

Exponential and Logarithmic equations

Today we will continue using *logarithmic properties* along with the *exponential properties* you already know to solve *exponential* and *logarithmic equations*.

Exponential and Logarithmic equations

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$$b^M = b^N$$

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- use logarithms:

$$\log (b^M) = \log (b^N)$$

$$M \cdot \cancel{\log b} = N \cdot \cancel{\log b}$$

$M = N$ then finish solving the equation ⁵

Exponential and Logarithmic equations

Examples: Let's solve

$$1) 3^{2x+1} = 27$$

$$2) 5^{2-x} = 1 / 125$$

$$3) 9^x = \frac{1}{\sqrt[3]{3}}$$

Exponential and Logarithmic equations

What if we cannot “express each side as a power of the same base” ?

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- isolate the exponential expression (already done)
- take common (**log**) or natural (**ln**) logarithm of both sides of the equation

$$\log (10^x) = \log 8.07$$

- simplify using properties of logarithms
 $x \log 10 = \log 8.07 \quad \rightarrow \quad x = \log 8.07$
- solve for the variable (already done)

$$x = \log 8.07 \approx 0.91$$

Exponential and Logarithmic equations

Examples: Let's solve

1) $4e^{7x} = 10,273$

2) $e^{4x-5} = 11,243$

3) $e^{4x} - 3e^{2x} - 18 = 0$

Exponential and Logarithmic equations

[Def] A *logarithmic equation* is an equation containing a variable in a logarithmic expression.

Examples: $\log_3(x-1)=5$ $\log(x+2)-\log x = \log\left(\frac{1}{x}\right)$

How to solve a logarithmic equation?

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1) re-write in exponential form

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2) express each side as a single logarithm with coefficient 1 and with the same bases

$$\log_b M = \log_b N$$

then $M = N$, solve it and **check solutions!**

Exponential and Logarithmic equations

Examples: Let's solve

1) $\log_5 x = 3$

2) $7 + 3 \ln(x+1) = 6$

3) $\log_5 x + \log_5 (4x-1) = 1$

4) $2\log_3 (x+4) = \log_3 9 + 2$

$$\log_b (MN) = \log_b M + \log_b N$$

$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$

$$\log_b (M^p) = p \log_b M, p \in \mathbb{R}$$

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

Exponential and Logarithmic equations

5) $\log \sqrt{x+4} = 1$

6) $\log_2 (x-3) + \log_2 x - \log_2 (x+2) = 2$

7) $\log (x+7) - \log 3 = \log (7x+1)$

8) $\log_2 (x-1) - \log_2 (x+3) = \log_2 (1 / x)$

Homework assignment

1) zyBooks: *review* Section 4.6

or

Textbook: *review* Section 3.4

2) We will have **Quiz 13** based on today's topics in the beginning of our next meeting.

3) WeBWorK: **HW 13** (due date is in one week)