Inverse Functions

Learning objectives: today we will

- Verify inverse functions.
- Determine the domain and range of an inverse function, and restrict the domain of a function to make it one-to-one.
- Find or evaluate the inverse of a function.
- Use the graph of a one-to-one function to graph its inverse function on the same axes.

Inverse Functions

Consider function f(x):

Inverse Functions

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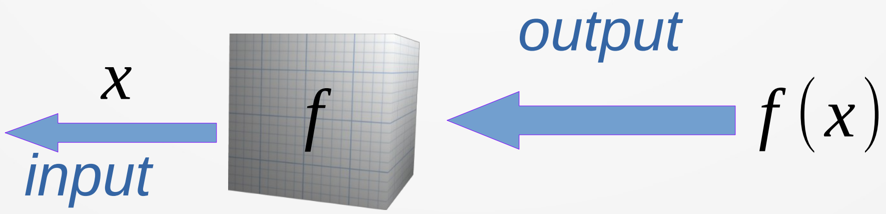
Can we reverse the process?



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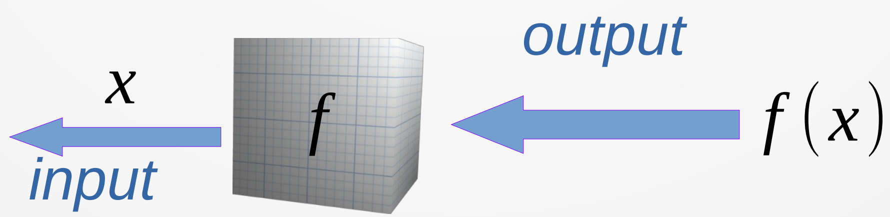

Sometimes yes, sometimes no.

Inverse Functions

Consider two functions $f(x) = 2x$ and $g(x) = \frac{x}{2}$

- they are *inverse functions*

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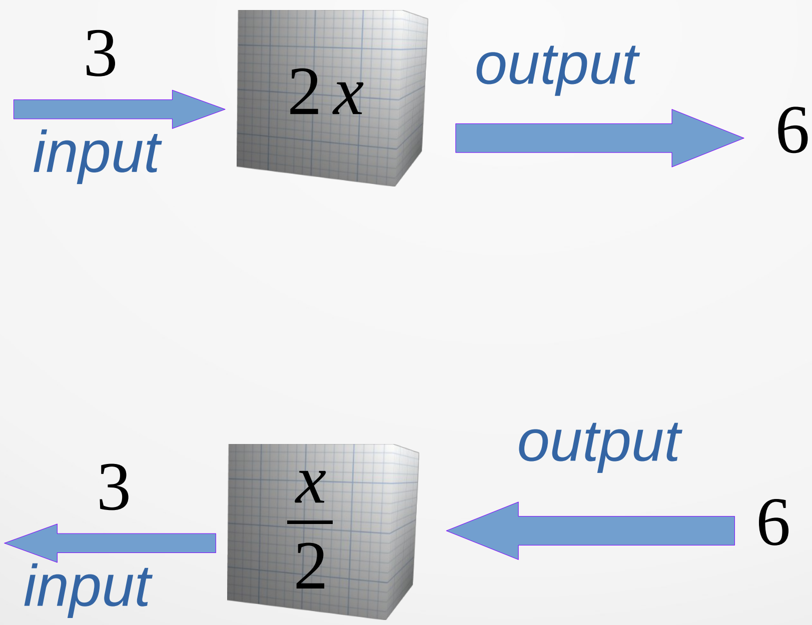

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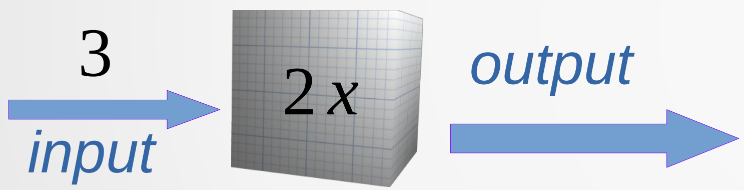

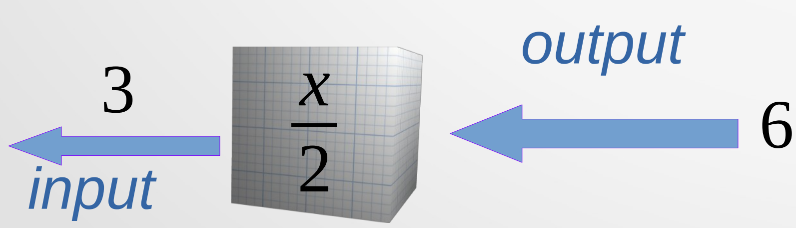

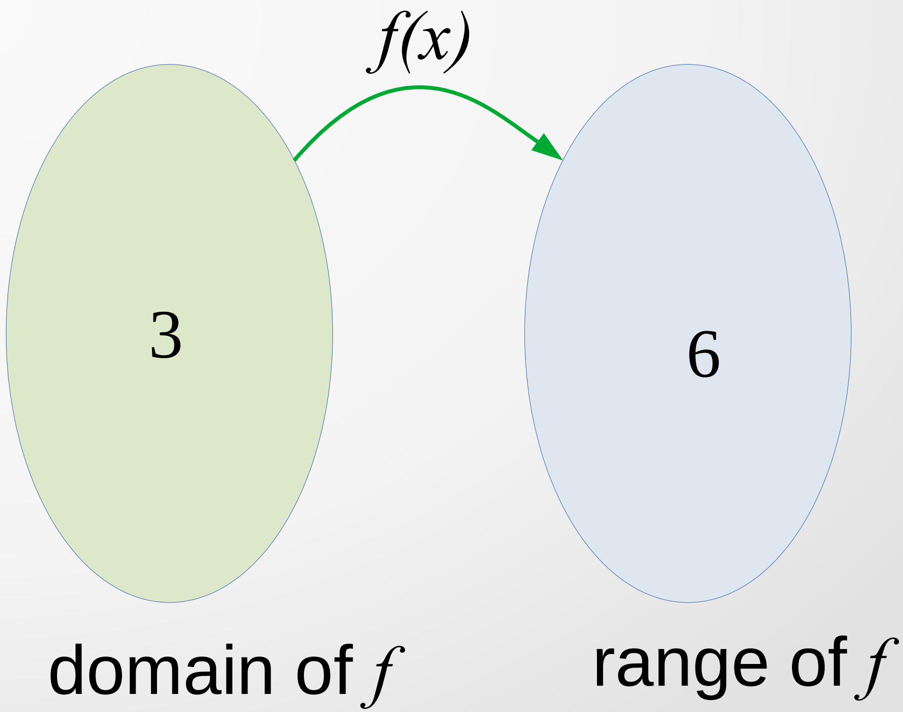

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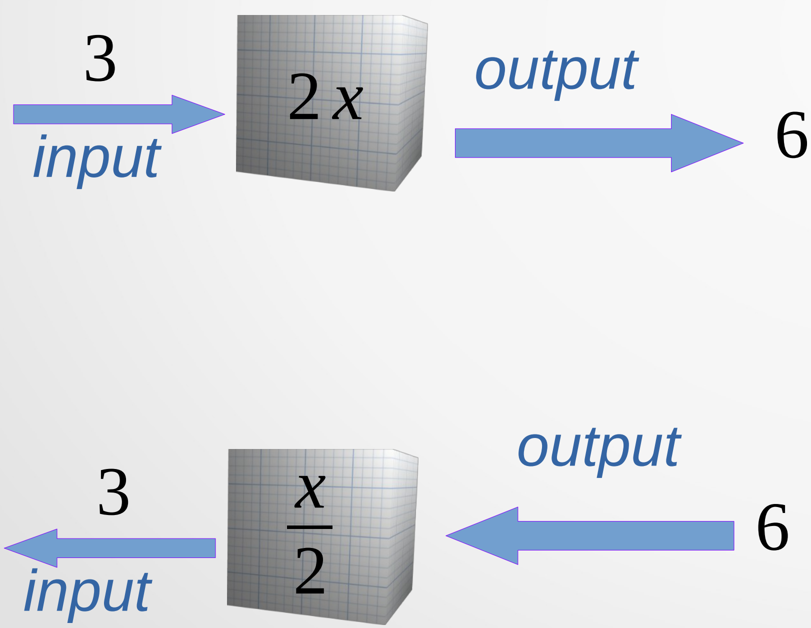

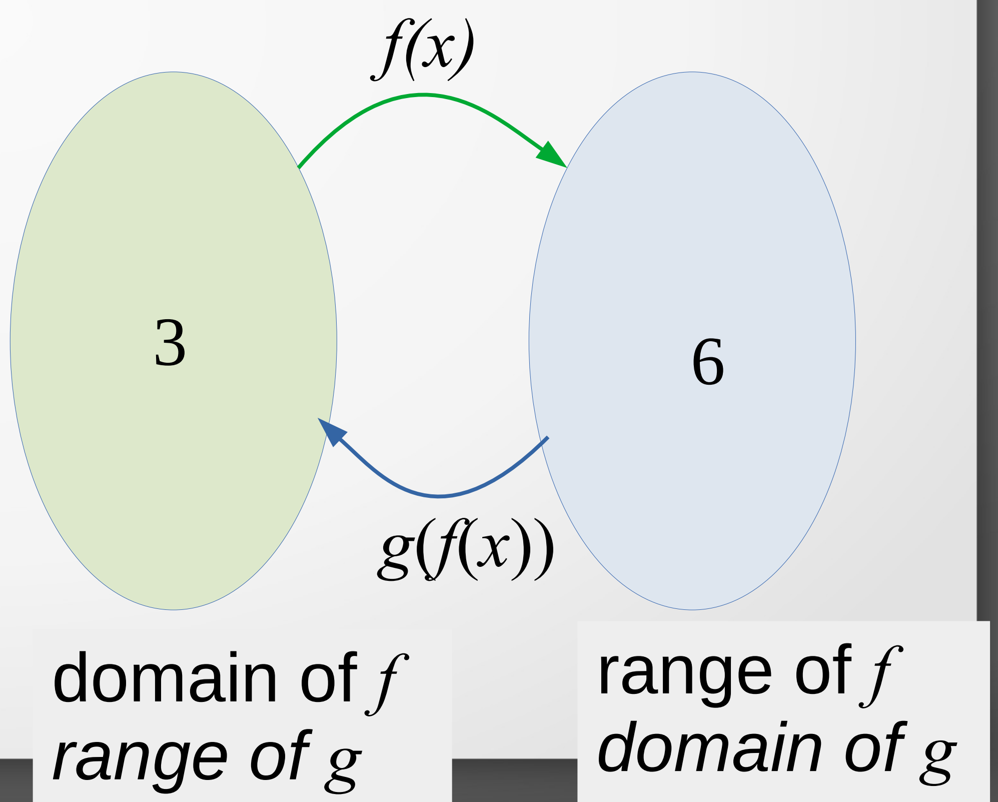

Inverse Functions

Consider two functions $f(x) = 2x$ and $g(x) = \dfrac{x}{2}$

Let’s check their compositions:

$(f \circ g)(x)$

and

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Inverse Functions

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Let's check their compositions:

$$(f \circ g)(x) = f(g(x)) = f(\frac{x}{2}) = 2\,\frac{x}{2} = x$$

and

$$(g \circ f)(x) = g(\mathrm{f}(x)) = g(2x) = \frac{2x}{2} = x$$

Inverse Functions

[Def] Let f and g be the two functions such that

$$f(g(x)) = x \text{ for every } x \in D_g$$

and

$$g(f(x)) = x \text{ for every } x \in D_f.$$

The function g is the inverse of the function f and is denoted by f^{-1} ("*f-inverse*").

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The function g is the inverse of the function f and is denoted by f^{-1} (“*f-inverse*”).

Thus $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

The domain D_f = range of f^{-1} and vise versa.

Inverse Functions

Example: let’s verify that each function is inverse of the other:

$$f(x) = 3x + 8 \qquad g(x) = \frac{x - 8}{3}$$

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Example: let's verify that each function is inverse of the other:

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We need to check that $f(g(x))=x$ and $g(f(x))=x$:

$$f(g(x))=f\left(\frac{x-8}{3}\right)=3\frac{x-8}{3}+8=x-8+8=x$$

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$$g(f(x)) = g(3x+8) = \frac{(3x+8) - 8}{3} = \frac{3x}{3} = x$$

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$$g(f(x))=g(3x+8)=\frac{(3x+8)-8}{3}=\frac{3x}{3}=x \quad \checkmark$$

Answer: yes, they are inverses of each other

In-class practice

Exercise 1:

Determine if f(x) and g(x) are inverse functions.

$$f(x)=\frac{1}{x}-7 \qquad g(x)=\frac{1}{x+7}$$

In-class practice

Exercise 2:

Determine if f(x) and g(x) are inverse functions.

$f(x) = x - 7$ $\qquad$ $g(x) = 7x$

Inverse Functions

Functions given by tables

Sometimes functions are given in a tabular format to us.

x	-2	0	1	4	7
$f(x)$	28	49	-9	15	6

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$f^{-1}(x)$	1	7	4	-2	0

Inverse Functions

Finding the inverse of a function

If we are given an equation of a function $f(x)$, then we can find the inverse of $f(x)$ following these steps:

1) in the equation for $f(x)$, replace $f(x)$ with y
2) interchange x and y
3) solve for y
   * if the equation doesn't define y as a function of x, then function f doesn't have an inverse function
   * otherwise, the resulting equation defines an inverse function f^{-1}
4) if f has inverse function, replace y in step 3) by $f^{-1}(x)$
5) check: we can verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Inverse Functions

Example: let’s find an inverse of $f(x)=5x-9$

$$f(x)=5x-9$$

replace *f(x)* with *y*

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Example: let's find an inverse of $f(x)=5x-9$

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$y=5x-9$ replace *f(x)* with *y*

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$x = 5y - 9$ interchange *x* and *y*

$x + 9 = 5y$ solve for y

$\frac{x+9}{5} = y$

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$y=5x-9$ replace *f(x)* with *y*

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$y=5x-9$ replace *f(x)* with *y*

$x=5y-9$ interchange *x* and *y*

$x+9=5y$ solve for y

$\frac{x+9}{5}=y$ defines a function of *x*

$f^{-1}(x)=\frac{x+9}{5}$ replace *y* with $f^{-1}(x)$

In-class practice

Exercise 1: find an inverse, if it exists, of $f(x) = \sqrt[3]{2x+3}$

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replace *f(x)* with *y*
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solve for y
defines a function of *x* ?
replace *y* with $f^{-1}(x)$

In-class practice

Exercise 2: find an inverse, if it exists, of $f(x)=3x^2-10$

replace *f(x)* with *y*
interchange *x* and *y*
solve for y
defines a function of *x* ?
replace *y* with $f^{-1}(x)$

Inverse Functions

The horizontal line test and one-to-one functions

A function f, has an inverse function f^{-1} if there is no horizontal line that intersects the graph of f at more than one point.

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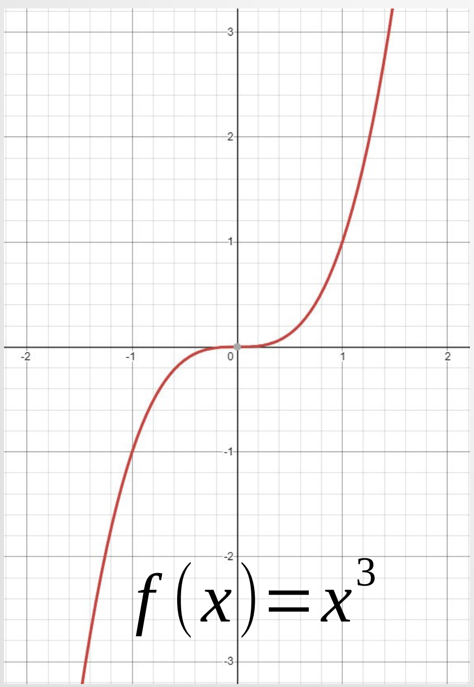
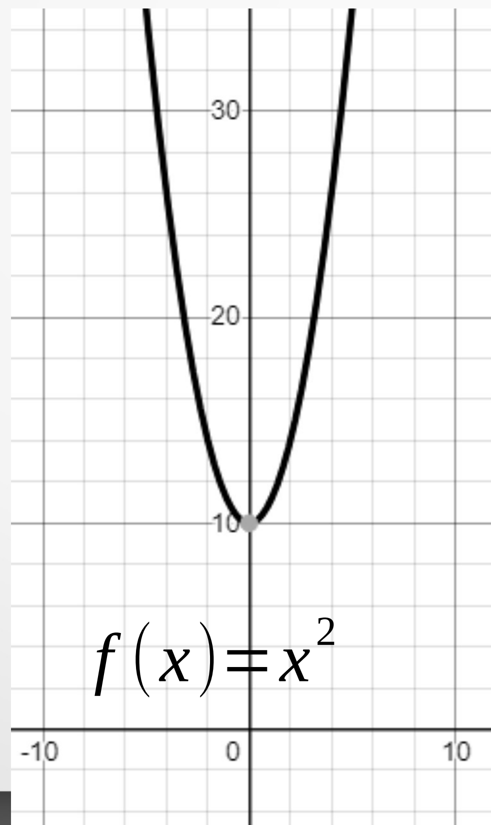
The horizontal line test and one-to-one functions

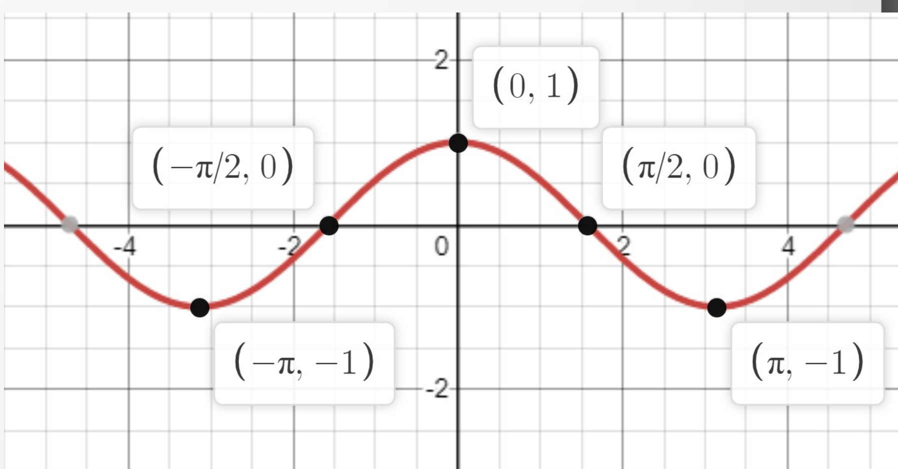
A function *f*, has an inverse function f^{-1} if there is <u>no horizontal line</u> that intersects the graph of *f* at <u>more than one point</u>.



$f(x) = x^3$



$f(x) = x^2$



$f(x) = \cos x$

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The horizontal line test and one-to-one functions

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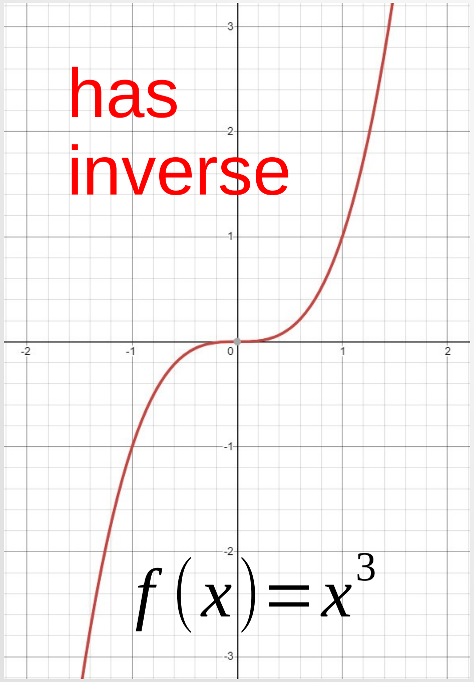
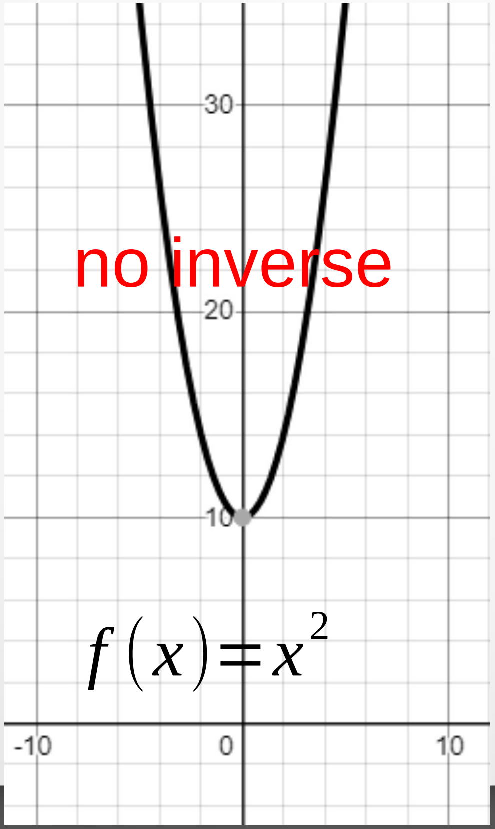
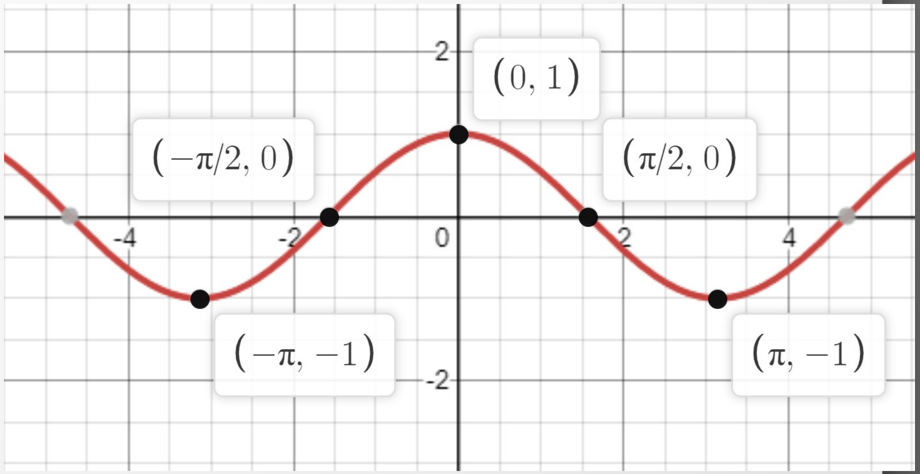




no inverse



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[Def] A *one-to-one function* is a function in which no two different ordered pairs have the same second component, i.e. if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

Inverse Functions

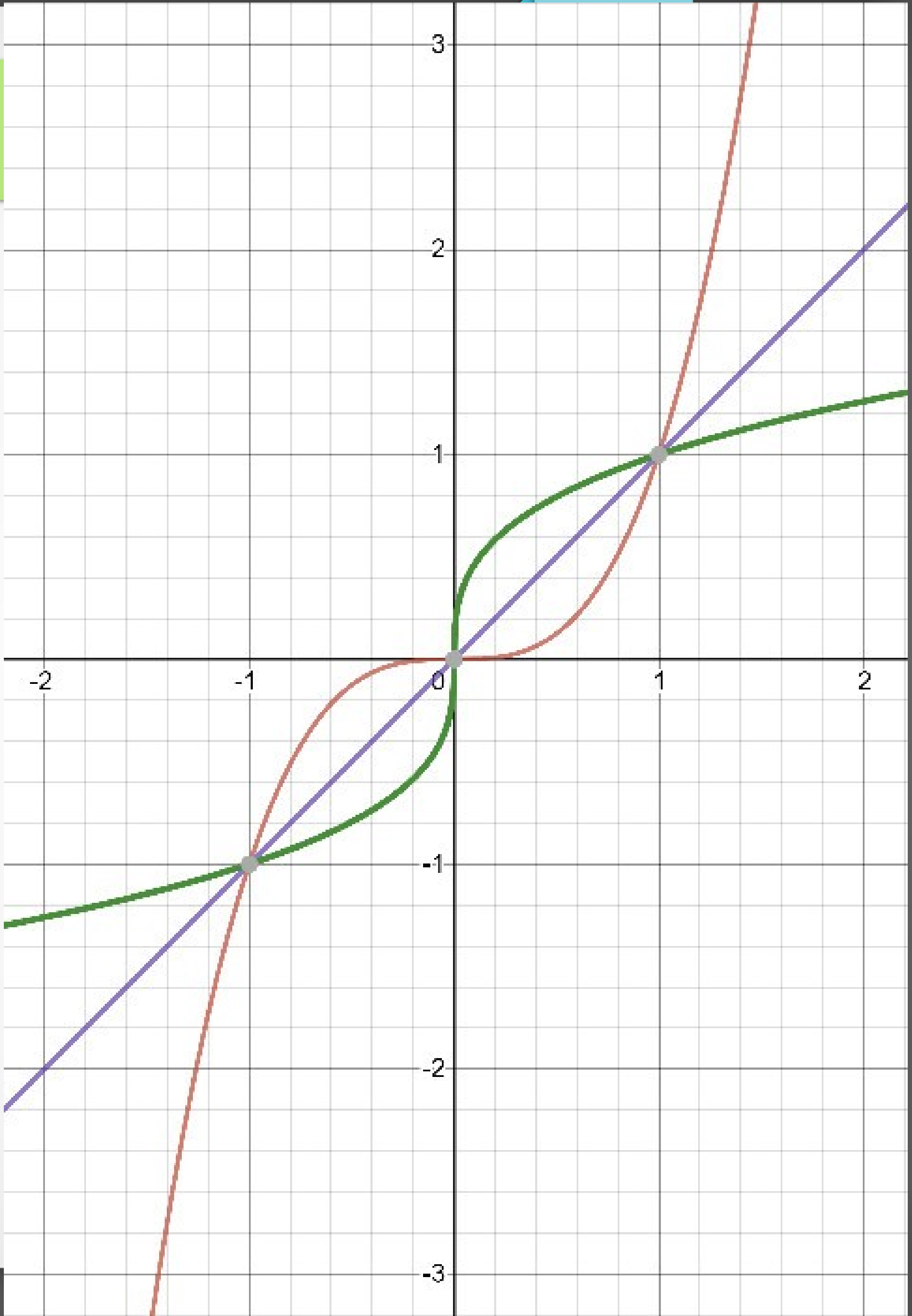
Graphs of f and f^{-1}

Consider the graphs of

$f(x) = x^3$ and

$g(x) = \sqrt[3]{x}$

the graphs of inverse functions are reflections of each other about the line $y = x$.



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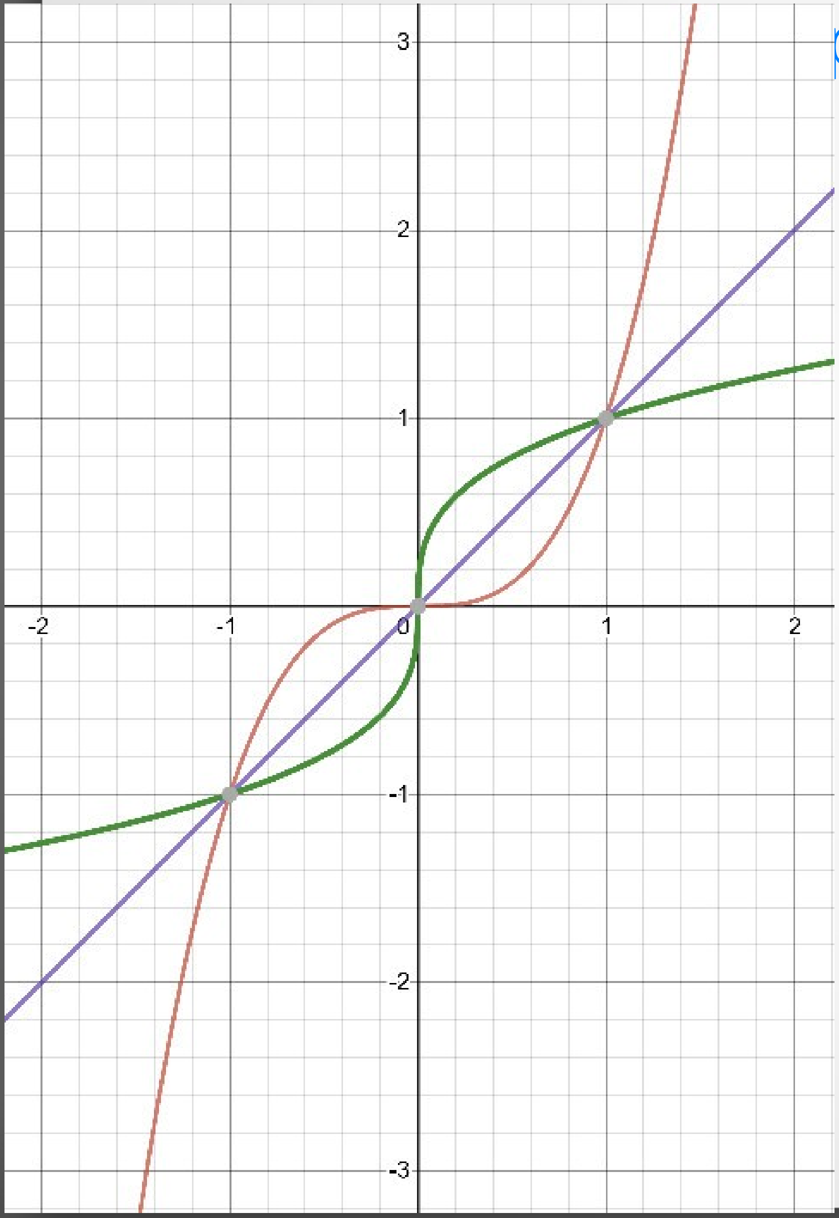

phs of f and f^{-1}

$f(x)=x^3$ and $g(x)=\sqrt[3]{x}$

The coordinates of points in graphs of inverse functions are switched: see the point (1.26,2) on the graph of x^3 and the point (2,1.26) on the graph of $\sqrt[3]{x}$.

x	y = x^3
0	0
1	1
2	8
3	27

x	y = $\sqrt[3]{x}$
0	0
1	1
8	2
27	3

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- Verify inverse functions.
- Determine the domain and range of an inverse function, and restrict the domain of a function to make it one-to-one.
- Find or evaluate the inverse of a function.
- Use the graph of a one-to-one function to graph its inverse function on the same axes.