Section 1.4 Composition of Functions

Combinations of Functions

Today we will:

- Combine functions using algebraic operations.
- Create a new function by composition of functions.
- Evaluate composite functions.
- Find the domain of a composite function.
- Decompose a composite function into its component functions.

Section 1.4 Composition of Functions

Combinations of Functions

We can *combine functions* using algebraic operations!

Section 1.4 Composition of Functions

Combinations of Functions

We can *combine functions* using algebraic operations!

Given two functions, $f(x)$ with the domain $\mathrm{D_f}$ and $g(x)$, with the domain $\mathrm{D_g}$,

the sum $(f+g)(x)=f(x)+g(x)$

the difference $(f-g)(x)=f(x)-g(x)$

the product $(fg)(x)=f(x)g(x)$

the quotient $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}, if\; g(x)\neq 0$

and their domain is $\mathrm{D_f} \cap \mathrm{D_g}$

Section 1.4 Composition of Functions

Combinations of Functions

Example: given two functions, $f(x)=2x-7$ and $g(x)=\frac{1}{x}$ we can combine them using multiplication operation:

$$h(x)=(fg)(x)=f(x)g(x)$$

Section 1.4 Composition of Functions

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Example: given two functions, $f(x)=2x-7$ and $g(x)=\frac{1}{x}$ we can combine them using multiplication operation:

$$h(x)=(fg)(x)=f(x)g(x)=(2x-7)\cdot\frac{1}{x}=\frac{2x-7}{x}$$

Section 1.4 Composition of Functions

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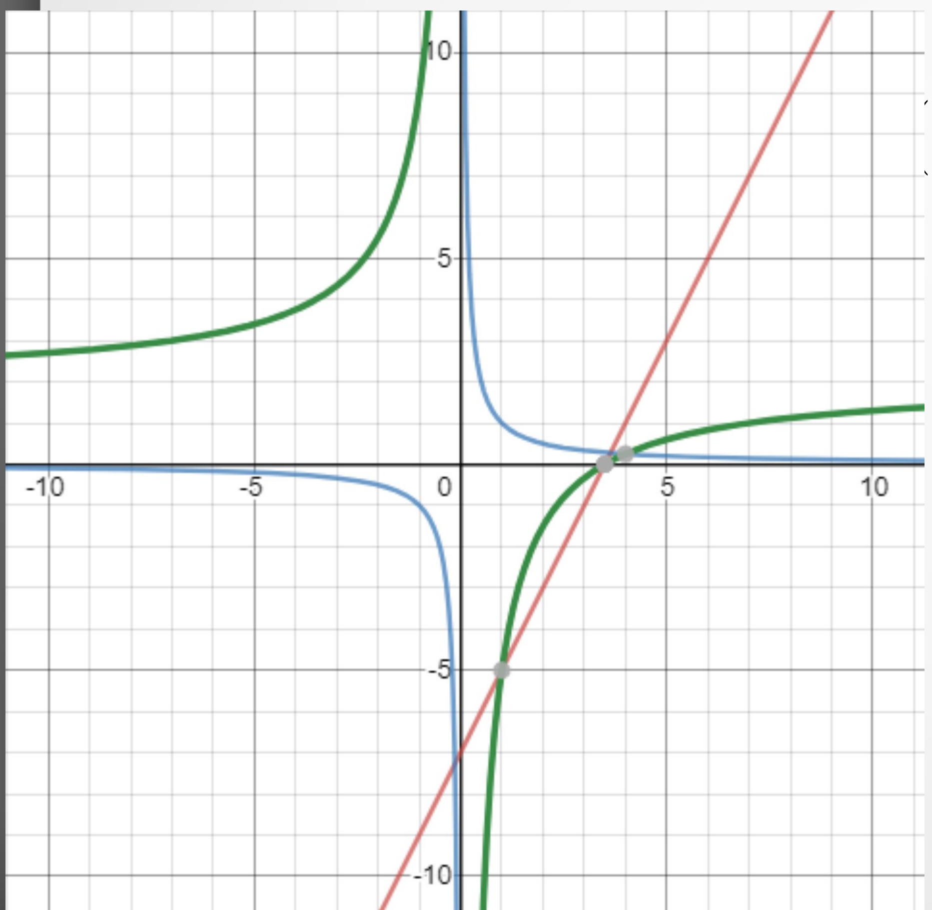
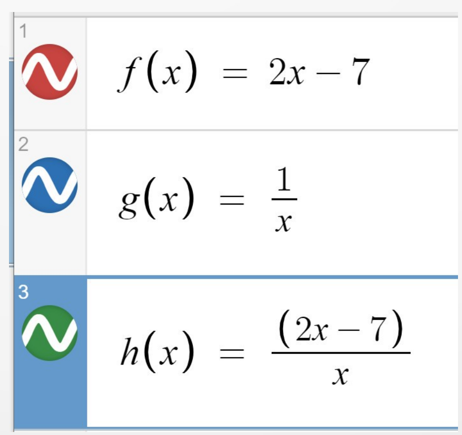


$$(2x-7)\cdot\frac{1}{x}=\frac{2x-7}{x}$$



Section 1.4 Composition of Functions

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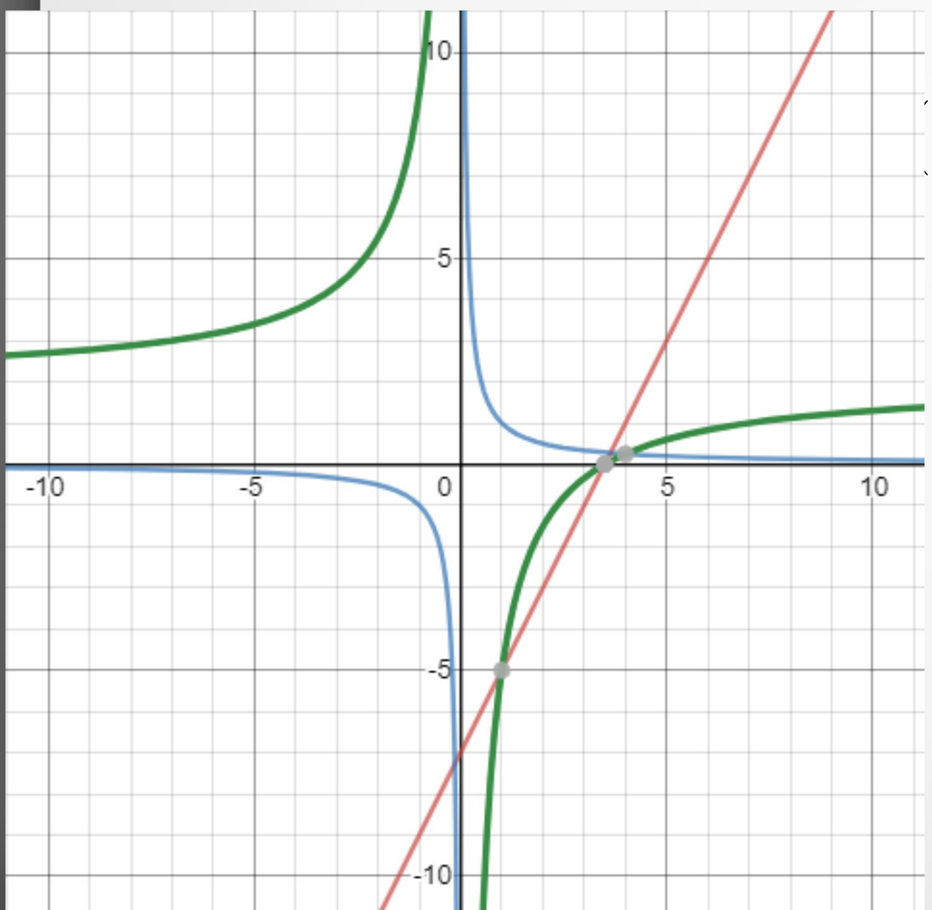

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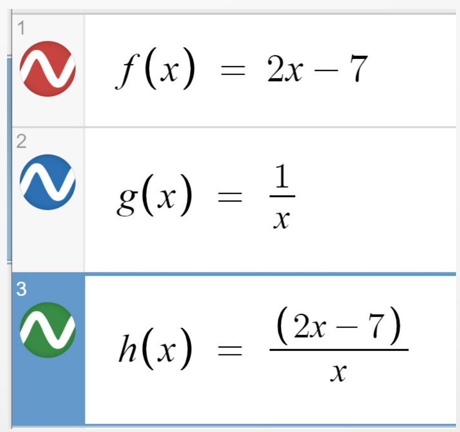

domain of h(x):
$(-\infty,0) \cup (0,\infty)$

Section 1.4 Composition of Functions

Combinations of Functions

We can *combine functions* using algebraic operations!

Example: given two functions, $f(x)=2x-7$ and $g(x)=\frac{1}{x}$ we can combine them using multiplication operation:

$$h(x)=(fg)(x)=f(x)g(x)=(2x-7)\cdot\frac{1}{x}=\frac{2x-7}{x}$$

Is the domain of the new function different from the domains of functions *f(x)* and *g(x)*? yes

Is the range of the new function different from the ranges of the functions *f(x)* and *g(x)*? yes

Do the domain and range necessarily change? no

In-class Practice

Exercise 1: For the functions $f(x)=x^2+5$ and $g(x)=2x-3$ find

(a) $(f-g)(x)$ and its domain

(b) $\left(\frac{f}{g}\right)(x)$ and its domain

In-class Practice

Exercise 2: For the functions $f(x)=\sqrt{x+2}$ and $g(x)=\sqrt{x-7}$ find

(a) $(fg)(x)$

(b) $(f+g)(x)$

(c) $\left(\frac{f}{g}\right)(x)$

Section 1.4 Composition of Functions

Compositions of Functions

There is another way of combining functions, called *composition of functions*.

Example: consider two functions: $f(x)=2x-7$ and $g(x)=\frac{1}{x}$

We can provide the expression for function g as a parameter/argument for function f :

$$f(g(x))=$$

Section 1.4 Composition of Functions

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$$f(g(x))=f\left(\frac{1}{x}\right)=2\left(\frac{1}{x}\right)-7=$$

Section 1.4 Composition of Functions

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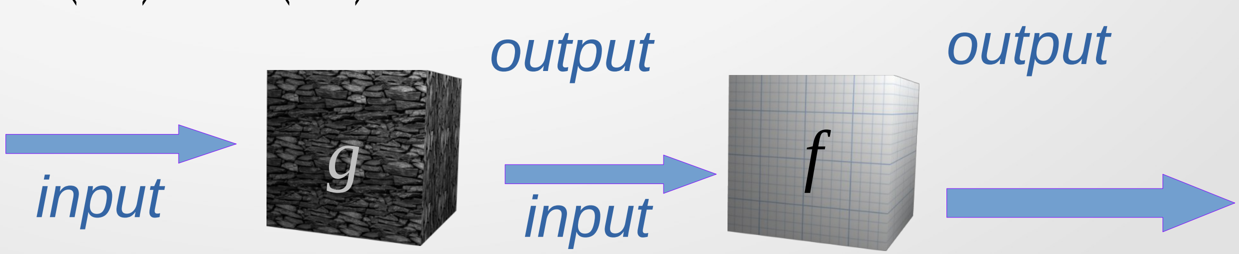

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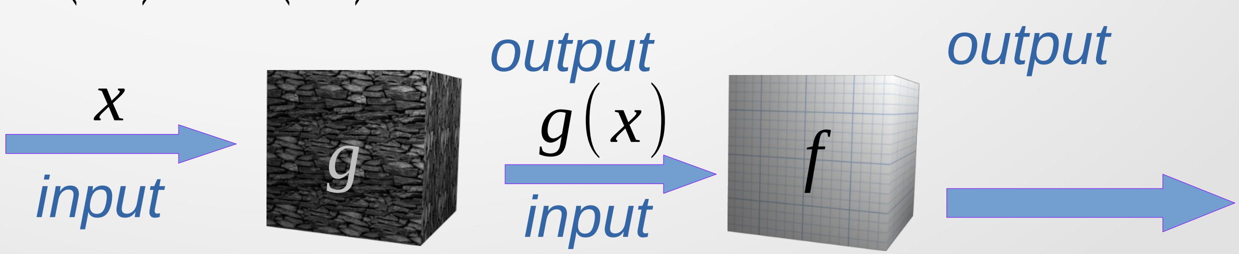

Section 1.4 Composition of Functions

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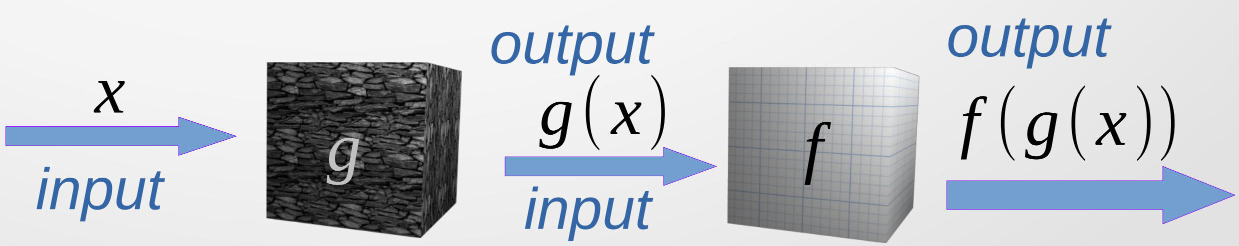

Section 1.4 Composition of Functions

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Example: consider two functions: $f(x)=2x-7$ and $g(x)=\frac{1}{x}$

What about the domain of the resulting function?

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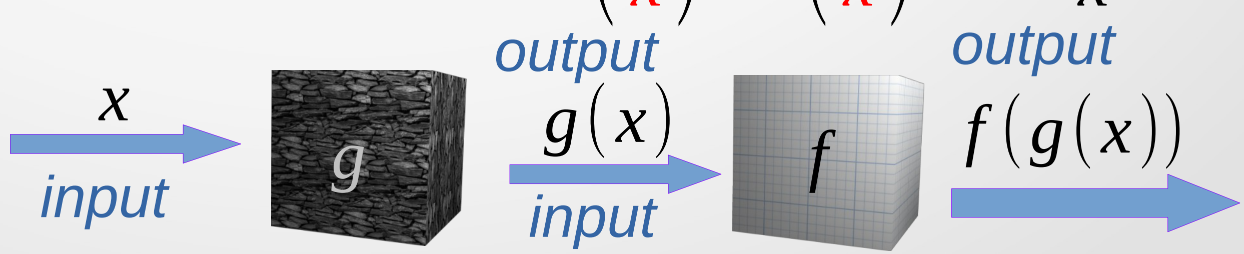

Section 1.4 Composition of Functions

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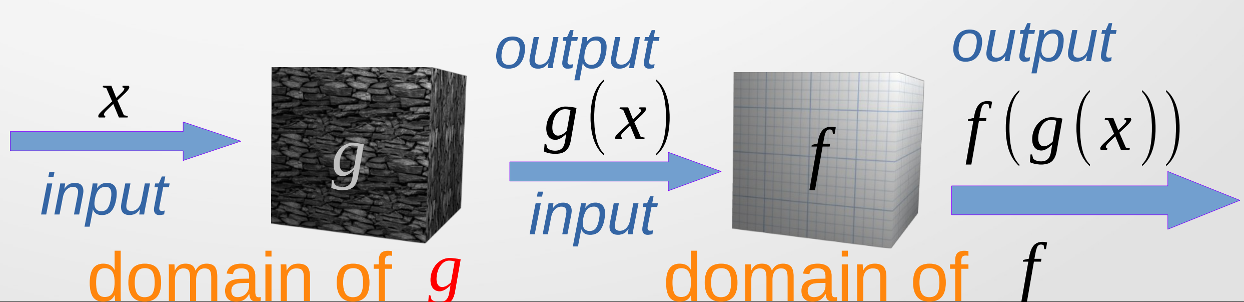
Example: consider two functions: $f(x) = 2x - 7$ and $g(x) = \frac{1}{x}$

What about the domain of the resulting function?

$$(f \circ g)(x) = f(g(x))$$



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Example: consider two functions: $f(x)=2x-7$ and $g(x)=\frac{1}{x}$

What about the domain of the resulting function?

The domain of the composite function $f\circ g$ is all x such that x is in the domain of *g(x)* and *g(x)* is in the domain of f

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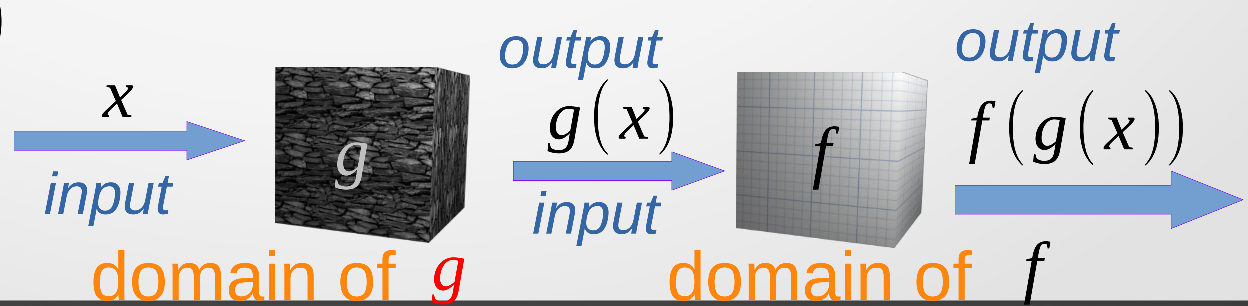

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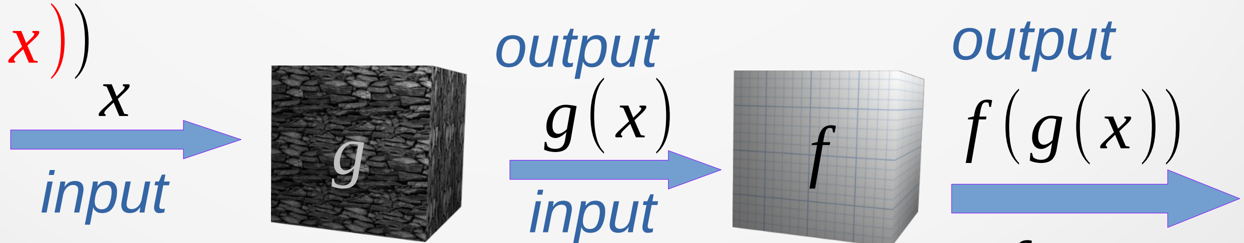

domain of g:
(-∞,0) ∪ (0,∞)

domain of f :
all real numbers, i.e. (-∞, ∞)

Therefore, the domain of (f°g)(x) is (-∞,0) ∪ (0,∞).

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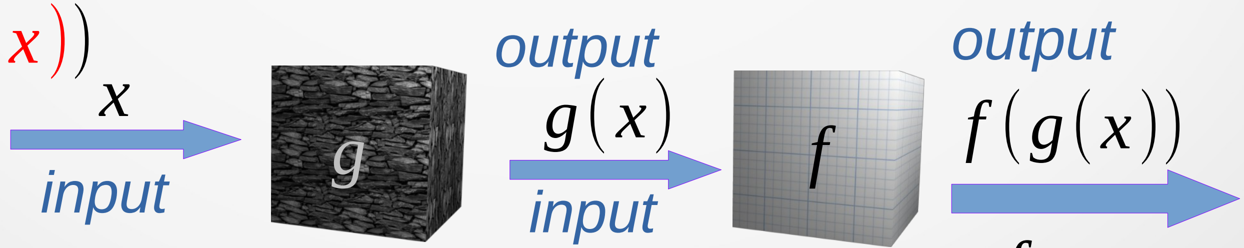

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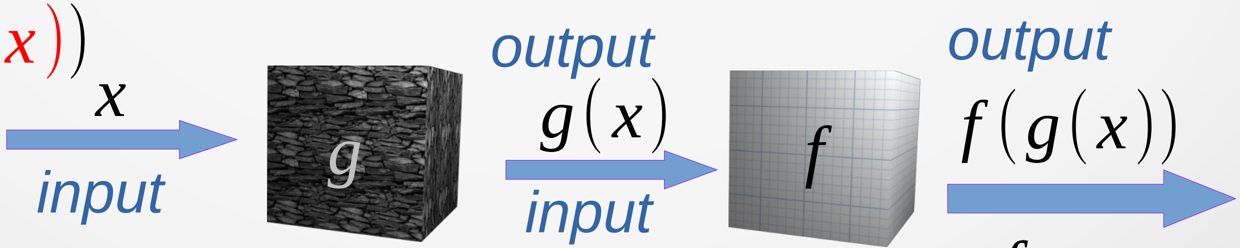

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- What expression will $(g\circ f)(x)$ have?

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$(g \circ f)(x) = g(f(x)) =$

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$$(g\circ f)(x)=g(f(x))=g(2x-7)=\frac{1}{2x-7}$$

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What is the domain?

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- What expression will $(g\circ f)(x)$ have?

$$(g\circ f)(x)=g(f(x))=g(2x-7)=\frac{1}{2x-7}$$

What is the domain? (-∞,7/2) ∪ (7/2, ∞)

Section 1.4 Composition of Functions

Compositions of Functions

There is another way of combining functions, called *composition of functions*.

Example: given two functions: $f(x) = 2x - 7$ and $g(x) = \frac{1}{x}$

$$(f \circ g)(x) = \frac{2}{x} - 7$$

$$(g \circ f)(x) = \frac{1}{2x - 7}$$

Therefore, the order of functions is IMPORTANT!

Compositions of Functions

We can evaluate composite functions, using their graphs and algebraic expressions.

Example 1: given two functions: $f(x)=2x-7$ and $g(x)=\frac{1}{x}$

Let's find

(a) $(f\circ g)(4)$

(b) $(g\circ f)(5)$

Section 1.4 Composition of Functions

Compositions of Functions

We can evaluate composite functions, using their graphs and algebraic expressions.

Example 1: given two functions: $f(x)=2x-7$ and $g(x)=\frac{1}{x}$

Let's find

(a) $(f\circ g)(4)=f(g(4))=f(\frac{1}{4})=2\cdot\frac{1}{4}-7=\frac{1}{2}-7=-6\frac{1}{2}=-6.5$

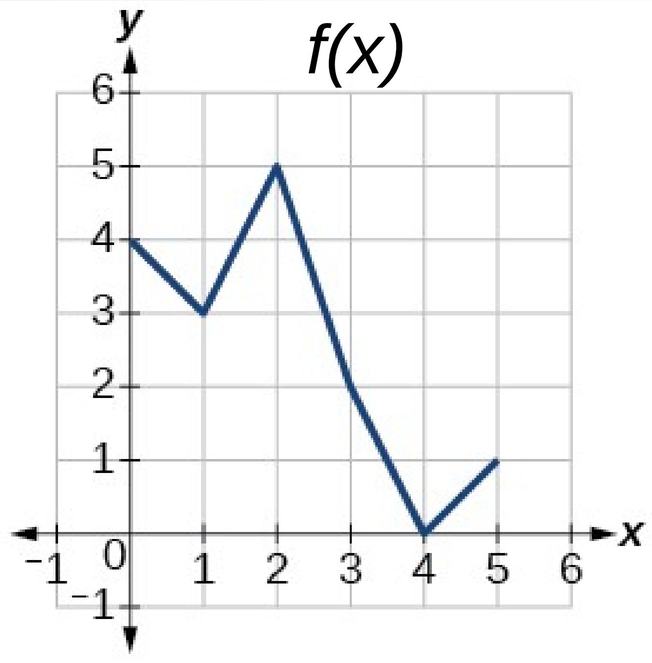
(b) $(g\circ f)(5)=g(f(5))=g(2\cdot 5-7)=g(10-7)=g(3)=\frac{1}{3}$

Section 1.4 Composition of Functions

Compositions of Functions

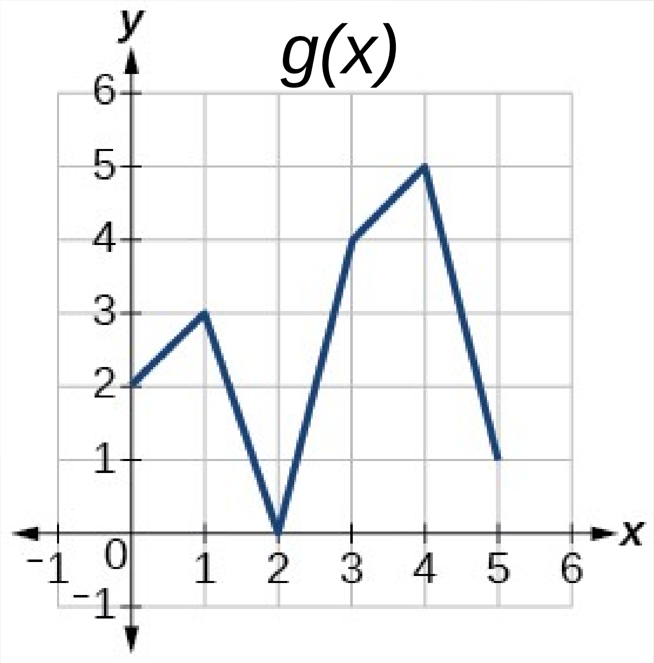
We can evaluate composite functions, using their graphs and algebraic expressions.

Example 2: given the graphs of two functions, *f(x)* and *g(x)*, let's find



(a) $(g \circ f)(5)$

(b) $(f \circ g)(3)$



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In-class Practice

Compositions of Functions

Exercise 1: Let $f(x)=2x^2-7$ and $g(x)=x-2$. Find

(a) $(f\circ g)(x)$ and its domain

(b) $(g\circ f)(x)$ and its domain

In-class Practice

Compositions of Functions

Exercise 2: Let $f(x)=\dfrac{7}{x-3}$ and $g(x)=\sqrt{x-1}$. Find

(a) $(f\circ g)(x)$ and its domain

(b) $(g\circ f)(x)$ and its domain

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

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Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 1: consider function $h(x) = \sqrt[3]{|x+5| - 10}$

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 1: consider function $h(x)=\sqrt[3]{|x+5|-10}$

What if we define two functions: $g(x)=|x+5|$

$f(x)=\sqrt[3]{x-10}$?

$(f\circ g)(x)=f(g(x))$

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 1: consider function $h(x)=\sqrt[3]{|x+5|-10}$

outer inner

What if we define two functions: $g(x)=|x+5|$

$f(x)=\sqrt[3]{x-10}$?

inner

$(f\circ g)(x)=f(g(x))$

outer

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 1: consider function $h(x)=\sqrt[3]{|x+5|-10}$

What if we define two functions: $g(x)=|x+5|$

$f(x)=\sqrt[3]{x-10}$?

inner

$$(f\circ g)(x)=f(g(x))=f(|x+5|)=\sqrt[3]{|x+5|-10}$$

outer

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 2: consider function $h(x)=\frac{1}{\sqrt{2x-3}}$

What should be the inner and outer functions?

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 2: consider function $h(x)=\frac{1}{\sqrt{2x-3}}$

Let's define two functions: $g(x)=\sqrt{2x-3}$ ← inner

$f(x)=\frac{1}{x}$ ← outer

$(f\circ g)(x)=f(g(x))$

inner: $g(x)$

outer: f

Section 1.4 Composition of Functions

Decomposing a Function

It is possible to reverse the process of functions compositions!

Example 2: consider function $h(x)=\frac{1}{\sqrt{2x-3}}$

Let's define two functions: $g(x)=\sqrt{2x-3}$ inner

$f(x)=\frac{1}{x}$ outer

inner

$(f\circ g)(x)=f(g(x))=f(\sqrt{2x-3})=\frac{1}{\sqrt{2x-3}}$

outer

In-class Practice

Exercise 1: Express the function $h(x) = |x^2 - 4x + 8|$ as a composition of two functions, *f* and *g*, so that $h(x) = (f \circ g)(x)$

In-class Practice

Exercise 2: Express the function $h(x)=(\sqrt{x+5}-1)^3$ as a composition of two functions, *f* and *g*, so that $h(x)=(f \circ g)(x)$

In-class Practice

Exercise 3: Find the domains of the given functions:

(a) $h(x)=\sqrt{x^2-25}$

(b) $f(x)=x^5$

(c) $g(x)=\frac{1}{\sqrt{x+3}}$

(d) $t(x)=\frac{2x}{x^2-4}$

In-class Practice

Exercise 4: Let $f(x)=5+\dfrac{1}{x+1}$ and $g(x)=3x+4$. Find

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $(fg)(x)$

(d) $\left(\dfrac{f}{g}\right)(x)$

and determine their domain

In-class Practice

Exercise 5: Let $f(x) = 2x + \frac{3}{x}$ and $g(x) = \frac{1}{x}$.

(a) Find $(f \circ g)(x)$ and determine its domain

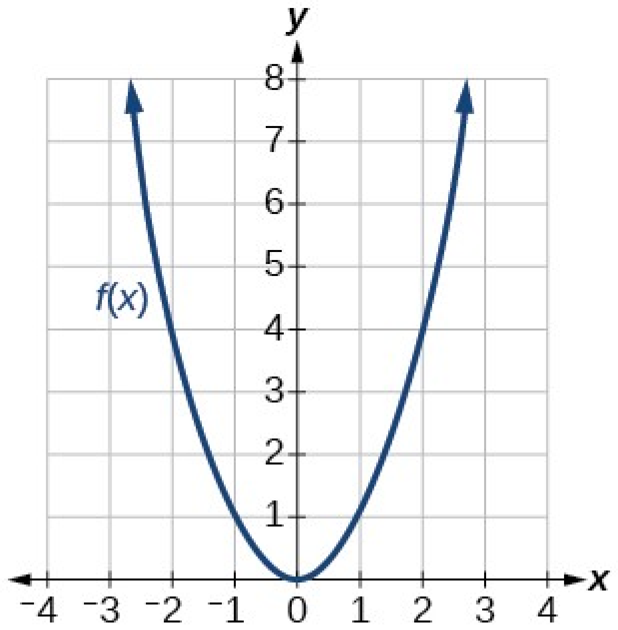
(b) $(f \circ g)(2)$

In-class Practice

Exercise 6: Use the graphs of *f(x)*, to the left, and *g(x)*, to the right, to find

(a) $(g \circ f)(2)$

(b) $(f \circ g)(4)$



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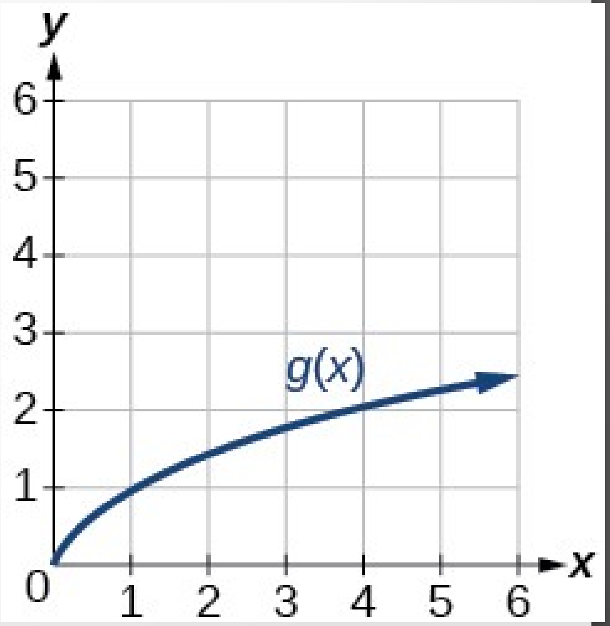

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Homework assignment

Today we:

- Combined functions using algebraic operations.
- Created a new function by composition of functions.
- Evaluated composite functions.
- Found the domain of a composite function.
- Decomposed a composite function into its component functions.