Dividing Polynomials. Remainder and Factor Theorems.

Today we will discuss:

- Long Division of Polynomials
- Synthetic Division of Polynomials
- The Division Algorithm
- Remainder Theorem
- Factor Theorem

Dividing Polynomials

Long Division of Polynomials

Let’s divide $6x^2 - 10x + 21$ by $2x + 3$:

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$2x+3 \overline{\smash{\big)}\, 6x^2 - 10x + 21}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$2x+3\overline{\smash{\big)}\,6x^2 - 10x + 21}$$

$$6x^2 \div 2x = 3x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

$$\begin{array}{r} 3x \\ 2x+3 \overline{)\, 6x^2 - 10x + 21} \end{array}$$

$$6x^2 \div 2x = 3x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

multiply

$$
\begin{array}{r}
\color{magenta}{3x} \\
2x+3\overline{\big)\,6x^2-10x+21} \\
-\left(6x^2+9x\right) \\
\hline
\end{array}
$$

$$6x^2 \div 2x = \color{magenta}{3x}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

multiply

$$\begin{array}{r|l} & 3x \\ 2x+3 & 6x^2-10x+21 \\ & -(6x^2+9x) \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

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\begin{array}{r}
3x \\
2x+3\,\overline{\smash{)}\,6x^2 - 10x + 21} \\
-(6x^2 + 9x) \\
\hline
-19x
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r l}
 & \color{magenta}{3x} \\
2x+3 \overline{)} & 6x^2 - 10x + 21 \\
 & -(6x^2 + 9x) \\
\hline
 & \color{green}{-19x} + 21
\end{array}
$$

$6x^2 \div 2x = \color{magenta}{3x}$

$6x^2 - 10x - 6x^2 - 9x = \color{green}{-19x}$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2-10x+21$ by $2x+3$:

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\begin{array}{r|l}
 & 3x \\
2x+3 & 6x^2-10x+21 \\
 & -(6x^2+9x) \\
\hline
 & -19x+21
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2-10x-6x^2-9x=-19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

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\begin{array}{r}
3x \quad -\frac{19}{2} \\
2x+3 \,\big)\, \overline{6x^2 - 10x + 21} \\
-(6x^2 + 9x) \\
\hline
-19x + 21
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

multiply

$$\begin{array}{r} 3x - \frac{19}{2} \\ 2x+3 \overline{\big) 6x^2 - 10x + 21} \\ -(6x^2 + 9x) \\ \hline -19x + 21 \\ -(2(-\frac{19}{2})x + 3(-\frac{19}{2})) \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

Dividing Polynomials

Long Division of Polynomials

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\begin{array}{r|l}
 & 3x - \frac{19}{2} \\
\hline
2x+3 & 6x^2 - 10x + 21 \\
 & -(6x^2 + 9x) \\
\hline
 & -19x + 21 \\
 & -(-19x - 28.5)
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

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Dividing Polynomials

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 & -(-19x - 28.5) \\
\hline
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$
\begin{array}{r l}
 & 3x - \frac{19}{2} \\
2x+3 \,\big)\!\! & \overline{6x^2 - 10x + 21} \\
 & -(6x^2 + 9x) \\
 & \overline{\qquad -19x + 21} \\
 & \qquad -(-19x - 28.5) \\
 & \qquad \overline{\qquad\qquad 49.5}
\end{array}
$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

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\begin{array}{r|l}
 & 3x - \frac{19}{2} \\
2x+3 & 6x^2 - 10x + 21 \\
 & -(6x^2 + 9x) \\
\hline
 & -19x + 21 \\
 & -(-19x - 28.5) \\
\hline
 & 49.5
\end{array}
$$

quotient (of the division): $3x - \frac{19}{2}$

remainder (of the division): 49.5

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

divide
multiply
subtract
bring down the next term

$$
\begin{array}{r}
3x - \frac{19}{2} \\
2x+3 \overline{\smash{)}\, 6x^2 - 10x + 21} \\
-(6x^2 + 9x) \\
\hline
-19x + 21 \\
-(-19x - 28.5) \\
\hline
49.5
\end{array}
$$

quotient (of the division): $3x - \frac{19}{2}$

remainder (of the division): 49.5

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Dividing Polynomials

Long Division of Polynomials

divide
multiply
subtract
bring down the next term

Let's divide $6x^2 - 10x + 21$ by $2x + 3$:

$$\begin{array}{r} 3x - \frac{19}{2} \\ 2x+3 \overline{)\, 6x^2 - 10x + 21} \\ -(6x^2 + 9x) \\ \hline -19x + 21 \\ -(-19x - 28.5) \\ \hline 49.5 \end{array}$$

$$6x^2 \div 2x = 3x$$

$$6x^2 - 10x - 6x^2 - 9x = -19x$$

$$-19x \div 2x = -\frac{19}{2}$$

$$-19x + 21 + 19x + 28.5 = 49.5$$

Answer: $(6x^2 - 10x + 21) \div (2x + 3) = (3x - \frac{19}{2})\, R\, 49.5 = 3x - \frac{19}{2} + \frac{49.5}{2x+3}$

In-class practice

Exercise: divide $18x^4+9x^3+3x^2$ by $3x^2+1$:

$$3x^2+1\ \overline{)\,18x^4+9x^3+3x^2}$$

divide
multiply
subtract
bring down the next term

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend

divisor *quotient* *remainder*

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend → $f(x)$; *divisor* → $d(x)$; *quotient* → $q(x)$; *remainder* → $r(x)$

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

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dividend

divisor *quotient* *remainder*

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

$$6x^2 - 10x + 21 = (2x+3)\left(3x - \frac{19}{2}\right) + 49.5$$

Dividing Polynomials

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials with $d(x) \neq 0$ and *degree of* $d(x) \leq$ *degree of* $f(x)$, then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend *divisor* *quotient* *remainder*

Recall example: $6x^2 - 10x + 21 \div 2x + 3$

$$6x^2 - 10x + 21 = (2x+3)\left(3x - \frac{19}{2}\right) + 49.5$$

If $r(x) = 0$, then $d(x)$ *divides evenly into* $f(x)$, and $d(x)$ and $q(x)$ are *factors* of $f(x)$.

Dividing Polynomials

Synthetic Division of Polynomials

Synthetic division works well when the divisor is in the form $x - C$.

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form $x - C$, hence

$x+3 = x - (-3)$.

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form x – C, hence x+3 = x - (- 3). List all the coefficients of the *dividend* and C:

	x^2	x	*const*
-3	6	-10	21

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form x – C, hence x+3 = x - (- 3). List all the coefficients of the *dividend* and C:

	x^2	x	*const*
-3	6	-10	21
	↓	-18	
	6		

multiply

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$. List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	
	6	-28	

multiply

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2-10x+21$ by $x+3$:

1) the divisor must be in the form x – C, hence x+3 = x - (- 3). List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	84
	6	-28	

multiply

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form x – C, hence x+3 = x - (- 3). List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	84
	6	-28	105

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$. List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	$\downarrow$	-18	84
	6	-28	105
	x	$const$	r

Dividing Polynomials

Synthetic Division of Polynomials

Let's divide $6x^2 - 10x + 21$ by $x + 3$:

1) the divisor must be in the form $x - C$, hence $x+3 = x - (-3)$. List all the coefficients of the *dividend* and C:

	x^2	x	$const$
-3	6	-10	21
	↓	-18	84
	6	-28	105
	x	$const$	r

2) the result: $(6x^2 - 10x + 21) \div (x+3) = (6x - 28)\, R\, 105 = 6x - 28 + \frac{105}{x+3}$

In-class practice

Exercise: divide x^3+4x^2-5x+5 by $x-3$ using synthetic division.

In-class practice

Exercise: divide x^3+4x^2-5x+5 by $x-3$ using synthetic division.

	x^3	x^2	x	$const$
$?$	?	?	?	?
	$\downarrow$			
	x^2	x	$const$	r

Dividing Polynomials

Remainder Theorem

If the polynomial $f(x)$ is divided by x-C, then the remainder is $f(C)$.

Dividing Polynomials

Remainder Theorem

If the polynomial $f(x)$ is divided by x-C, then the remainder is $f(C)$.

Example from previous slides:

$$\left(6x^2-10x+21\right)\div\left(x+3\right)=\left(6x-28\right)R\,105=6x-28+\frac{105}{x+3}$$

Dividing Polynomials

Remainder Theorem

If the polynomial *f(x)* is divided by *x-C*, then the remainder is *f(C)*.

Example from previous slides:

$$(6x^2-10x+21)\div(x+3)=(6x-28)\,R\,105=6x-28+\frac{105}{x+3}$$

$$f(x)=6x^2-10x+21$$

$$f(C)=f(-3)=6\cdot(-3)^2-10\cdot(-3)+21=54+30+21=105$$

In-class practice

Using the *remainder theorem*, if possible, answer the following questions:

(1) find the remainder of the division

$$(x^3 - 4x^2 + 5x + 3) \div (x - 3)$$

(2) find the remainder of the division

$$(x^2 + 10x + 21) \div (x + 7)$$

(3) find the remainder of the division

$$(18x^4 + 9x^3 + 3x^2) \div (3x^2 + 1)$$

Dividing Polynomials

Factor Theorem

Let $f(x)$ be a polynomial
a) if $f(C)=0$ then x-C is a factor of $f(x)$
b) if x-C is a factor of $f(x)$ then $f(C)=0$

In-class practice

Exercise 1: Solve the equation $2x^3-3x^2-11x+6=0$ given that -2 is a zero of $f(x)=2x^3-3x^2-11x+6$.

In-class practice

Exercise 2: Use long or synthetic division to divide $x^3 - 4x^2 + x + 6$ by $x + 1$. Then use the result to find all the zeros of $f(x) = x^3 - 4x^2 + x + 6$.

Homework assignment

1) zyBooks: *review* Sections
3.5 *Dividing polynomials*, and
3.6 *Zeros of polynomial functions* (everything until **Using the rational zero theorem to find rational zeros**)

or

Textbook: *review* Section 2.4 *Dividing Polynomials, Remainder and Factor Theorems*.

2) We will have Quiz 7 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 7 (due date is in one week)