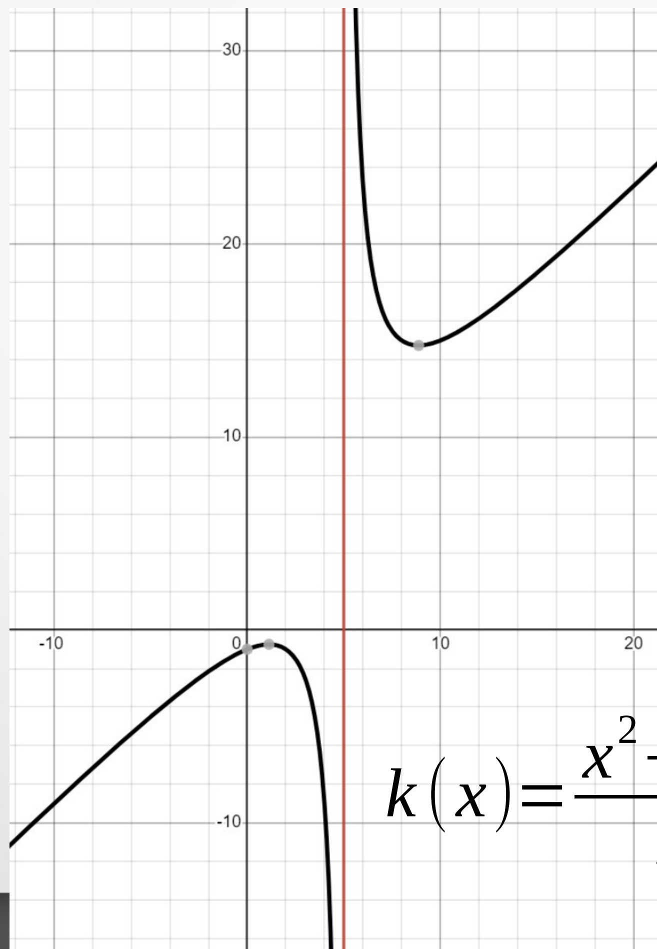
$$t(x)=\frac{(x-3)(x+6)}{(x-2)(x+1)}$$

Rational Functions

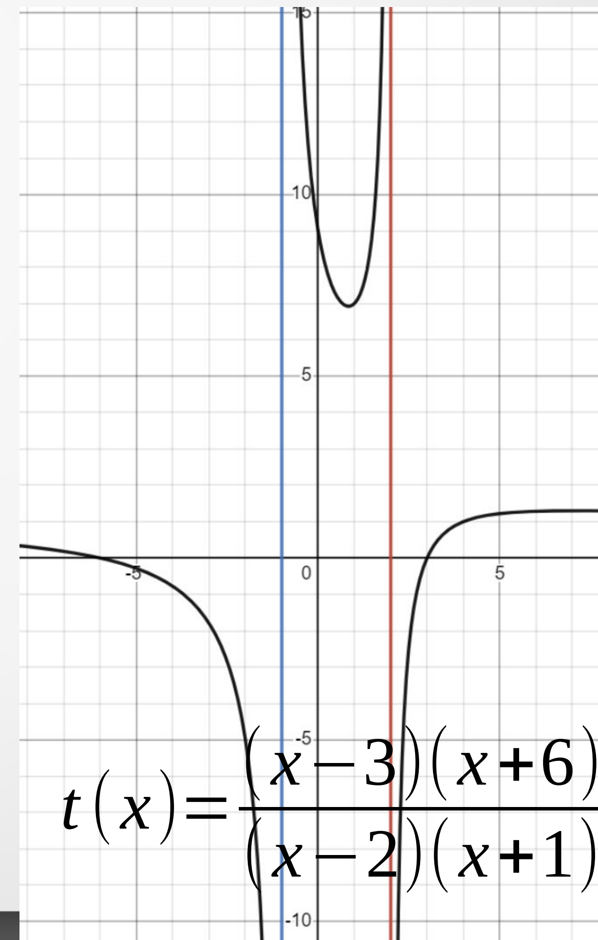
Vertical asympotes

Zeros of rational function’s denominator that are not factors of numerator are vertical asymptotes of the function.

Examples:



$$k(x)=\frac{x^2-3x+5}{x-5}$$



$$t(x)=\frac{(x-3)(x+6)}{(x-2)(x+1)}$$

Rational Functions

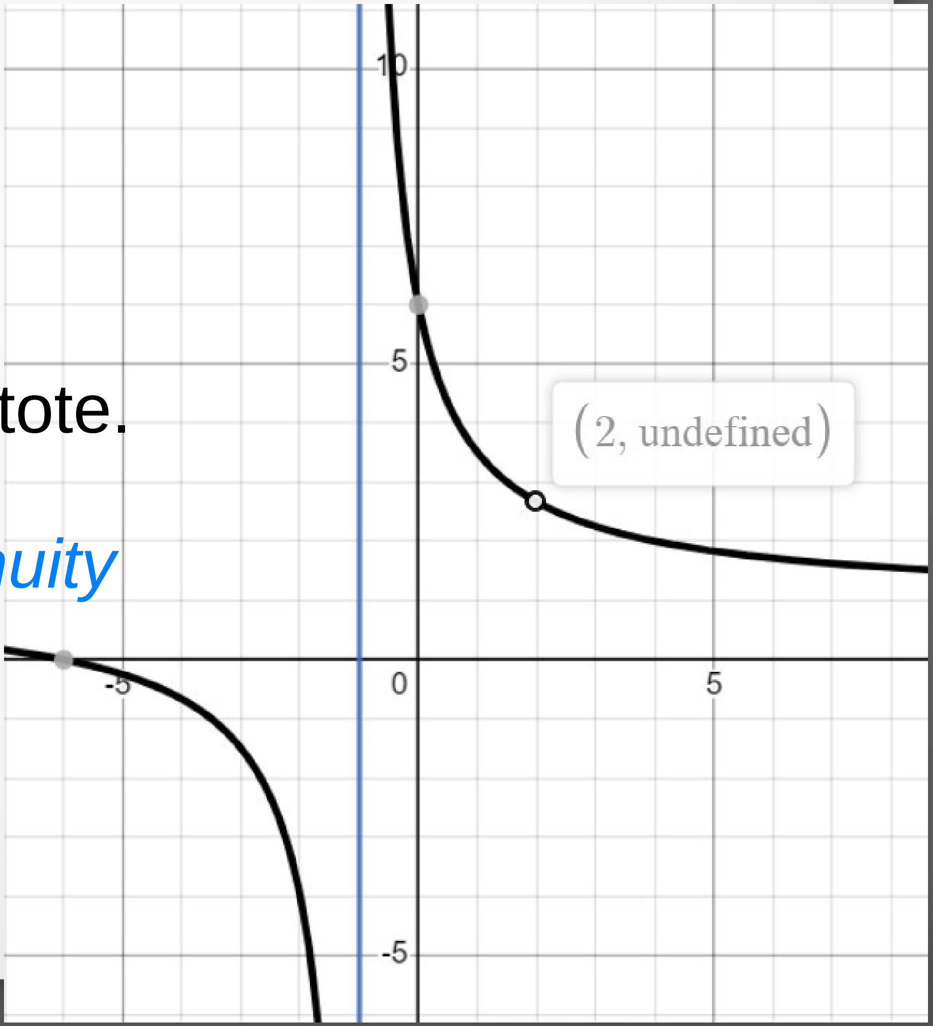
Removable discontinuities

Sometimes, a factor of the denominator is a facror of a numerator:

$$m(x)=\frac{(x-2)(x+6)}{(x-2)(x+1)}$$

In this case, x = 2 is not an asymptote.

x = 2 is called *removable discontinuity*



Rational Functions: Horizontal asympotes

The horizontal asymptote of a rational function can be determined by looking at the degrees of the numerator and denominator.

- If the degree of numerator is less than degree of denominator: horizontal asymptote at y = 0.

$$k(x)=\frac{7x^2-3x+5}{x^3-5x+10}$$

- If the degree of numerator is greater than degree of denominator by one: no horizontal asymptote; slant asymptote. To find it: divide, get the quotient.

$$t(x)=\frac{(x-3)(x+6)(x+7)}{(x-2)(x+1)}$$

- If the degree of numerator is equal to degree of denominator: horizontal asymptote at ratio of leading coefficients.

$$n(x)=\frac{3x^2-3x+6}{5x^2-2x+1}$$

In-class practice

For the following function: $f(x) = \frac{x^2 - 4x + 3}{x^2 - 4x - 5}$

Find:
a) its x- and y-intercepts, if possible
b) its horisontal and vertical asymptotes
c) its end behavior
d) its domain.
Then sketch its graph

In-class practice

For the following function: $f\left(x\right)=\frac{\left(x-8\right)\left(x+3\right)}{x-2}$

Find:

a) its x- and y-intercepts, if possible
b) its horisontal and vertical asymptotes, if they exist
c) its end behavior
d) its domain.
Then sketch its graph

Rational Functions

Learning Objectives

Today we

- used arrow notation to show that x or $f(x)$ is approaching a particular value.
- Found the domains of rational functions.
- Identified vertical and horizontal asymptotes.
- Graphed rational functions.