

# Polynomial and Rational Inequalities

Today we will discuss:

- Polynomial Inequalities
- Rational Inequalities

# Polynomial and Rational Inequalities

## Polynomial Inequality

is an inequality that can be put into one of the forms:

$$f(x) < 0, f(x) > 0, f(x) \leq 0, f(x) \geq 0$$

where  $f(x)$  is a polynomial function

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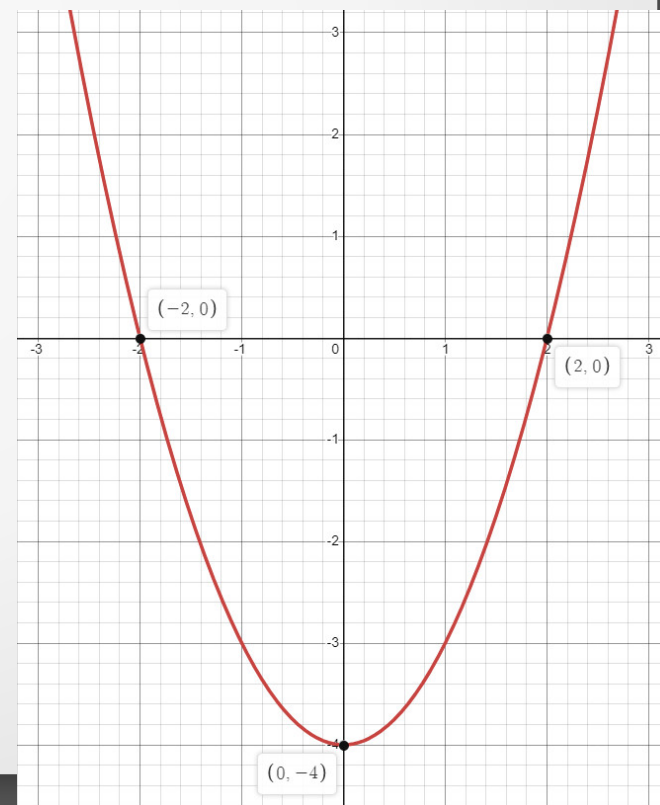
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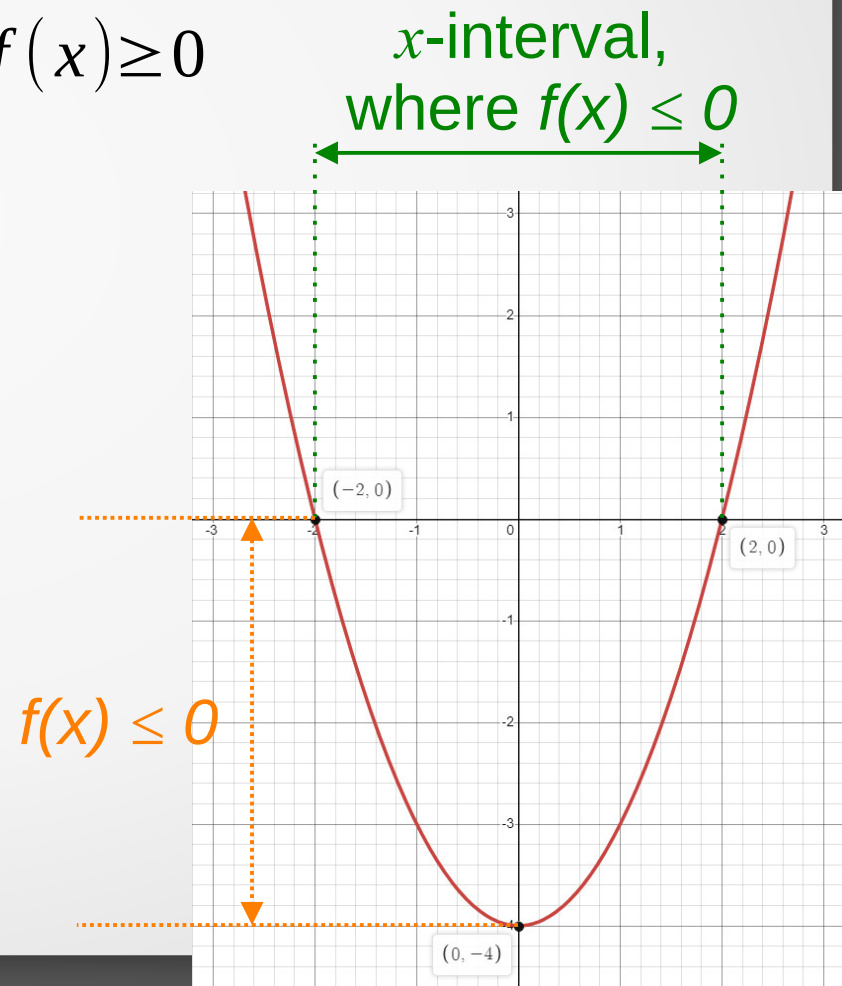
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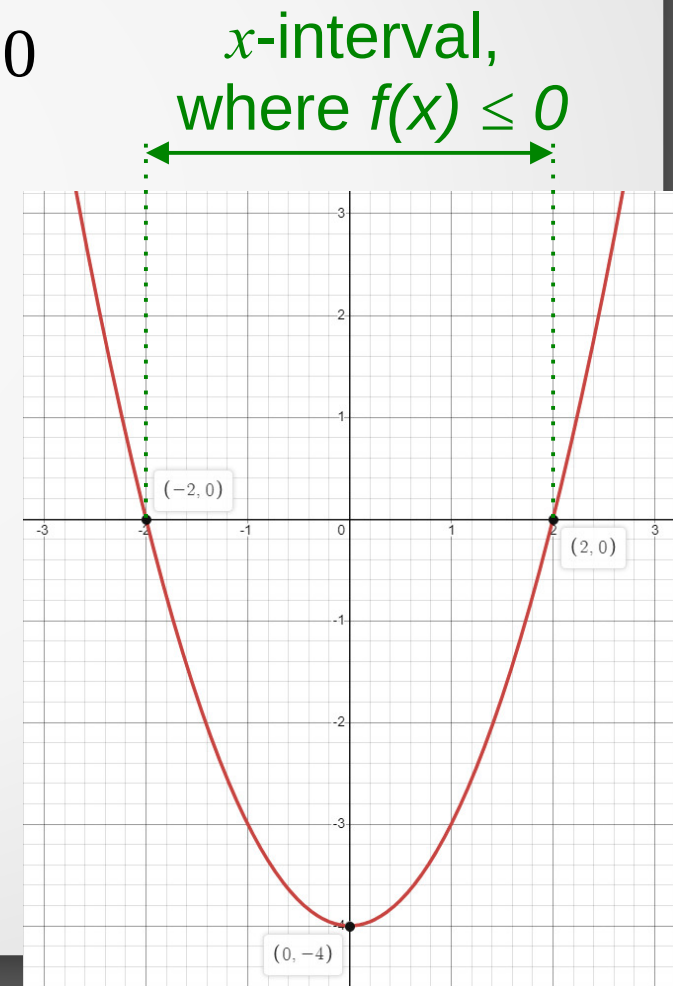
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Example: consider  $x^2 - 4 \leq 0$

1. solve the equation  $x^2 - 4 = 0$ 
  - *boundary points* (included/excluded)



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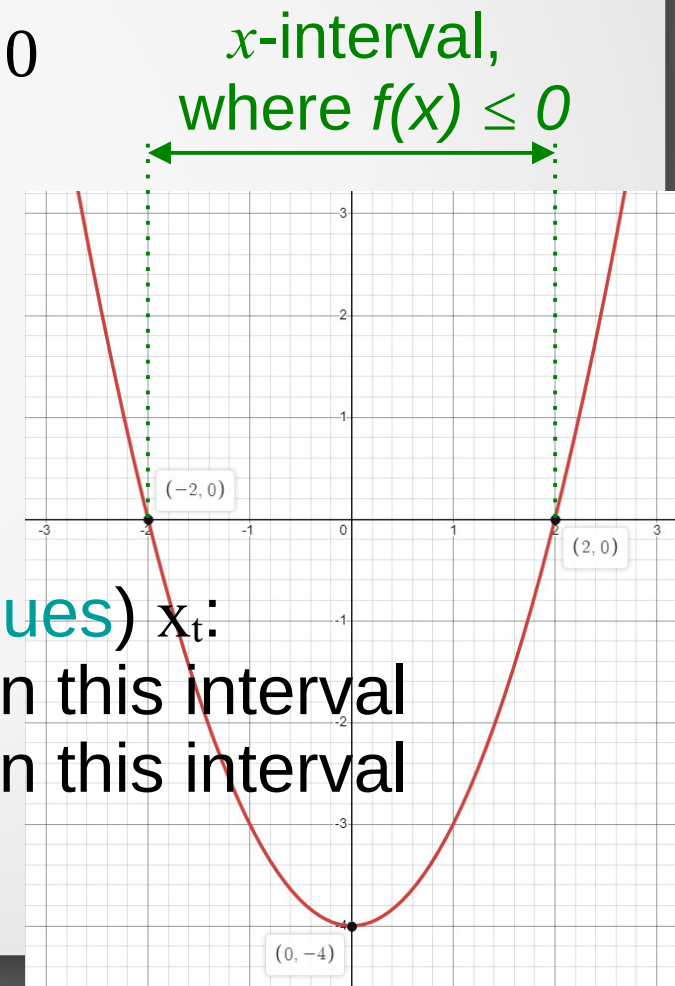
Example: consider  $x^2 - 4 \leq 0$

1. solve the equation  $x^2 - 4 = 0$

- *boundary points* (included/excluded)

2. choose *representative values* (test values)  $x_t$ :

- if  $f(x_t) < 0$  then  $f(x) < 0$  for all values in this interval
- if  $f(x_t) > 0$  then  $f(x) > 0$  for all values in this interval





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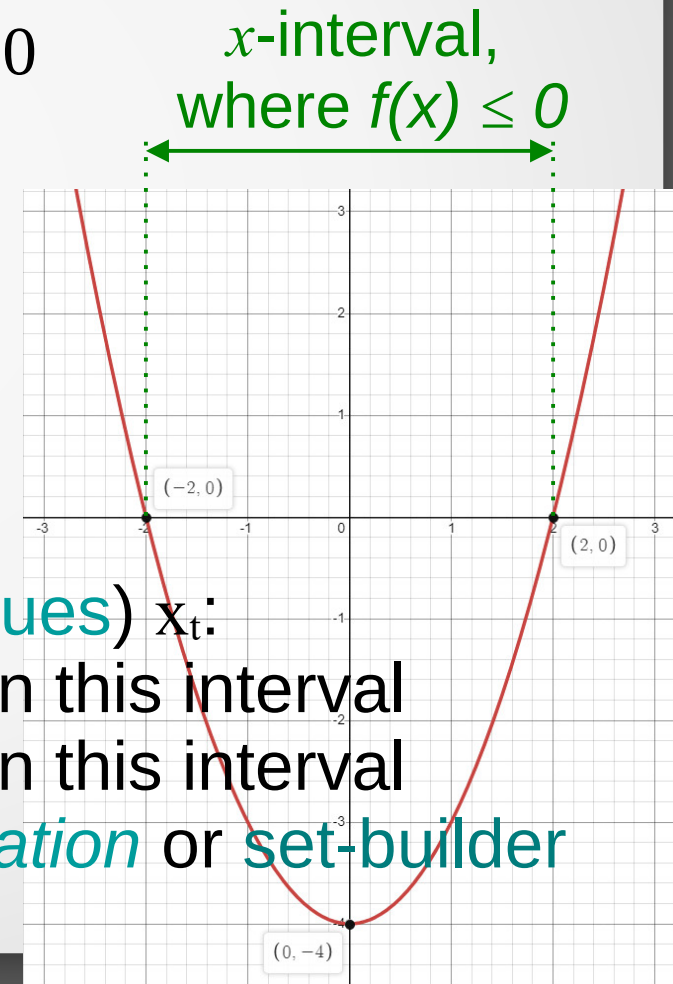
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3. write the solution set using *interval notation* or *set-builder notation*



# Polynomial and Rational Inequalities

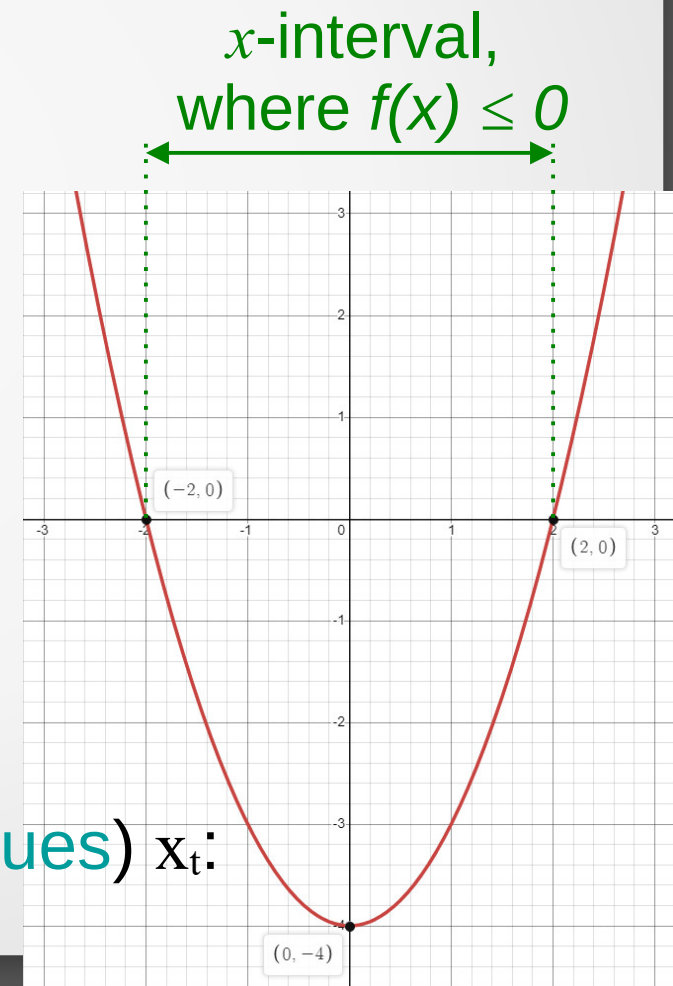
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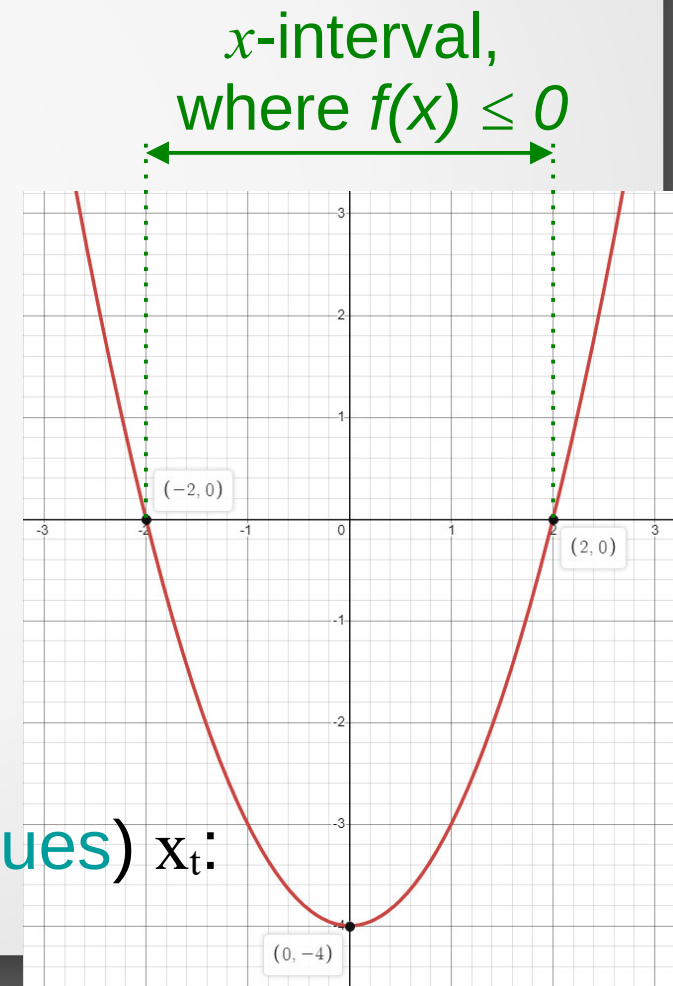
1. solve the equation  $x^2 - 4 = 0$ 
  - *boundary points* (included/excluded)

$$(x-2)(x+2) = 0$$

$$x-2=0 \quad \text{or} \quad x+2=0$$

$$x=2 \quad \quad \quad x=-2$$

2. choose *representative values* (test values)  $x_t$ :
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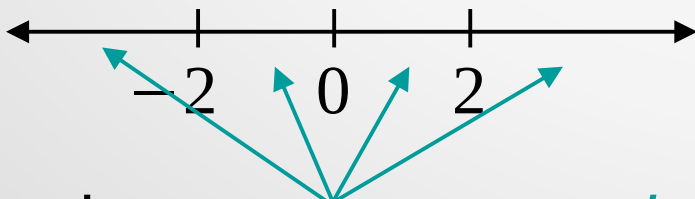
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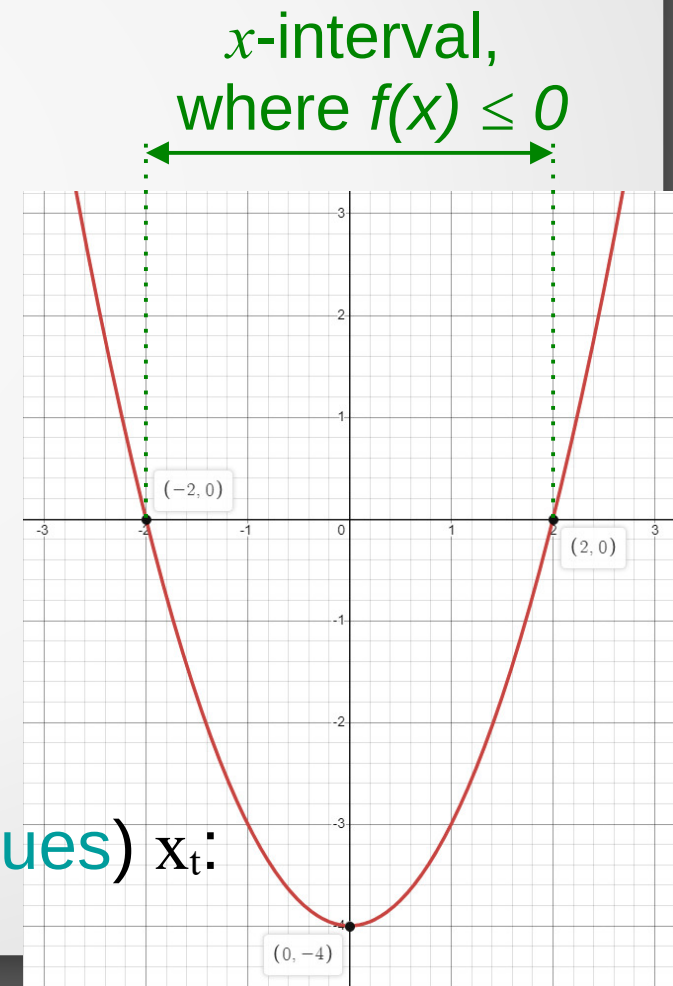
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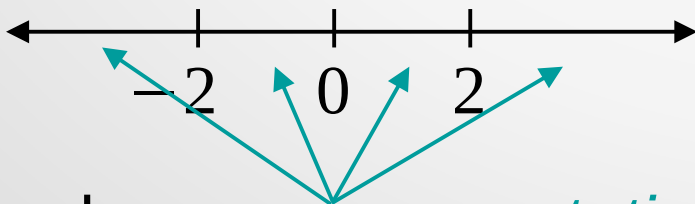
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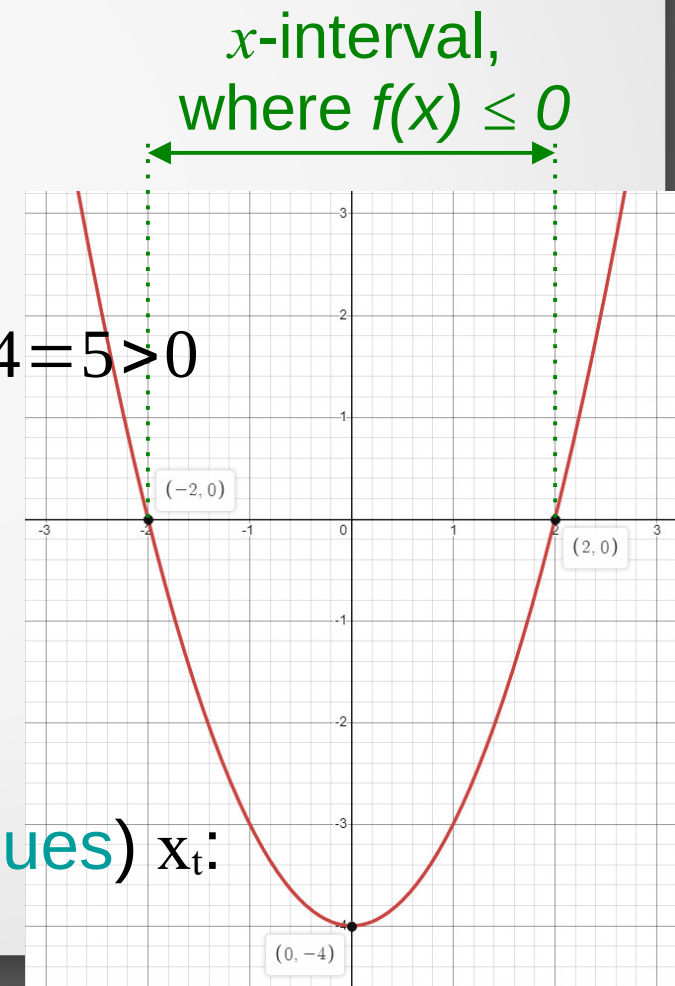
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$$x^2 - 4 \Big|_{x=-3} = 9 - 4 = 5 > 0$$

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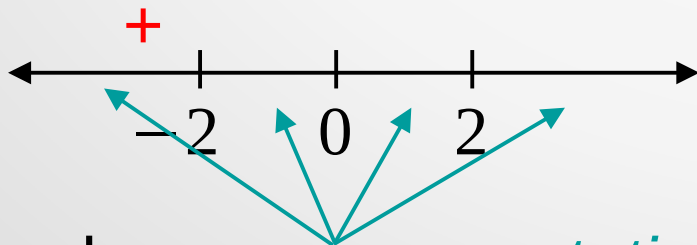
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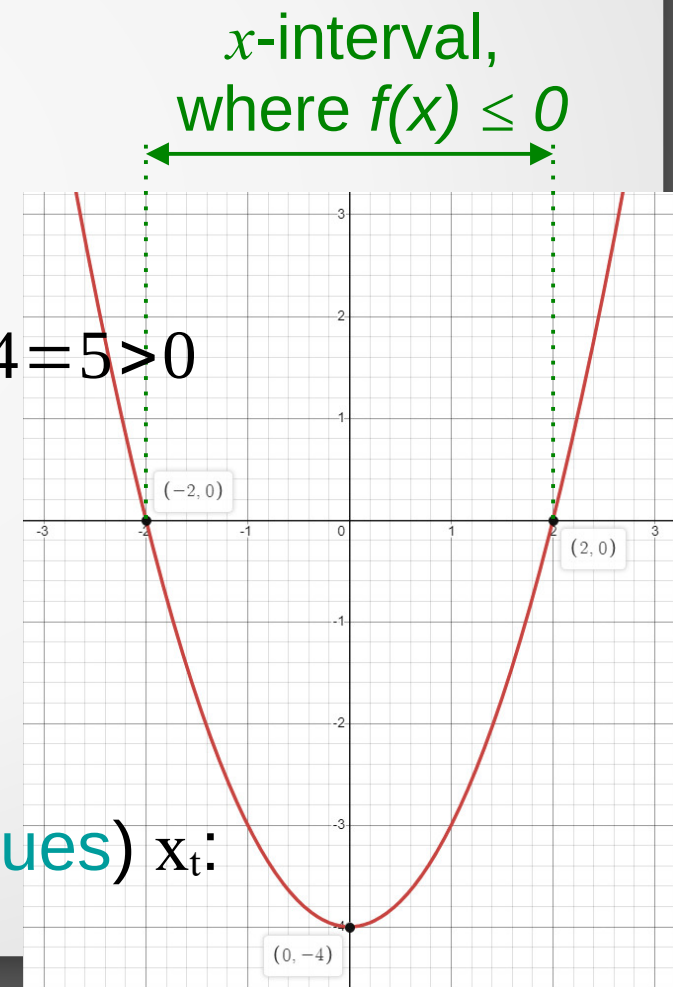
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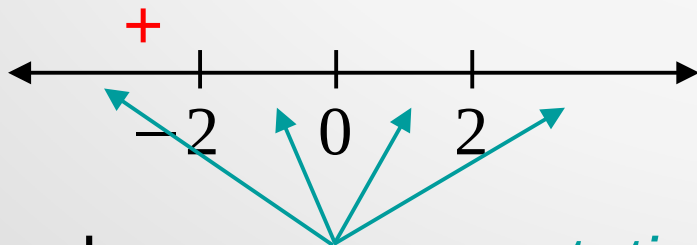
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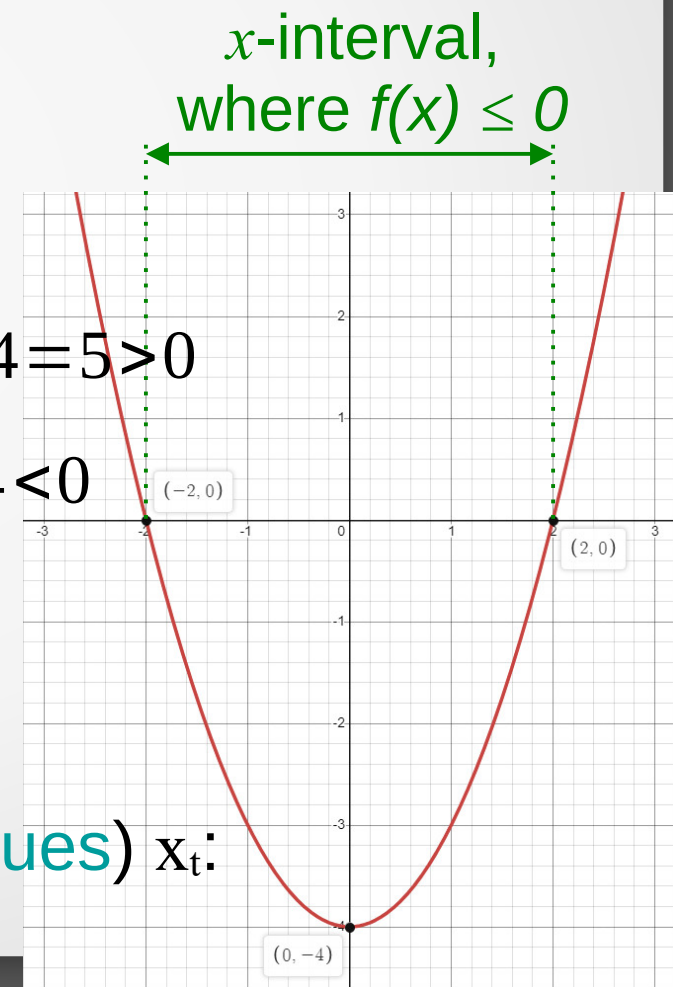
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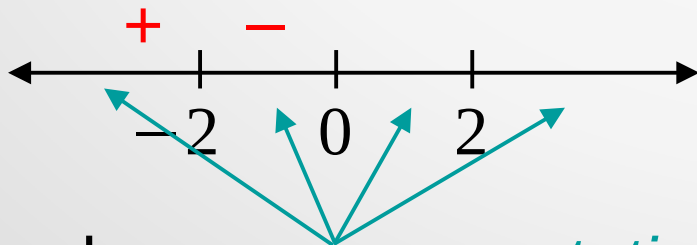
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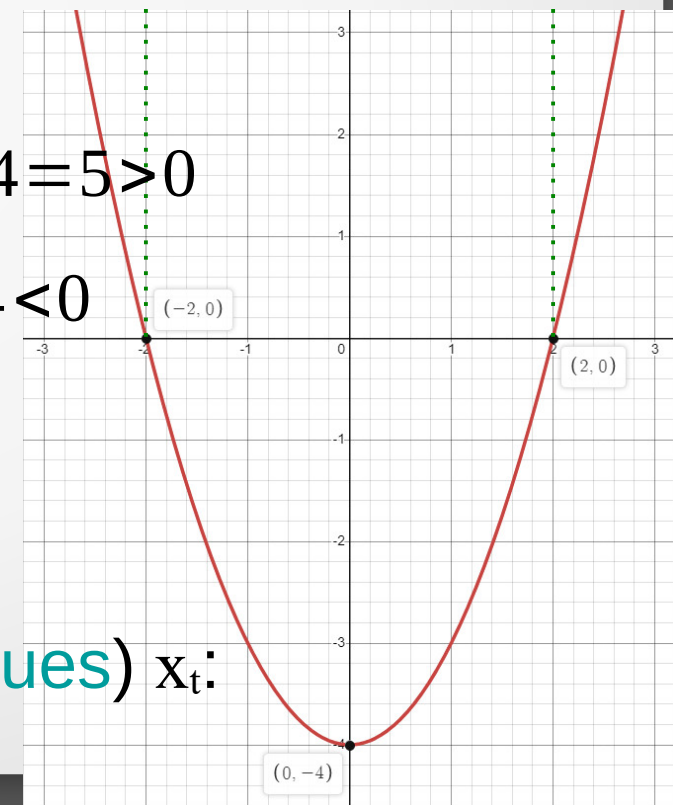


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$x$ -interval,  
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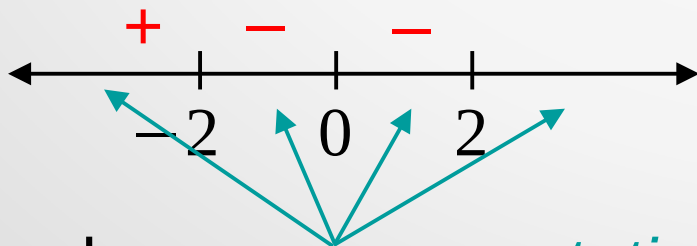
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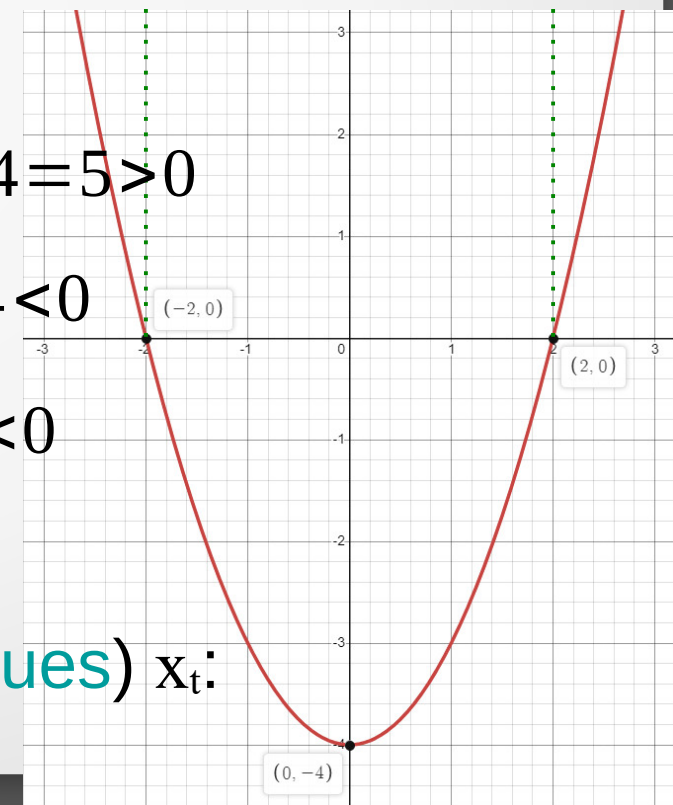
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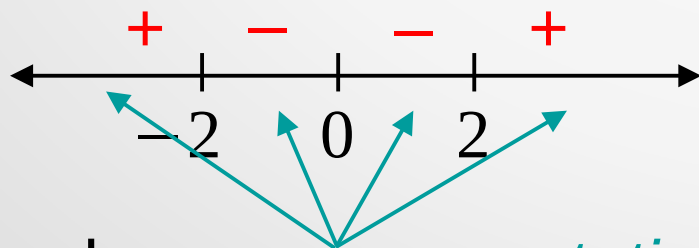
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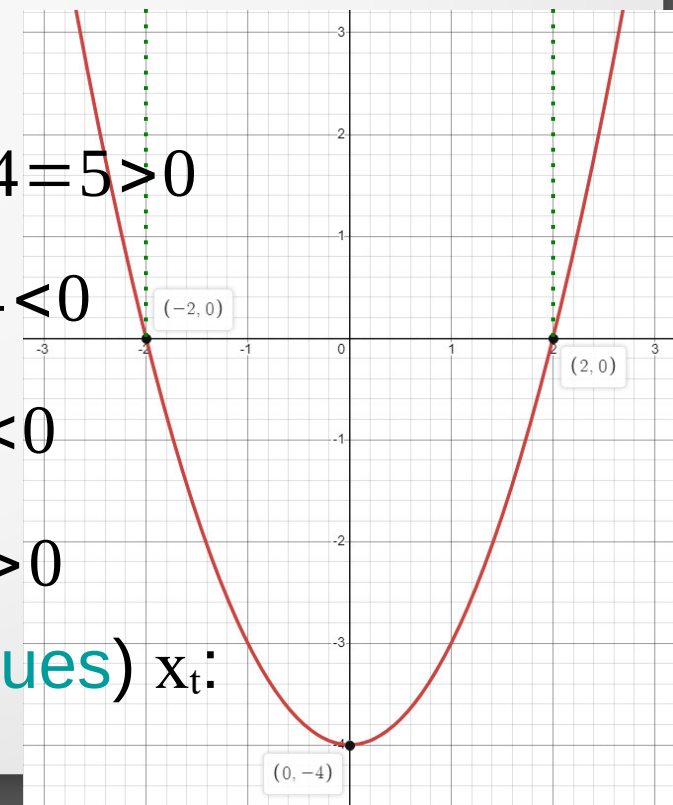
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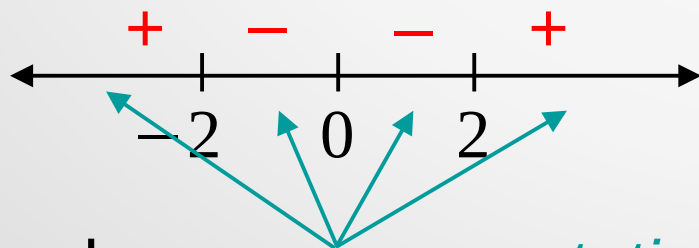
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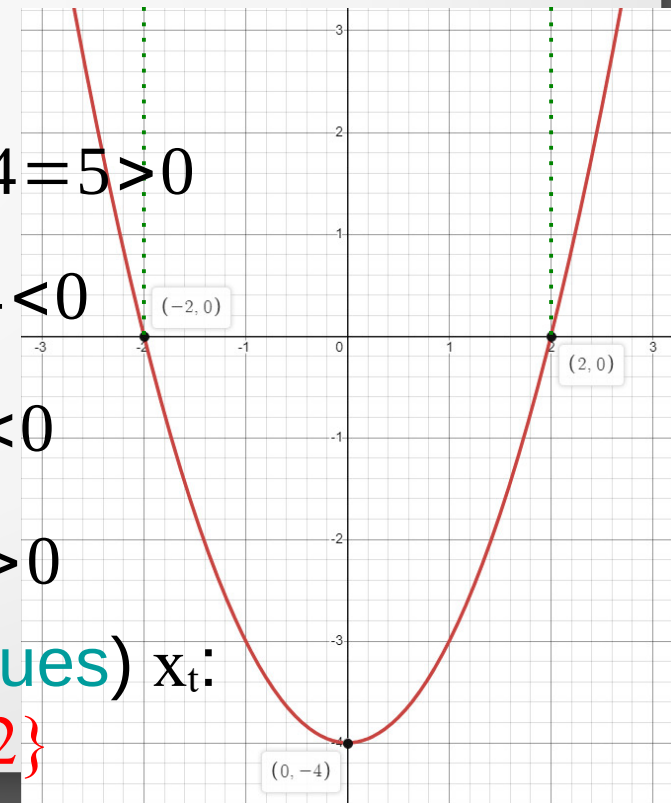
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2. choose *representative values* (test values)  $x_t$ :

3. write the solution:  $[-2, 2]$  or  $\{x \mid -2 \leq x \leq 2\}$

$x$ -interval,  
where  $f(x) \leq 0$



## In-class practice

**Exercise:** solve the inequality  $x^3 + x^2 > -4x - 4$

1. solve the equation ( *boundary points*, included/excluded)

2. choose *representative values* (test values)  $x_t$

3. write the solution:

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is any inequality that can be put into one of the forms:

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Let's move everything to the left:

$$\frac{x+5}{x+2} + 1 < 0 \quad \text{and graph} \quad f(x) = \frac{x+5}{x+2} + 1$$



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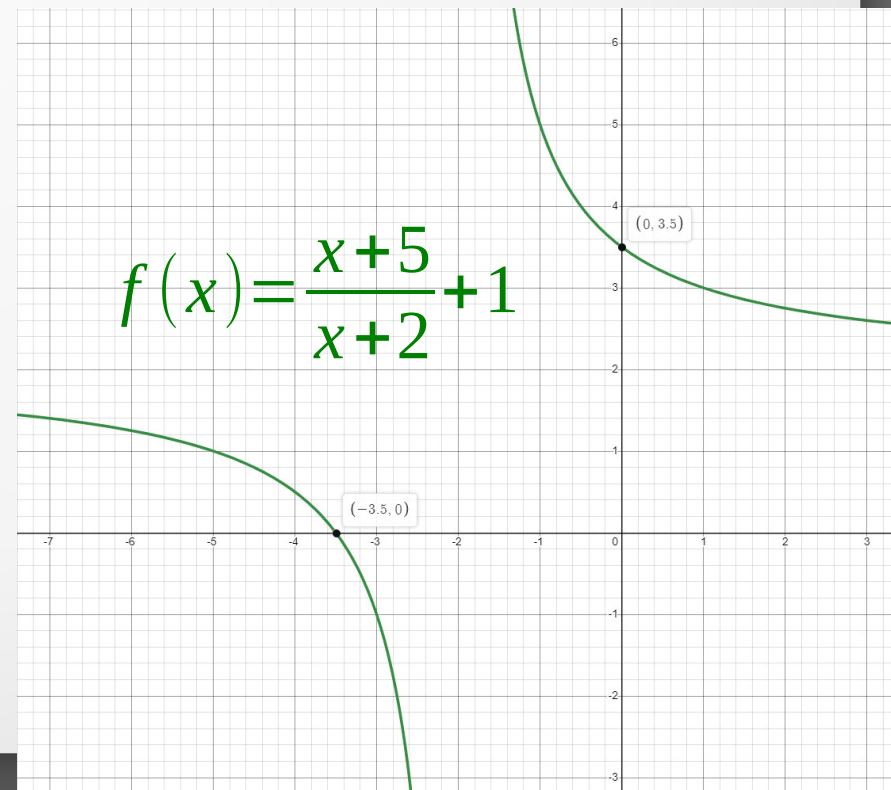
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$$\frac{2x+7}{x+2} = 0$$

$$x = -\frac{7}{2} = -3.5$$

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3. Find where the function is undefined (set denominator equal to 0):  $x+2=0$  or  $x=-2$  : *boundary points* (*excluded*)

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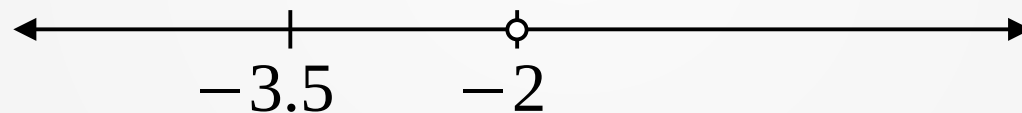
4. Set-up the number line:



# Polynomial and Rational Inequalities

**Example:** solve rational inequality  $\frac{x+5}{x+2} < -1$

5. Choose *test values*  $x_t$  and evaluate the rational function at them:

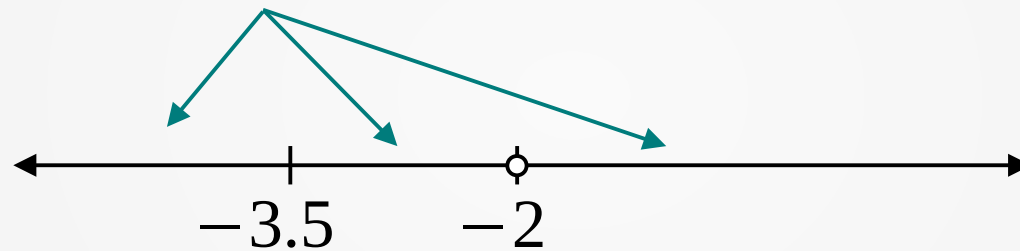




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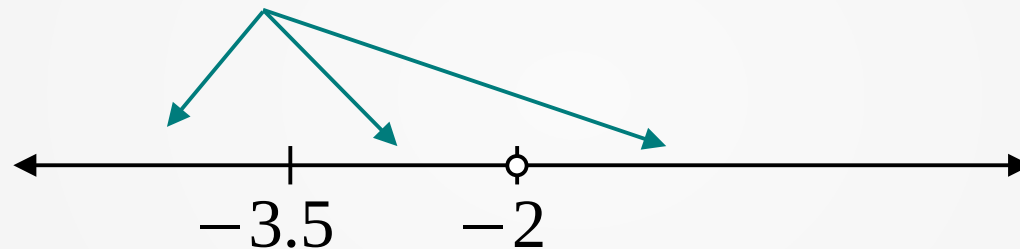
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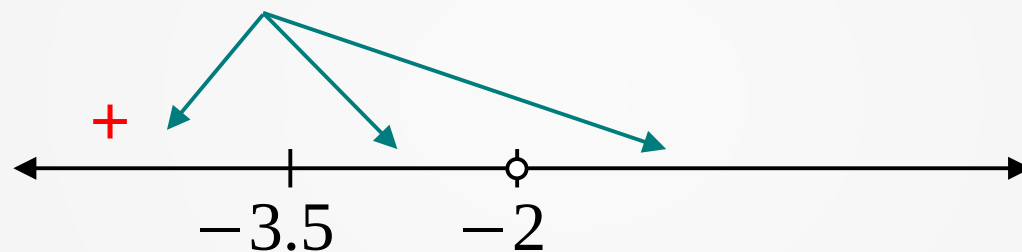


$$\frac{x+5}{x+2} + 1 \Big|_{x=-4} = \frac{1}{-2} + 1 > 0$$

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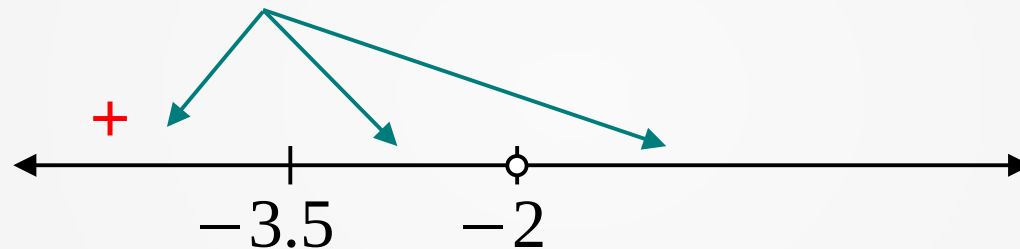


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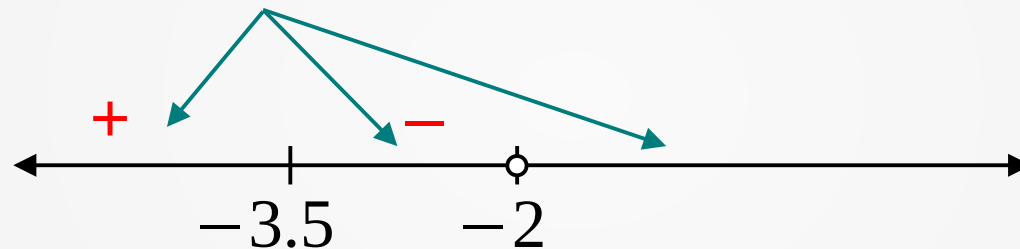
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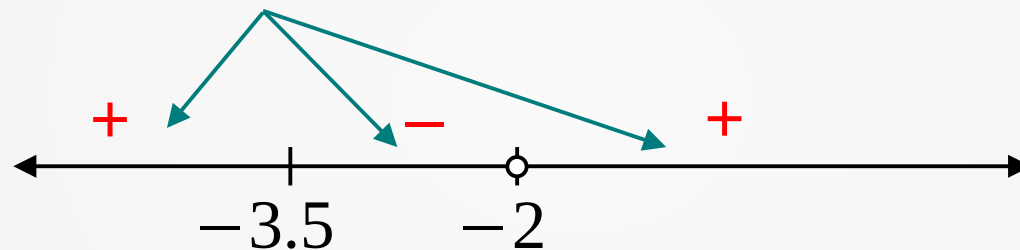
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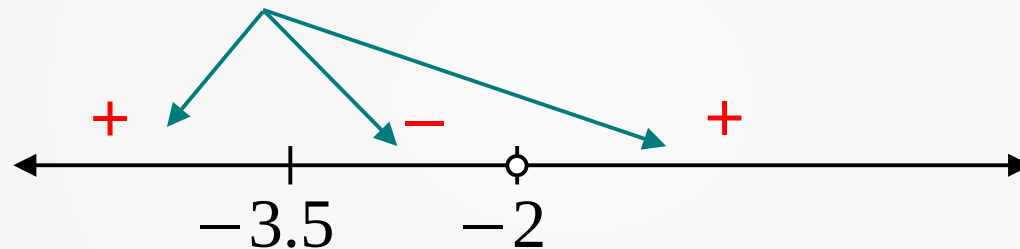
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$$\frac{x+5}{x+2} + 1 \Big|_{x=-4} = \frac{1}{-2} + 1 > 0$$

$$\frac{x+5}{x+2} + 1 \Big|_{x=-3} = \frac{2}{-1} + 1 < 0$$

$$\frac{x+5}{x+2} + 1 \Big|_{x=0} = \frac{5}{2} + 1 > 0$$

6. Answer:  $(-3.5, -2)$  or  $\{x \mid -3.5 < x < -2\}$

## In-class practice

**Exercise 1:** Solve the inequality  $\frac{4-2x}{3x+4} \geq 1$

1. Move everything on the left side:

2-3. Find *zeros* and where the function (on the left of inequality) is *undefined*.

4-5. Set up the number line and pick *test points*. Test them.

6. Put the answer:



## In-class practice

**Exercise 2:** Solve the inequality  $(x+3)(x-5) \geq 0$

# Homework assignment

**1) zyBooks:** *review* lecture slides, look through the additional materials listed on our website if you feel that you need additional practice

or

**Textbook:** *review* Section 2.7 *Polynomial and Rational Inequalities*.

**2)** We will have **Quiz 10** based on today's topics in the beginning of our next meeting.

**3) WeBWorK:** **HW 10** (due date is in one week)