Quadratic Functions and Polynomial Functions

Quadratic Functions

$y = ax^2+bx+c$ a,b,c are real numbers, $a \neq 0$.

Quadratic Functions and Polynomial Functions

Quadratic Functions

$y = ax^2+bx+c$

completing the square

$f(x) = ax^2 + bx + c = a(x-h)^2 + k$, *quadratic function in standard form*

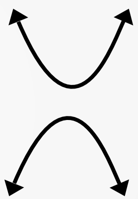
a, b, c, h, k are real numbers, $a \neq 0$.

Quadratic Functions and Polynomial Functions

Quadratic Functions

$f(x) = a(x-h)^2 + k$, a, h, k are real numbers, $a \neq 0$.

- if $a > 0$, parabola opens *upward*
  if $a < 0$, parabola opens *downward*



- the *vertex* is at the point (h,k)

- it is *symmetric* with respect to the line $x = h$

Quadratic Functions and Polynomial Functions

Quadratic Functions

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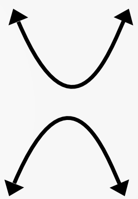


- the *vertex*'s x-coordinate is $-\frac{b}{2a}$, use it to find the y-coordinate
- it is *symmetric* with respect to the line $x = -\frac{b}{2a}$

Quadratic Functions and Polynomial Functions

Example: graph function $f(x) = -2(x+1)^2 + 5$,

Quadratic Functions and Polynomial Functions

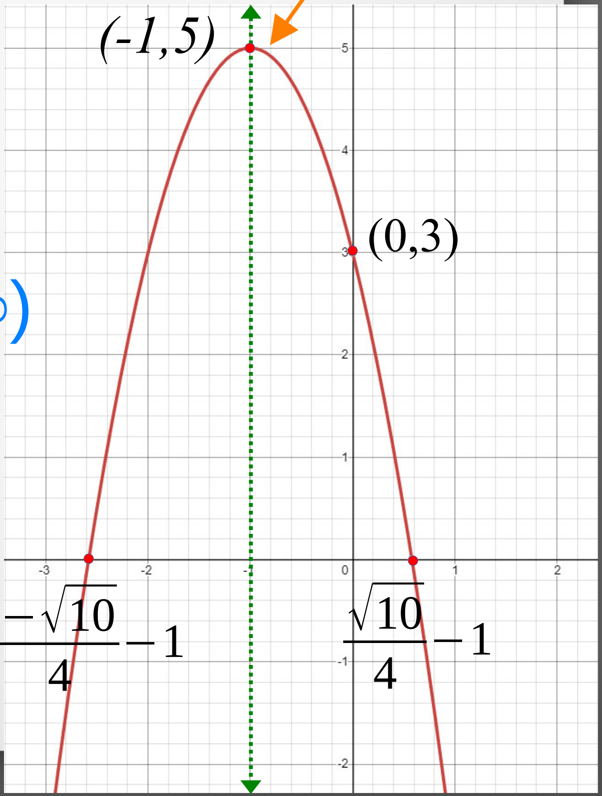
Example: graph function $f(x) = -2(x+1)^2 + 5$,

k

absolute max

$a = -2 < 0$, *hence parabola opens downward*

$-h$



vertex is at the point $(-1,5)$

range: $(-\infty, 5]$
domain: $(-\infty, \infty)$

parabola is *symmetric* with respect to the line $x = -1$

x-intercepts: $-2(x+1)^2 + 5 = 0$

$$x = \frac{\pm\sqrt{10}}{4} - 1$$

y-intercept: $f(0) = 3$ $(0,3)$

Quadratic Functions and Polynomial Functions

Example: graph function $g(x) = 3x^2 - 12x + 5$

Quadratic Functions and Polynomial Functions

Example: graph function $g(x) = 3x^2 - 12x + 5$

c

b

range: $[-7, \infty)$
domain: $(-\infty, \infty)$

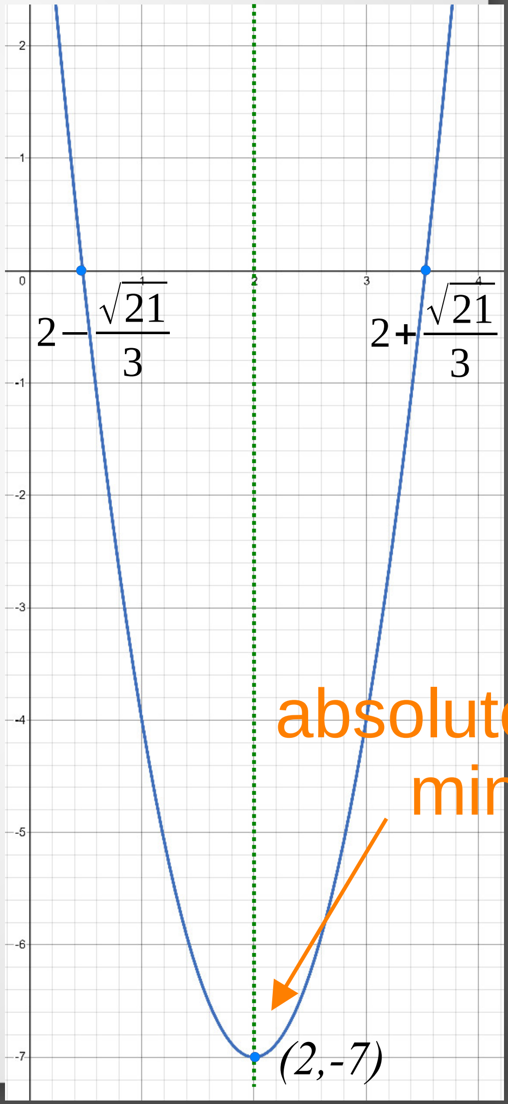
$a = 3 > 0$, hence parabola opens upward

the *vertex*'s x-coordinate is $-\dfrac{b}{2a} = -\dfrac{-12}{2 \cdot 3} = 2$,
let's use $x = 2$ to find
the y-coordinate : $g(2) = 3 \cdot 2^2 - 12 \cdot 2 + 5 = -7$.
Hence vertex: $(2,-7)$

symmetric with respect to the line $x = 2$

x-intercepts: $3x^2 - 12x + 5 = 0$, $x = 2 \pm \dfrac{\sqrt{21}}{3}$

y-intercept: $g(0) = 5$



Quadratic Functions and Polynomial Functions

Polynomial Functions

[Def] The function defined by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0, \text{ where } a_n, \ldots a_0 \in R,\ n \geq 0$$

is called a *polynomial function of degree n*

a_n is called the *leading coefficient*.

Quadratic Functions and Polynomial Functions

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Examples of polynomial functions:

$f(x) = -3x^5 + \sqrt{2}x^2 + 5$ *polynomial function of degree 5*

$g(x) = -3x^4(x-2)(x+3)$ *polynomial function of degree 6*

Quadratic Functions and Polynomial Functions

Polynomial Functions

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Examples of non-polynomial functions:

$$F(x) = -3\sqrt{x} + \sqrt{2}x - 10$$

$$G(x) = -3x^{-2} + 7x^2 + 6$$

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(1) *graphs of polynomial functions are smooth and continuous*

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(2) *the leading coefficient test*

As x increases/decreases without bound, the graph of

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0,\ a_n \neq 0$$

eventually rises or falls, in particular

Quadratic Functions and Polynomial Functions

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for odd n

• if $a_n > 0$ then the graph falls to the left, and rises to the right (↙, ↗)

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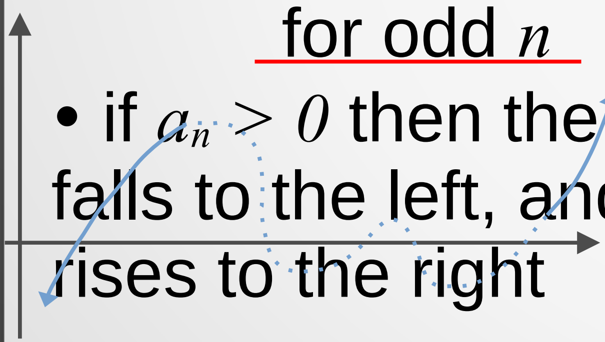

Quadratic Functions and Polynomial Functions

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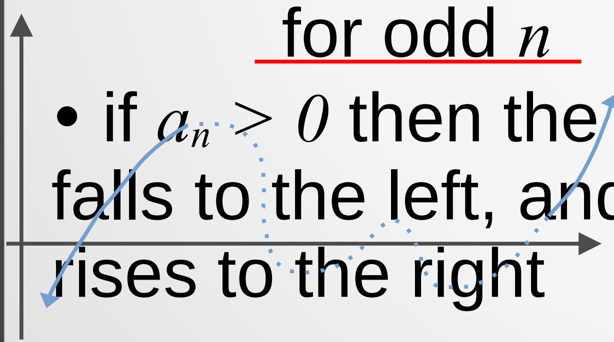

• if $a_n > 0$ then the graph falls to the left, and rises to the right $(\swarrow, \nearrow)$

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Quadratic Functions and Polynomial Functions

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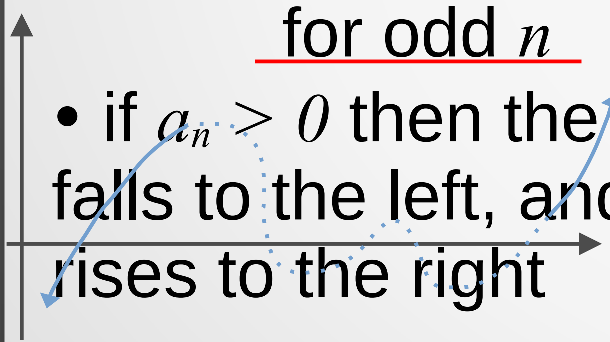

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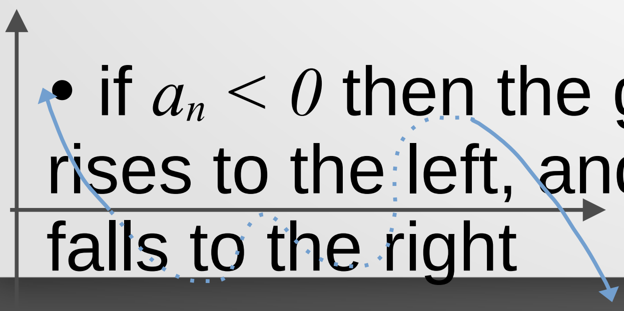

• if $a_n < 0$ then the graph rises to the left, and falls to the right (↖, ↘)

for even n

• if $a_n > 0$ then the graph rises to the left and right (↖, ↗)

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

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Quadratic Functions and Polynomial Functions

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Quadratic Functions and Polynomial Functions

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Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(3) *zeros of the polynomial function* are the values of x for which $f(x)$ is 0 (x*-intercepts, roots, solutions*).

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

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(4) *Descartes's rule of signs* will be covered later in the chapter.

Polynomial Functions and Their Graphs

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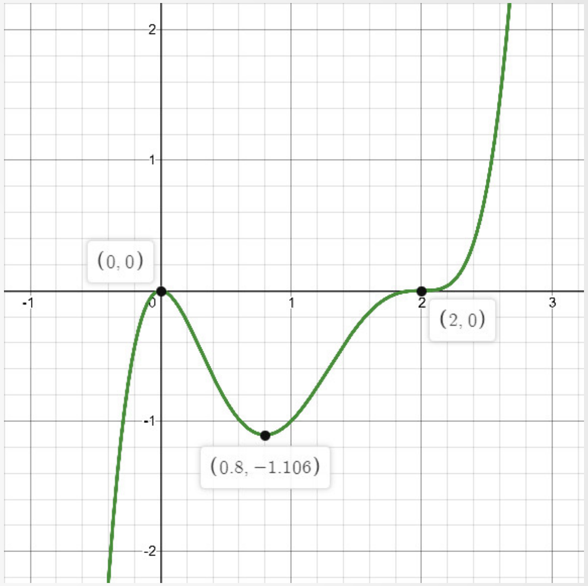
(5) y - intercept: when $x = 0$

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(6) *multiplicity of zeros*



Consider $f(x) = x^2 (x-2)^3$

$x = 0$
multiplicity 2

$x = 2$
multiplicity 3

- if r is a zero of even multiplicity, then the graph touches the x-axis and turns around at r.
- if r is a zero of odd multiplicity, then the graph crosses the x-axis at r.
- regardless of odd or even multiplicity, graphs tend to flatten out near zeros with multiplicity > 1.

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(6) *multiplicity of zeros*

Example: let's look at the zeros of $g(x) = (3x^2 - 8x)(x^2 - 16)^3$ and their multiplicities.

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

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First, let's re-write the expression for $g(x)$:

$$g(x) = x(3x - 8)(x-4)^3(x+4)^3$$

Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

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Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

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- zeros at $x = 0$ and at $x = 8/3$ with *multiplicity 1*, so the graph of g(x) crosses the x-axis
- a zeros at $x = 4$ and at $x = 4$ of *multiplicity 3*, so the graph of g(x) crosses the x-axis, but tents to flatten out near the zeros

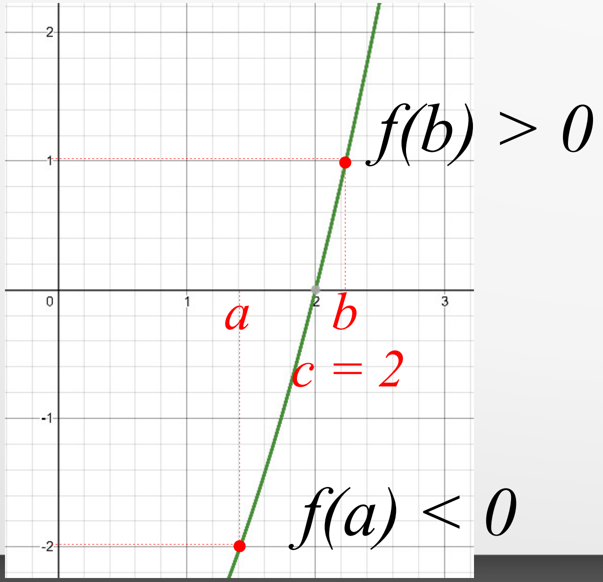
Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(7) *The Intermediate Value Theorem*

Let f be a polynomial function with real coefficients.

If $f(a)$ and $f(b)$ have opposite signs, then there is at least one value c, between a and b, such that $f(c) = 0$.



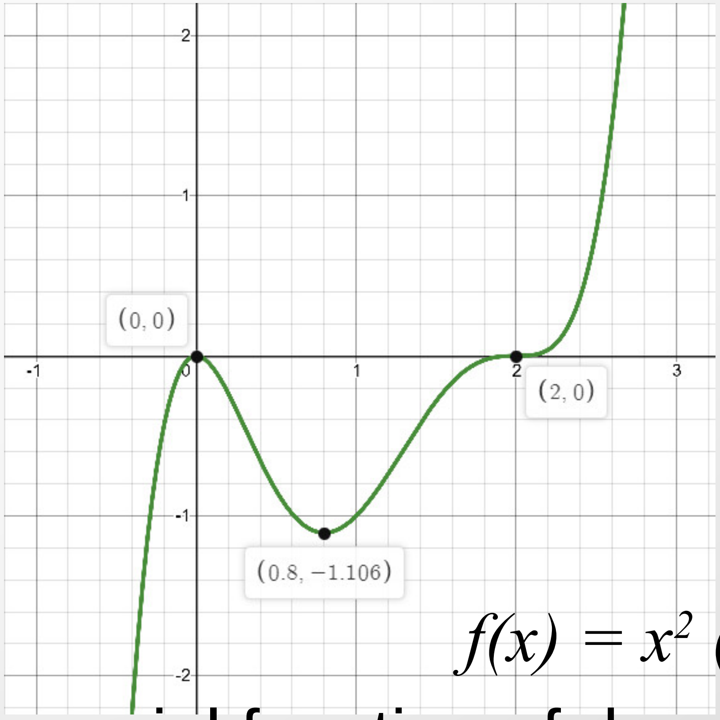
Quadratic Functions and Polynomial Functions

Polynomial Functions and Their Graphs

(8) *Turning points o*f polynomial functions

If f is a polynomial function of degree n, then the graph of f has at most $(n-1)$ turning points.

A *turning point* is a point at which the function values change from increasing to decreasing or decreasing to increasing.



$$f(x) = x^2\,(x-2)^3$$

a polynomial function of degree 5

Quadratic Functions and Polynomial Functions

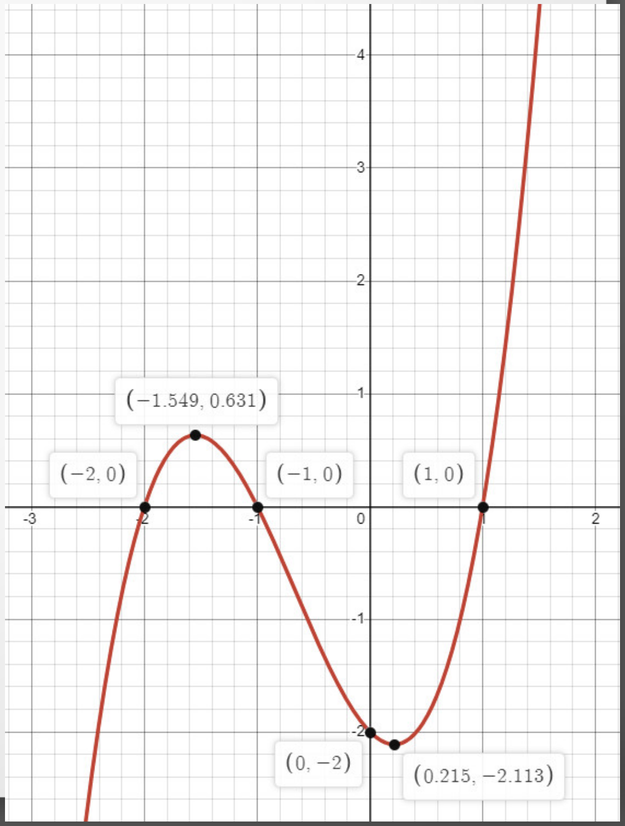
Example: let's graph function $f(x) = x^3 + 2x^2 - x - 2$

Quadratic Functions and Polynomial Functions

Example: let's graph function $f(x) = x^3 + 2x^2 - x - 2$

1) y-intercept: -2 or (0,-2)
2) leading coefficient $a_3 = 1 > 0$, ($\swarrow$, $\nearrow$)
degree of the polynomial $= 3$.
3) zeros of polynomial (x-intercepts):
$x = -2, -1, 1$ or (-2,0), (-1,0), (1,0)
Multiplicity of each zero is one, hence the graph is crossing the x-axis.
4) degree of the polynomial – 1 = 2 turning points
5) additional points: (pick x, find y)

x	-3	-1.5	-0.5	0.5	2	...
y						



Homework assignment

1) zyBooks: *review* Sections 3.2 *Quadratic Functions,*
3.3 *Power functions and polynomial functions (skip the power function)*
3.4 *Graphs of Polynomial Functions*

or

Textbook: *review* Sections 2.2 *Quadratic Functions*
2.3 *Polynomial Functions and Their Graphs*

2) We will have Quiz 6 based on today's topics in the beginning of our next meeting.

3) WeBWorK: HW 6 (due date is in one week)