Dividing Polynomials

Learning Objectives

In this section we will use

- long division of polynomials
- synthetic division of polynomials

Dividing Polynomials

Long Division of Polynomials

Let’s divide $6x^2 - 10x + 21$ by $2x + 3$:

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$2x+3 \overline{\smash{)}\, 6x^2 - 10x + 21}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$2x+3\overline{\smash{)}\,6x^2 - 10x + 21}$$

$$6x^2 \div 2x = 3x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

$$
\begin{array}{r}
3x \\
2x+3 \overline{\big) 6x^2 - 10x + 21}
\end{array}
$$

$6x^2 \div 2x = 3x$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

multiply

$$\begin{array}{r} 3x \\ 2x+3 \overline{\smash{)}\,6x^2-10x+21} \\ -\left(6x^2+9x\right) \\ \hline \end{array}$$

$6x^2 \div 2x = 3x$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

$$\begin{array}{r} 3x \\ 2x+3 \overline{)\, 6x^2 - 10x + 21} \\ -(6x^2 + 9x) \\ \hline \end{array}$$

$6x^2 \div 2x = 3x$

$6x^2 - 10x - 6x^2 - 9x = -19x$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r@{}l}
 & 3x \\
2x+3 & \overline{)\,6x^2 - 10x + 21} \\
 & -(6x^2 + 9x) \\
\hline
 & \quad -19x
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r}
3x \\
2x+3 \overline{\big) 6x^2 - 10x + 21} \\
-(6x^2 + 9x) \\
\hline
-19x + 21
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

$$\begin{array}{r} 3x \\ 2x+3\overline{\smash{)}\,6x^2-10x+21} \\ -(6x^2+9x) \\ \hline -19x+21 \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

$$
\begin{array}{r}
3x \quad -\frac{19}{2} \\
2x+3 \overline{)\, 6x^2 - 10x + 21} \\
-(6x^2 + 9x) \\
\hline
-19x + 21
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

multiply

$$\begin{array}{r} 3x \quad -\frac{19}{2} \\ 2x+3\overline{)6x^2-10x+21} \\ -(6x^2+9x) \\ \hline -19x+21 \\ -\left(2\left(-\frac{19}{2}\right)x+3\left(-\frac{19}{2}\right)\right) \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

$$\begin{array}{r|l} & 3x - \frac{19}{2} \\ \hline 2x+3 & 6x^2-10x+21 \\ & -(6x^2+9x) \\ \hline & -19x+21 \\ & -(-19x-28.5) \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

$$
\begin{array}{r l}
 & 3x \quad -\frac{19}{2} \\
2x+3 \big) & 6x^2-10x+21 \\
 & -(6x^2+9x) \\
\hline
 & -19x+21 \\
 & -(-19x-28.5) \\
\hline
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x+21+19x+28.5=49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r l}
 & 3x - \frac{19}{2} \\
2x+3 \,\big) & 6x^2 - 10x + 21 \\
 & -(6x^2 + 9x) \\
\hline
 & -19x + 21 \\
 & -(-19x - 28.5) \\
\hline
 & 49.5
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

quotient (of the division): $3x-\frac{19}{2}$

$$
\begin{array}{r}
3x-\frac{19}{2} \\
2x+3\,\overline{)\,6x^2-10x+21} \\
-(6x^2+9x) \\
\hline
-19x+21 \\
-(-19x-28.5) \\
\hline
49.5
\end{array}
$$

remainder (of the division): 49.5

$$6x^2\div 2x=3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x\div 2x=-\frac{19}{2}$$

$$-19x+21+19x+28.5=49.5$$

Dividing Polynomials

Long Division of Polynomials

divide
multiply
subtract
bring down the next term

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r l}
 & 3x - \frac{19}{2} \\
2x+3 \overline{)} & 6x^2 - 10x + 21 \\
 & -(6x^2 + 9x) \\
\hline
 & -19x + 21 \\
 & -(-19x - 28.5) \\
\hline
 & 49.5
\end{array}
$$

quotient (of the division): $3x - \frac{19}{2}$

remainder (of the division): 49.5

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

$$
\begin{array}{r}
3x \quad -\dfrac{19}{2} \\
2x+3\overline{\big)\,6x^2-10x+21} \\
-\left(6x^2+9x\right) \\
\hline
-19x+21 \\
-\left(-19x-28.5\right) \\
\hline
49.5
\end{array}
$$

divide
multiply
subtract
bring down the
next term

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x+21+19x+28.5=49.5$$

$$\left(6x^2-10x+21\right)\div\left(2x+3\right)=\left(3x-\frac{19}{2}\right)R\,49.5=3x-\frac{19}{2}+\frac{49.5}{2x+3}$$

In-class practice

Exercise: divide $18x^4+9x^3+3x^2$ by $3x^2+1$:

$$3x^2+1 \overline{\big)\, 18x^4+9x^3+3x^2}$$

divide
multiply
subtract
bring down the next term

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend → $f(x)$

divisor → $d(x)$

quotient → $q(x)$

remainder → $r(x)$

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend → $f(x)$; *divisor* → $d(x)$; *quotient* → $q(x)$; *remainder* → $r(x)$

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend

divisor *quotient* *remainder*

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

$$6x^2 - 10x + 21 = (2x+3)\left(3x - \frac{19}{2}\right) + 49.5$$

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend → $f(x)$; *divisor* → $d(x)$; *quotient* → $q(x)$; *remainder* → $r(x)$

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

$$6x^2 - 10x + 21 = (2x+3)\left(3x - \frac{19}{2}\right) + 49.5$$

If $r(x) = 0$, then $d(x)$ *divides evenly into* $f(x)$, and $d(x)$ and $q(x)$ are *factors* of $f(x)$.

Dividing Polynomials

Synthetic Division of Polynomials

Synthetic division works well when the divisor is in the form $x - C$.

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2-10x+21$ by $x+3$:

the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$.

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x+3$:
the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21

Dividing Polynomials

Synthetic Division of Polynomials

Let’s divide $6x^2 - 10x + 21$ by $x + 3$:
the divisor must be in the form $x - C$, hence $x + 3 = x - (-3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	
	6		

multiply

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2-10x+21$ by $x+3$:
the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	
	6	-28	

multiply

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:
the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	84
	6	-28	

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2-10x+21$ by $x+3$:
the divisor must be in the form $x - C$, hence $x+3 = x - (- 3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	84
	6	-28	105

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:
the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$.
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	$\downarrow$	-18	84
	6	-28	105
	x	$const$	r

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2-10x+21$ by $x+3$:
the divisor must be in the form x – C, hence x+3 = x - (- 3).
List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	$\downarrow$	-18	84
	6	-28	105
	x	$const$	r

2) the result: $(6x^2-10x+21)\div(x+3)=(6x-28)\,R\,105=6x-28+\frac{105}{x+3}$

In-class practice

Exercise: divide x^3+4x^2-5x+5 by $x-3$ using synthetic division.

In-class practice

Exercise: divide x^3+4x^2-5x+5 by $x-3$ using synthetic division.

	x^3	x^2	x	$const$
$?$	?	?	?	?
	$\downarrow$			
	x^2	x	$const$	r

In-class practice

Exercise 2: Use long or synthetic division to divide

$x^3 - 4x^2 + x + 6$ by $x + 1$.

Dividing Polynomials

Learning Objectives

In this section we saw

- long division of polynomials
- synthetic division of polynomials