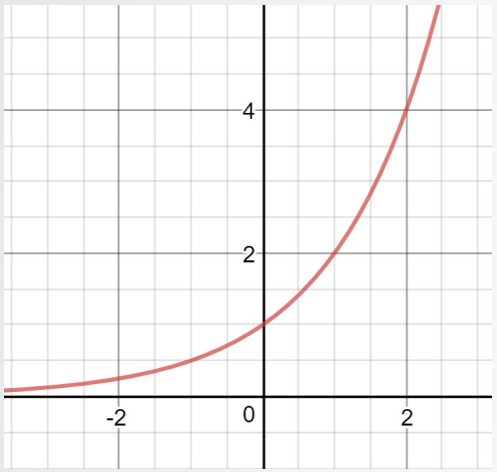
Plan for today's meeting:

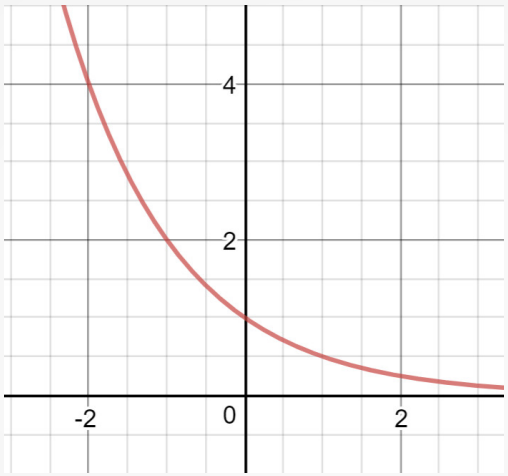
- a brief review
- *common* and *natural logarithms*
- properties of logarithms
- practice

Review

$f(x)=2^x$

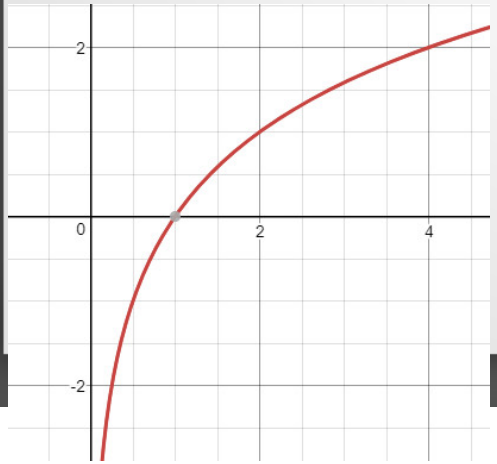


$f(x)=\left(\frac{1}{2}\right)^x$

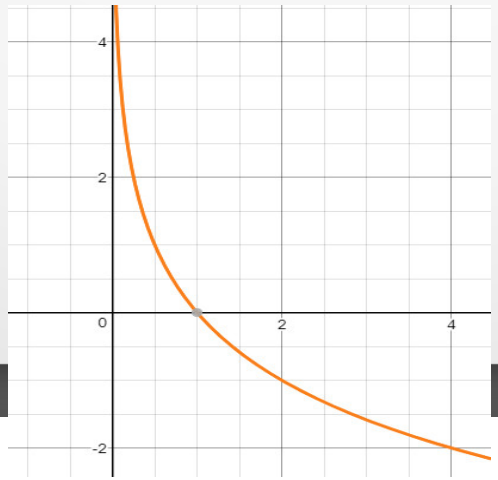


$y=\log_b x \longleftrightarrow b^y=x$

g(x) = log₂ x



g(x) = $\log_{1/2}$ x



properties of logarithms

1) $\log_b b = 1$
2) $\log_b 1 = 0$
3) $\log_b b^x = x$
4) $b^{\log_b x}=x$

Common and Natural Logarithms

Common logarithms

$f(x) = \log_{10} x = \log x$

Natural logarithms

$f(x) = \log_e x = \ln x$

Common and Natural Logarithms

Common logarithms

$f(x) = \log_{10} x = \log x$

Natural logarithms

$f(x) = \log_e x = \ln x$

Properties:

$\log 1 = 0$

$\log 10 = 1$

$\log 10^x = x$

$10^{\log x} = x$

Common and Natural Logarithms

Common logarithms

$f(x) = \log_{10} x = \log x$

Properties:

$\log 1 = 0$

$\log 10 = 1$

$\log 10^x = x$

$10^{\log x} = x$

Natural logarithms

$f(x) = \log_e x = \ln x$

Properties:

$\ln 1 = 0$

$\ln e = 1$

$\ln e^x = x$

$e^{\ln x} = x$

Properties of Logarithms

product rule

$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

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$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

quotient rule

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

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power rule

$$(b^m)^n = b^{mn}$$

$$\log_b(M^p) = p\log_b M$$

$$p \in R$$

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What can we do with their help?

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What can we do with their help?

- expand the logarithmic expression, or
- condense the logarithmic expression

Properties of Logarithms

Example 1: expand $\log_7 49a$

Example 2: condense (re-write as one logarithm)

$\log 20 + \log (x^2)$

product rule

$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

quotient rule

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

power rule

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$$p \in R$$

Properties of Logarithms

Example 3: expand $\log_5 \frac{125}{x^2}$

Example 4: condense (re-write as one logarithm)

ln 7 – ln a

product rule

$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

quotient rule

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

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Properties of Logarithms

Example 5: expand $\log_5 x^3$

Example 6: condense (re-write as one logarithm) 2log a

product rule

$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

quotient rule

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

power rule

$$(b^m)^n = b^{mn}$$

$$\log_b(M^p) = p\log_b M$$
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Properties of Logarithms

Common mistakes:

$$\log_5(x+1) \neq \log_5 x + \log_5 1$$

$$\log_5(5x) \neq \log_5 5 \cdot \log_5 x$$

$$\log_5(5x) \neq 5\log_5 x$$

product rule

$$b^m b^n = b^{m+n}$$

$$\log_b(MN) = \log_b M + \log_b N$$

quotient rule

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

power rule

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In-class practice

(1) expand logarithmic expressions as much as possible, evaluate if possible.

(a) $\log(100x)$

(b) $\log_5\left(25\sqrt{x^2}\right)$

(c) $\log_3 \dfrac{27}{\sqrt[3]{x+1}}$

(d) $\log_2 \sqrt[5]{\dfrac{xy^4}{16}}$

(e) $\log \dfrac{100x^3\sqrt{5-x}}{4(x+3)^3}$

$b^m b^n = b^{m+n}$ $\qquad \dfrac{b^m}{b^n} = b^{m-n}$

$(b^m)^n = b^{mn}$

$\log_b(MN) = \log_b M + \log_b N$

$\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$

$\log_b(M^p) = p\log_b M,\ p \in R$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_b b^x = x$

$b^{\log_b x} = x$

In-class practice

(2) condense each logarithmic expression; express as a single logarithm with coefficient 1; evaluate if possible,

(a) $\ln x + 2\ln 3$

(b) $2\log x - \frac{1}{2}\log y$

(c) $2\left(2\ln(x+3) - \ln y\right)$

(d) $\frac{1}{2}\left(\log_7 x + \log_7 y + 1\right) - 2\log_7(x-1)$

(e) $\log x + \log(x^2-4) - \log 15 - \log(x+2)$

$b^m b^n = b^{m+n}$ $\quad \frac{b^m}{b^n} = b^{m-n}$

$(b^m)^n = b^{mn}$

$\log_b(MN) = \log_b M + \log_b N$

$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b(M^p) = p\log_b M,\ p \in R$

$\log_b b = 1$

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Properties of Logarithms

Change of base property

$$\log_b M = \frac{\log_a M}{\log_a b}, a>0, a\neq 1$$

$$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Properties of Logarithms

Change of base property

$$\log_b M = \frac{\log_a M}{\log_a b}, a>0, a\neq 1 \qquad \log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Example: let's use common or natural logarithms to evaluate given logarithms with a calculator.

1) $\log_5 17$

2) $\log_{0.1} 13$

3) $\log_\pi 200$

Properties of Logarithms

Change of base property

$$\log_b M = \frac{\log_a M}{\log_a b}, a>0, a\neq 1 \qquad \log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Example: let's use common or natural logarithms to evaluate given logarithms with a calculator.

1) $\log_5 17 = \frac{\log 17}{\log 5} \approx 1.76$

2) $\log_{0.1} 13 = \frac{\log 13}{\log 0.1} \approx -1.11$

3) $\log_\pi 200 = \frac{\log 200}{\log \pi} \approx 4.63$

Homework assignment

1) zyBooks: *review* Section 4.5
or
Textbook: *review* Section 3.3

2) We will have Quiz 12 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 12 (due date is in one week)