MTH 30: Pre-calculus mathematics

Plan for today

- discuss the structure of the class
- see the online textbook at openstax.org
- Cover *Section 1.1 Functions and Function Notation*

  Objectives:
  - *Determine whether a relation represents a function.*
  - *Find the value of a function.*
  - *Determine whether a function is one-to-one.*
  - *Use the vertical line test to identify functions.*
  - *Graph the functions listed in the library of functions.*

1.1 Functions and Function Notation

[Def] A *relation* is any set of ordered pairs.

domain/input: the set of all fist components of the ordered pairs.

range/output: the set of all second components of the ordered pairs.

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domain (input): {Anna, Maria, Debbie, Sophia}
range (output): {12, 13, 14}

1.1 Functions and Function Notation

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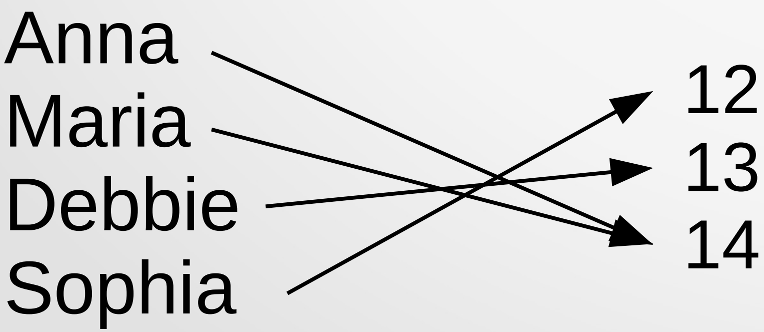

Anna
Maria
Debbie
Sophia

12
13
14

visual representation of the relation

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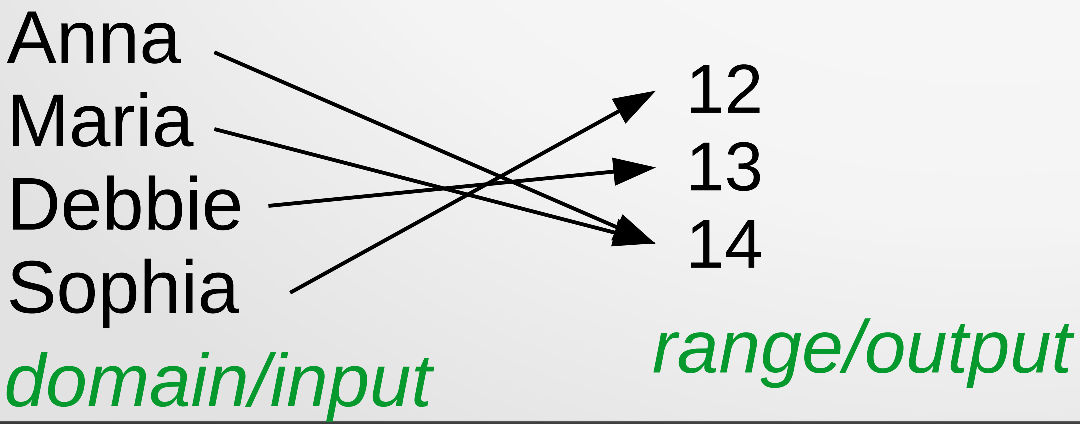

Anna
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domain/input

range/output

visual representation of the relation

1.1 Functions and Function Notation

[Def] A *relation* in which each member of the domain corresponds to exactly one member of the range is called a function.

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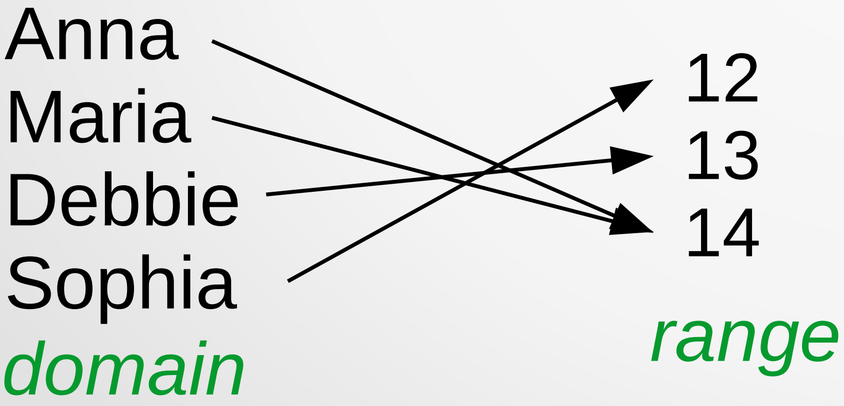

Anna
Maria
Debbie
Sophia
domain

12
13
14
range

The given relation is a function

1.1 Functions and Function Notation

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Example: consider the relation defined by:
(Anna,14), (Maria, 13), (Debbie, 17), (Anna, 10), (Sophia, 11)

The given relation is not a function

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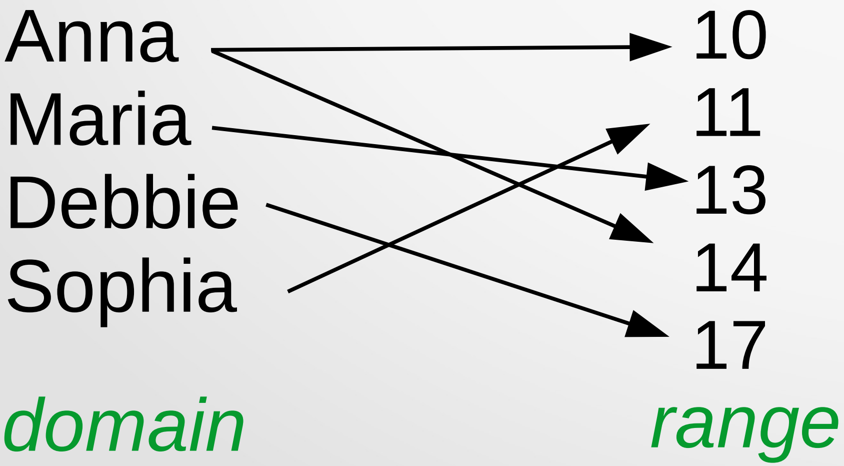

Anna
Maria
Debbie
Sophia

10
11
13
14
17

domain

range

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1.1 Functions and Function Notation

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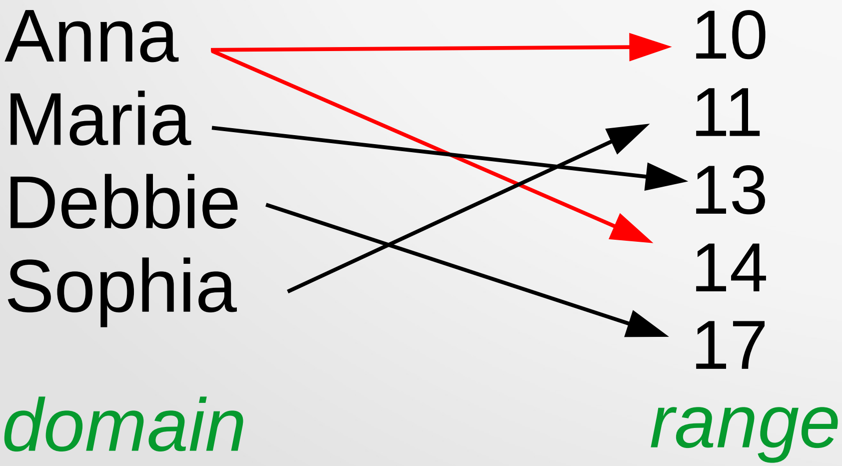

The given relation is not a function

In – class practice

State whether the given *relation* is a *function* or not. Explain why.

(a) (a,3), (b,4), (c,1), (d,2), (a,7), (f, 5)

(b) (a,3), (b,3), (c,3), (d,2), (f,10), (g,3)

In – class practice

State whether the given *relation* is a *function* or not. Explain why.

(a) (a,3), (b,4), (c,1), (d,2), (a,7), (f, 5)

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(b) (a,3), (b,3), (c,3), (d,2), (f,10), (g,3)

The given relation is a function

1.1 Functions and Function Notation

[Def] A *relation* in which each member of the domain corresponds to exactly one member of the range is called a function.

In other words, a function is a relation in which no two ordered pairs have the same first component and different second component.

1.1 Functions and Function Notation

Functions as equations

Consider the equation $y = 2x^2 - 5x + 7$

1.1 Functions and Function Notation

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value y depends of x, we call y *dependent variable*, and x is an *independent variable*.

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value y depends of x, we call y *dependent variable*, and
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This equation defines a function.
(0,7), (-1, 14), ...

1.1 Functions and Function Notation

Functions as equations

Consider the equation $y = 2x^2 – 5x + 7$
value y depends of x, we call y *dependent variable*, and x is an *independent variable*.

This equation defines a function.

However, not all equations with variables x and y define functions.

1.1 Functions and Function Notation

Functions as equations

Consider the equation $y^2 + x^2 = 4$ ← *equation of a circle*

1.1 Functions and Function Notation

Functions as equations

Consider the equation $y^2 + x^2 = 4$ ← *equation of a circle*

This equation does not define a function.

(0,2), (0, -2), ...

1.1 Functions and Function Notation

Functions as equations

If an equation is solved for y and more than one value of y can be obtained for a given x value, then the equation is *not a function*.

Examples:

(a) $y=\pm\sqrt{x^2+5}$

(b) $y=2x^2-5x+7$

(c) $x^2+y^2=9$

(d) $2x+2y=20$

1.1 Functions and Function Notation

Functions as equations

If an equation is solved for y and more than one value of y can be obtained for a given x value, then the equation is *not a function*.

Examples:

(a) $y = \pm\sqrt{x^2+5}$ is not a function

(b) $y = 2x^2 - 5x + 7$ is a function

(c) $x^2 + y^2 = 9$ is not a function

(d) $2x + 2y = 20$ is a function

1.1 Functions and Function Notation

Functions as equations

If an equation is solved for y and more than one value of y can be obtained for a given x value, then the equation is *not a function*.

Examples:

(a) $y=\pm\sqrt{x^2+5}$ is not a function, if $x = 2$, then $y=\pm\sqrt{9}=\pm 3$

(b) $y=2x^2-5x+7$ is a function

(c) $x^2+y^2=9$ is not a function, $y^2=\sqrt{9-x^2}$

(d) $2x+2y=20$ is a function

1.1 Functions and Function Notation

Function notation

In $y=x^2+5$ we can "replace" y by f(x),

"f of x" or "f at x" represents the value of the function at the number x".

Functions may have different names: f, h, g, F, G, …

$$f(x)=x^2+5$$

1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x)=x^2-2x+5$

1.1 Functions and Function Notation

Function notation

Examples:

consider function $f(x)$ defined by $f(x) = x^2 - 2x + 5$

(a) Let's evaluate function f at $x = 2$:

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Function notation

Examples:
consider function $f(x)$ defined by $f(x)=x^2-2x+5$

(a) Let's evaluate function f at $x = 2$:

$$f(2)=$$

domain value

range value

1.1 Functions and Function Notation

Function notation

Examples:

consider function $f(x)$ defined by $f(x) = x^2 - 2x + 5$

(a) Let’s evaluate function f at $x = 2$:

$$f(\underset{\substack{\uparrow \\ \textit{domain value}}}{2}) = 2^2 - 2 \times 2 + 5 = \underset{\substack{\uparrow \\ \textit{range value}}}{}$$

1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x) = x^2 - 2x + 5$

(a) Let's evaluate function f at $x = 2$:

$$f(2) = 2^2 - 2 \times 2 + 5 = 4 - 4 + 5 = 5$$

domain value

range value

Answer: $f(2) = 5$

1.1 Functions and Function Notation

Function notation

Examples:
consider function $f(x)$ defined by $f(x)=x^2-2x+5$

(b) Let's find $f(3x-1)$:

1.1 Functions and Function Notation

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1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x)=x^2-2x+5$

(b) Let’s find f(3x-1):

$$f(3x-1)=(3x-1)^2-2\times(3x-1)+5=$$

1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x)=x^2-2x+5$

(b) Let’s find f(3x-1):

$$f(3x-1)=(3x-1)^2-2\times(3x-1)+5=9x^2-6x+1-6x+2+5=$$

$$=9x^2-12x+8$$

Answer: $f(3x-1)=9x^2-12x+8$

1.1 Functions and Function Notation

Function notation

Examples:

consider function $f(x)$ defined by $f(x) = x^2 - 2x + 5$

(c) Let’s find $f(-x)$:

1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x)=x^2-2x+5$

(c) Let's find f(-x):

$$f(-x)=(-x)^2-2\times(-x)+5=$$

1.1 Functions and Function Notation

Function notation

Examples:

consider function f(x) defined by $f(x)=x^2-2x+5$

(c) Let’s find f(-x):

$$f(-x)=(-x)^2-2\times(-x)+5=x^2+2x+5$$

Answer: $f(-x)=x^2+2x+5$

1.1 Functions and Function Notation

Finding an Equation of a Function

Example:

consider the equation $2x-4y=12$

(1) We can rewrite the equation as if y is a function of x:

$$y=\frac{1}{2}x-3$$

(2) We can rewrite the equation as if x is a function of y:

$$x=2y+6$$

1.1 Functions and Function Notation

Graphing Functions

We can graph functions.

1.1 Functions and Function Notation

Graphing Functions

We can graph functions. Let’s graph three functions:

$$f(x)=3x$$

$$g(x)=3x+5$$

$$h(x)=3x-2$$

- these are *linear functions*.

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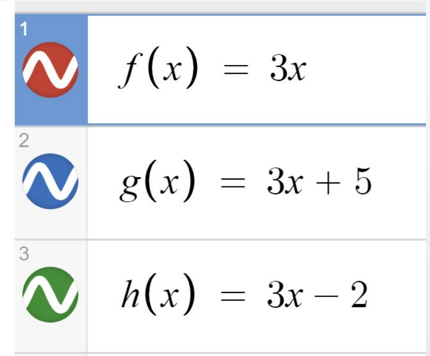

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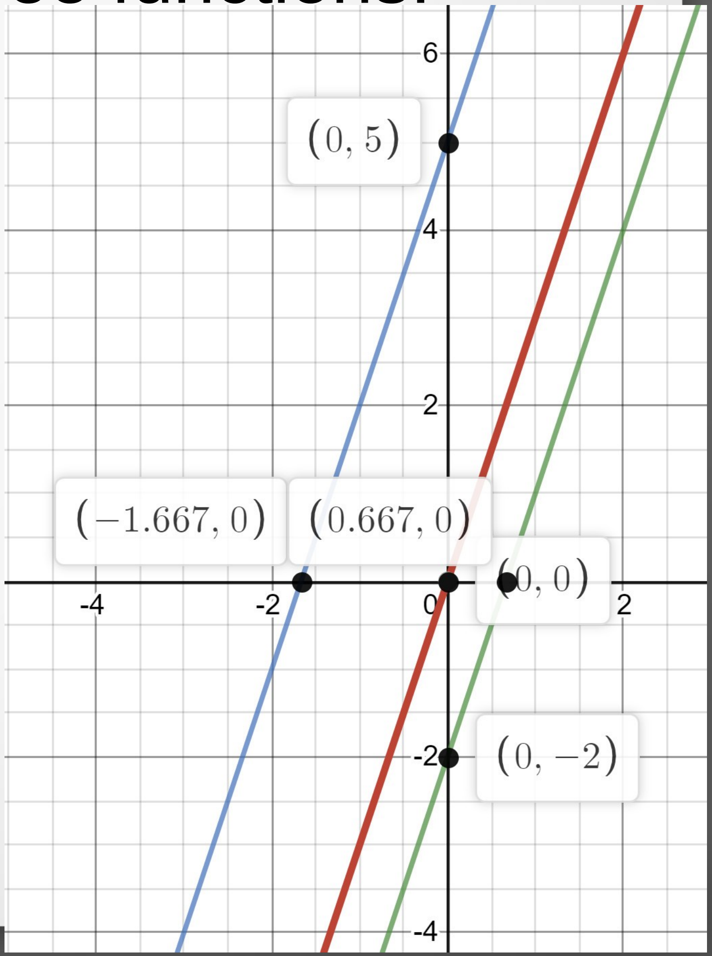

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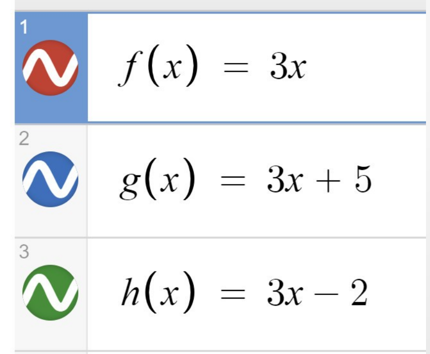
one value of y for a given value of x



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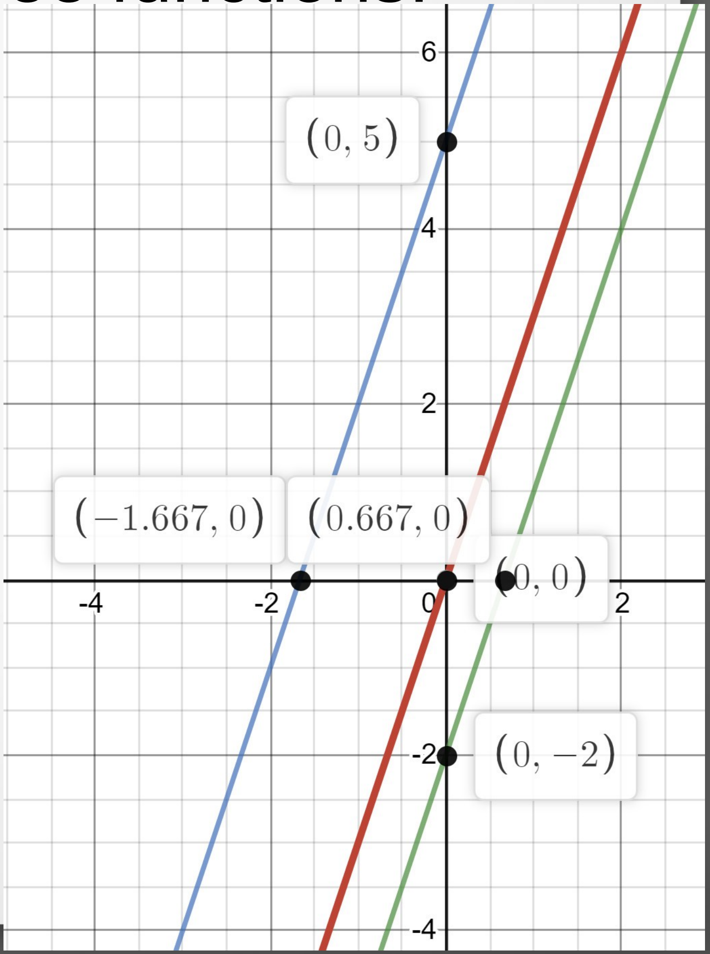

1.1 Functions and Function Notation

Vertical Line Test

Not every graph in the rectangular coordinate system is the graph of a function.

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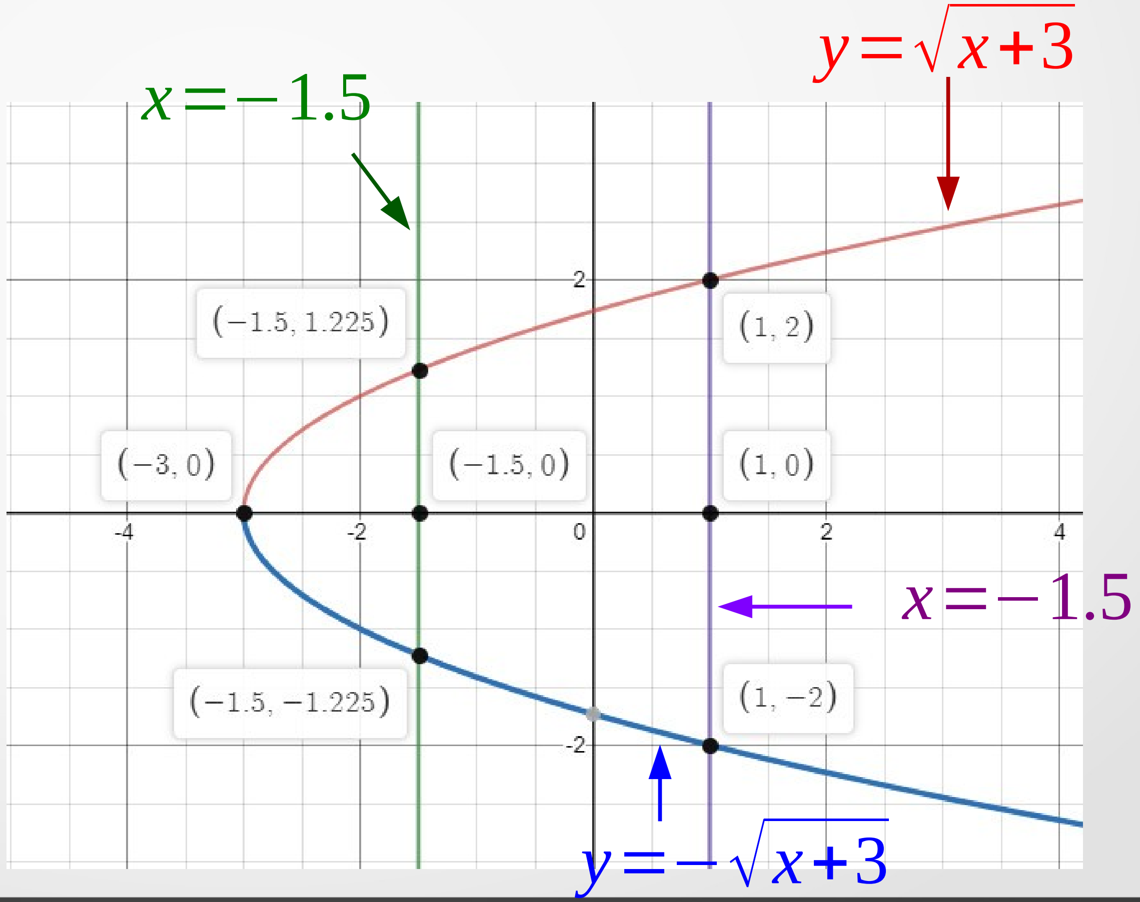

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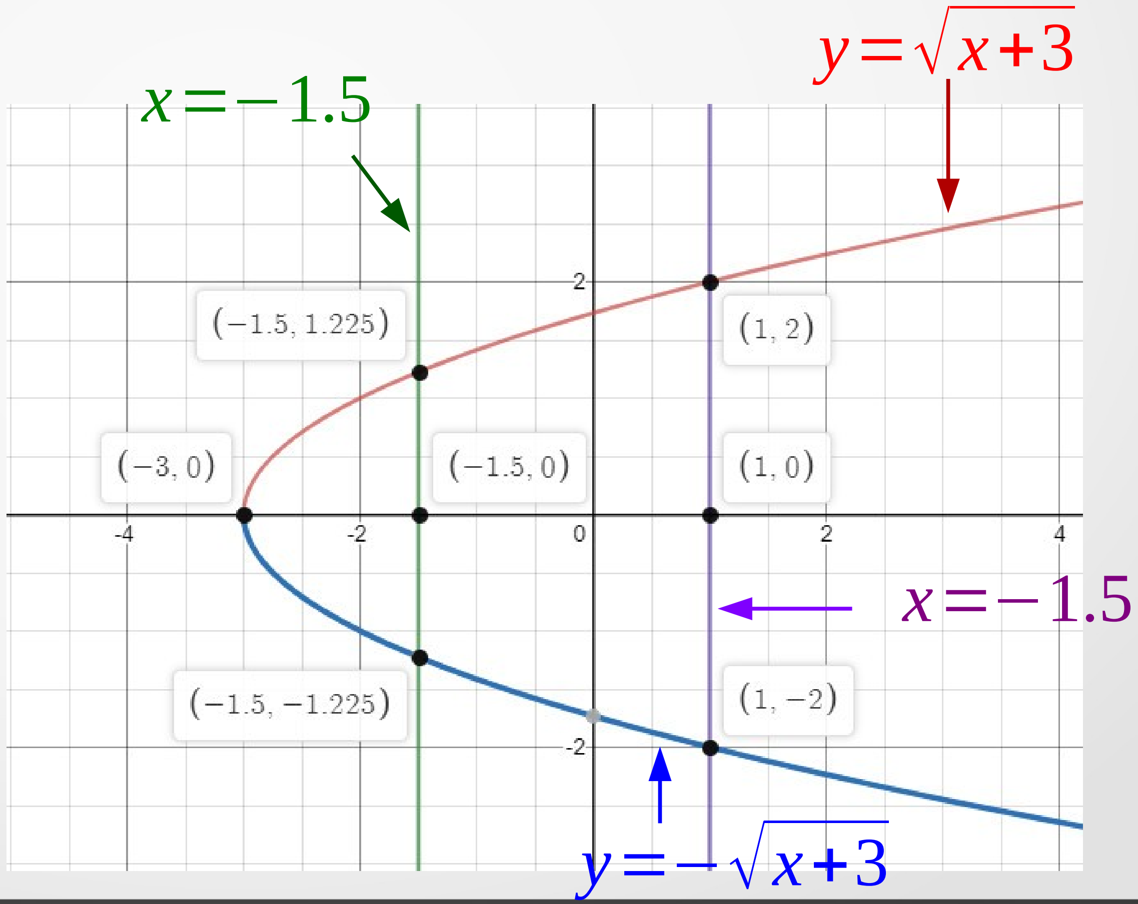

$y=\sqrt{x+3}$

$x=-1.5$

$x=-1.5$

$y=-\sqrt{x+3}$

two values
~~one value~~ of y for a given value of x

1.1 Functions and Function Notation

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If any *vertical line* intersects a graph *in more than one point*, then the graph does not define a function

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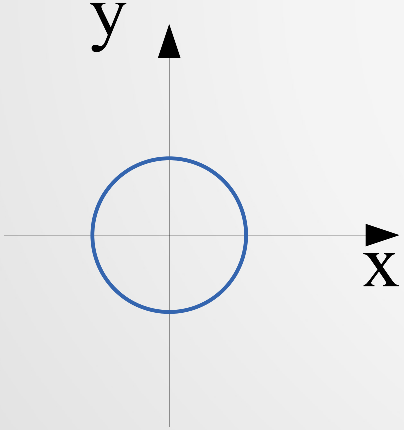

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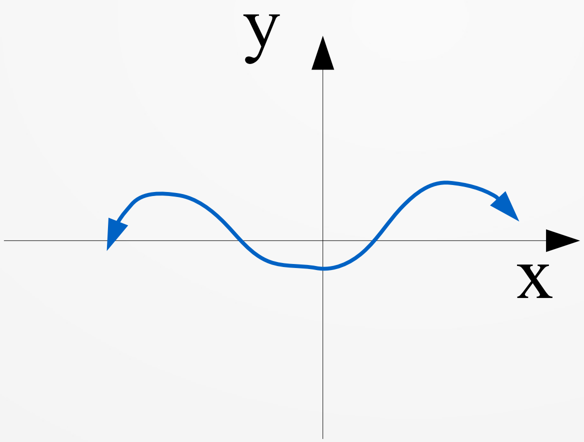

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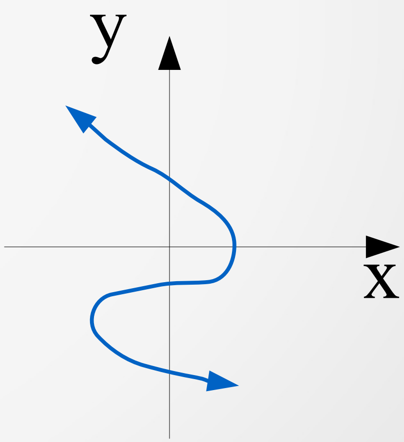

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1.1 Functions and Function Notation

Obtaining Information from Graphs

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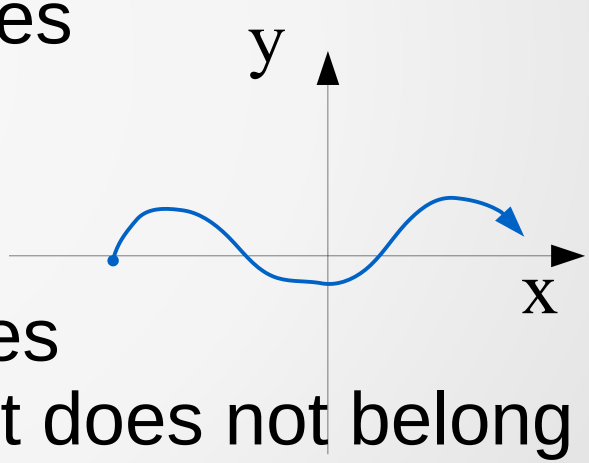


an open dot ° indicates that the graph does not extend beyond this point and the point does not belong to the graph

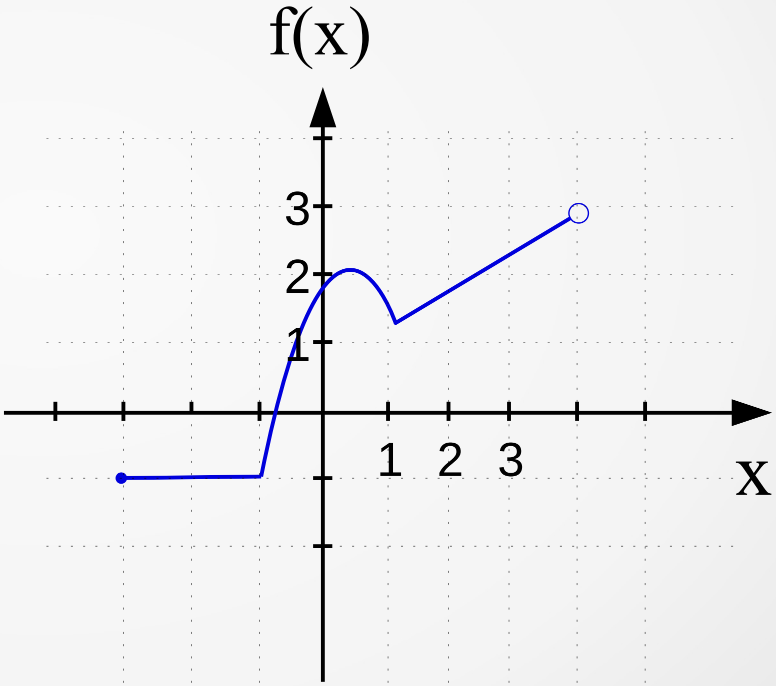
an arrow → indicates that the graph extends indefinitely in the direction the arrow points

1.1 Functions and Function Notation

Obtaining Information from Graphs

(2) Evaluate

- **(a)** f(0)
- **(b)** f(-1)
- **(c)** f(4)
- **(d)** f(-3)
- **(e)** f(-4)



1.1 Functions and Function Notation

Obtaining Information from Graphs

(2) Evaluate

- **(a)** f(0) ≈ 1.8
- **(b)** f(-1) = -1
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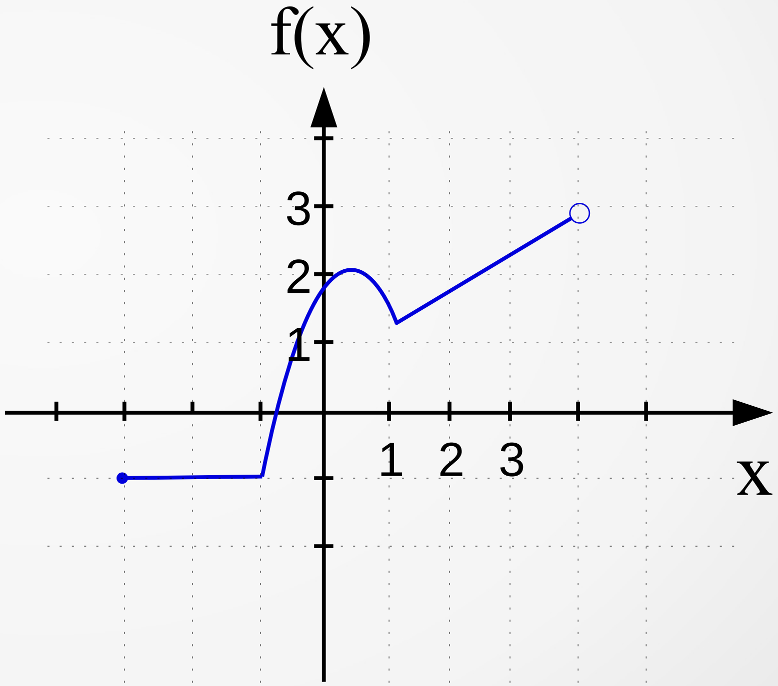

1.1 Functions and Function Notation

Obtaining Information from Graphs

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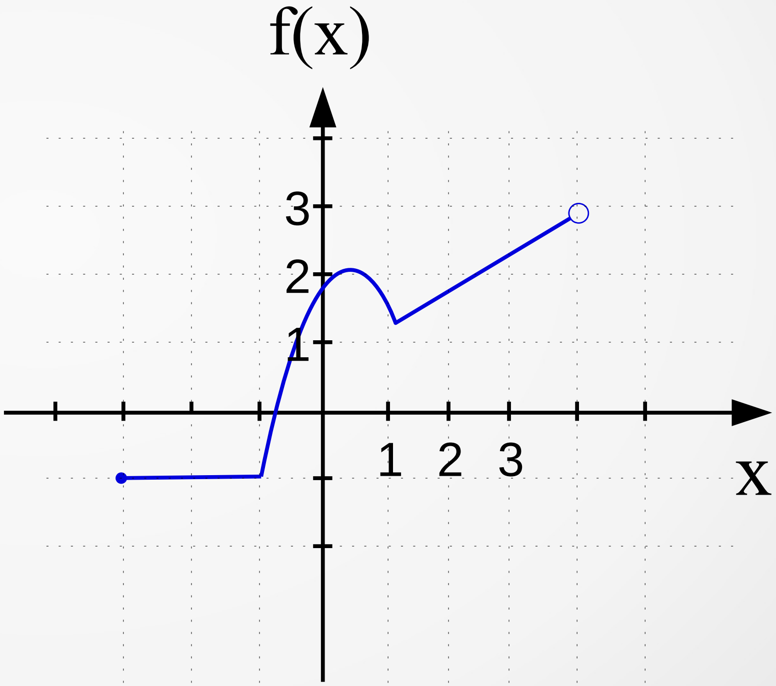
(3) Solve

- **(a)** f(x) = 0
- **(b)** f(x) = 2.1



1.1 Functions and Function Notation

Obtaining Information from Graphs

(2) Evaluate

- **(a)** $f(0) \approx 1.8$
- **(b)** $f(-1) = -1$
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- **(d)** $f(-3) = -1$
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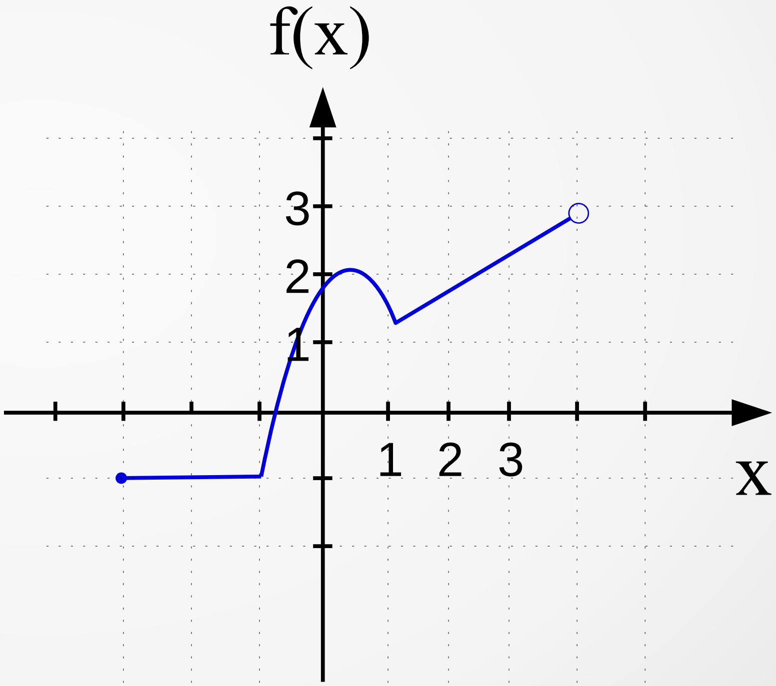


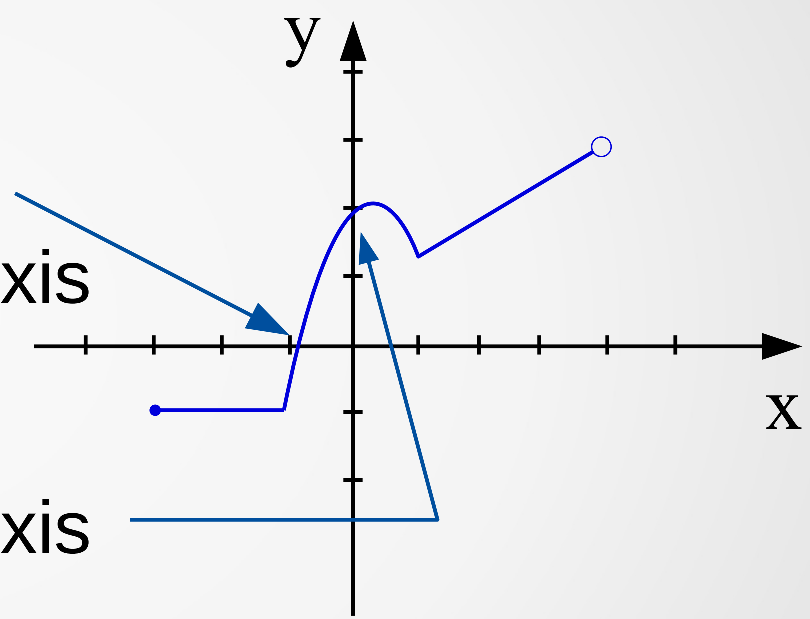
(3) Solve

- **(a)** $f(x) = 0$ $x \approx -0.75$
- **(b)** $f(x) = 2.1$ $x \approx 0.4, 2.5$

1.1 Functions and Function Notation

Obtaining Information from Graphs

(4) identify intercepts



x-intercept is the point where the graph intercepts or touches the x-axis

y-intercept is the point where the graph intercepts or touches the x-axis

In-class practice

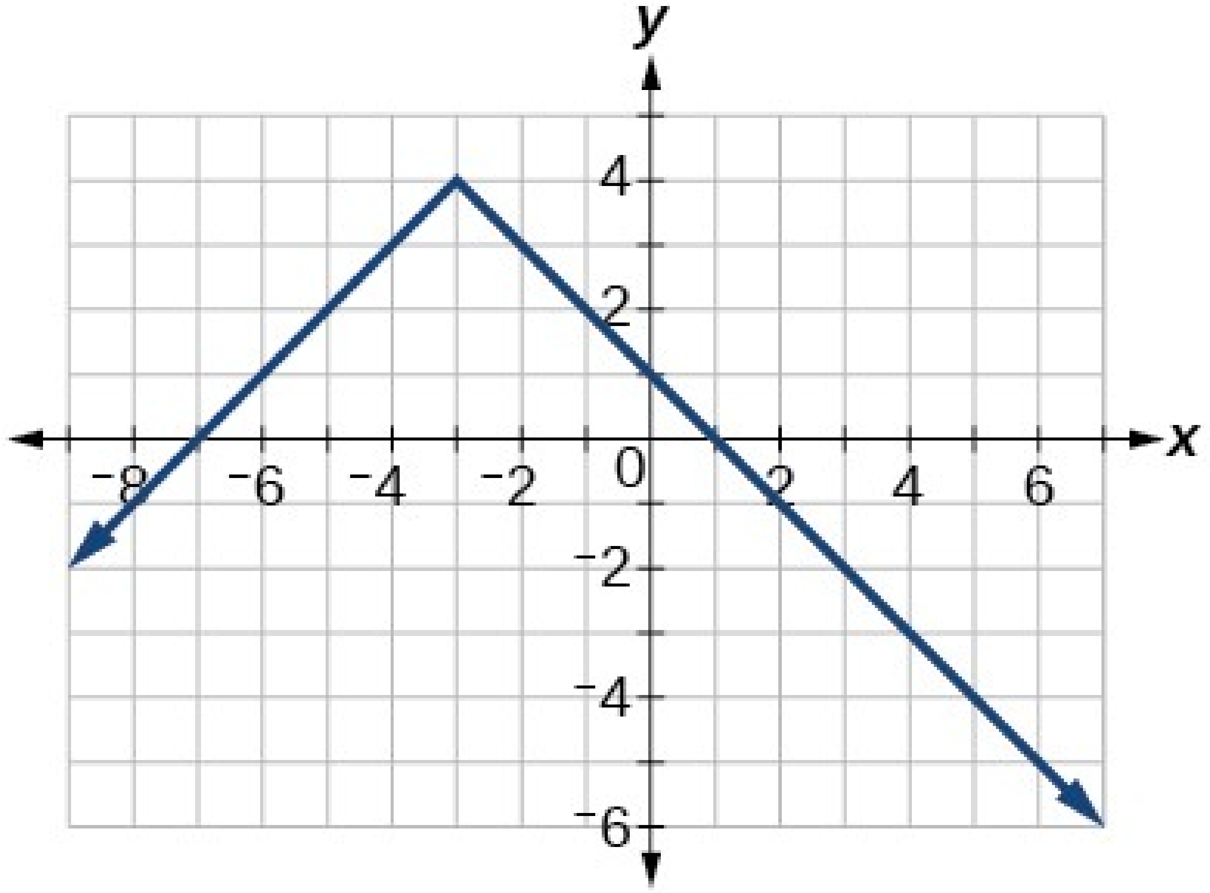
Exercise 1: Given the following graph,

(a) evaluate $f(4)$

(b) solve for $f(x) = 1$

(c) find the x-intercept

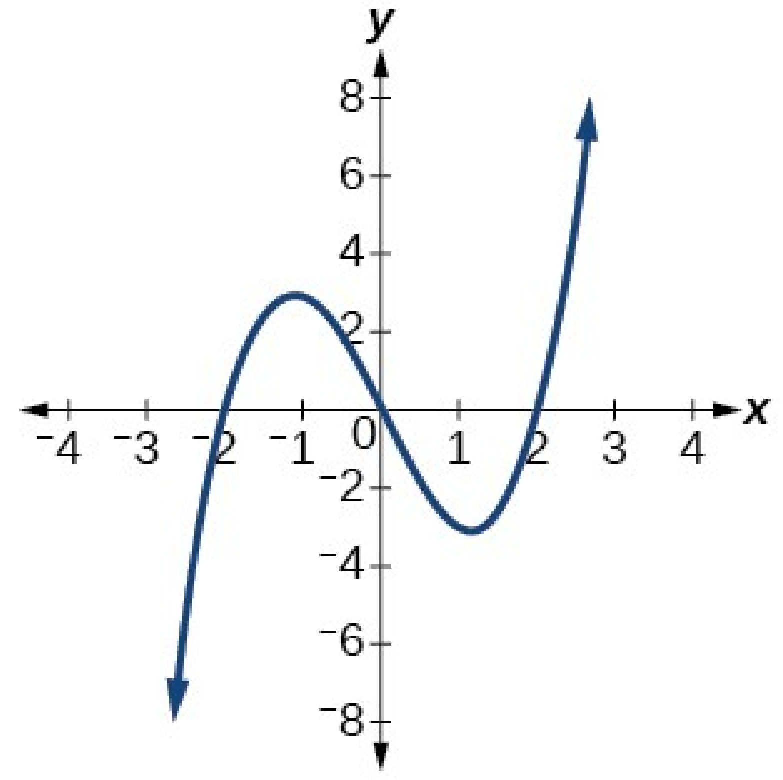
(d) find the y-intercept



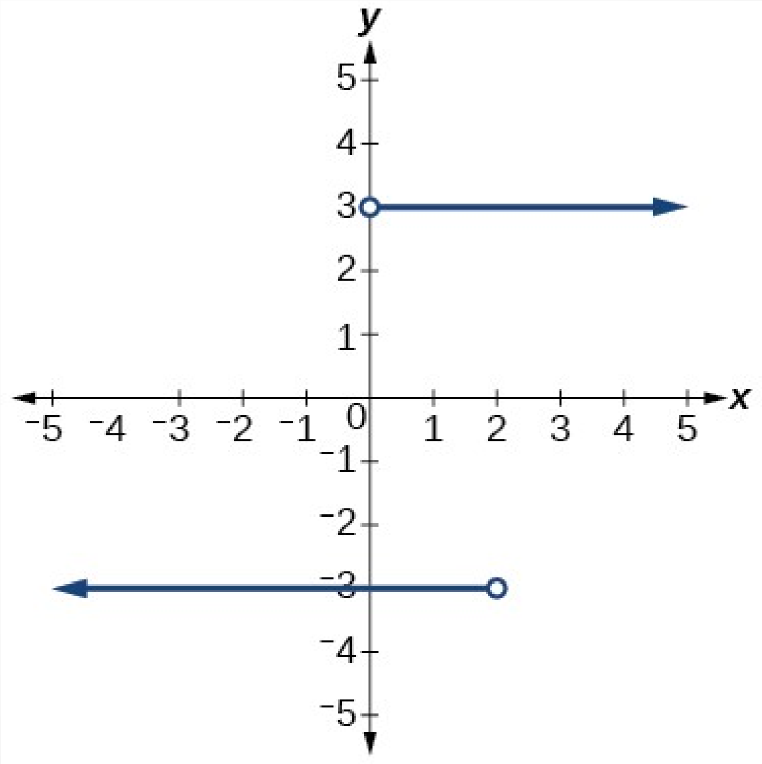
In-class practice

Exercise 2: use vertical line test to determine which graphs show relations that are *functions.*

(a)



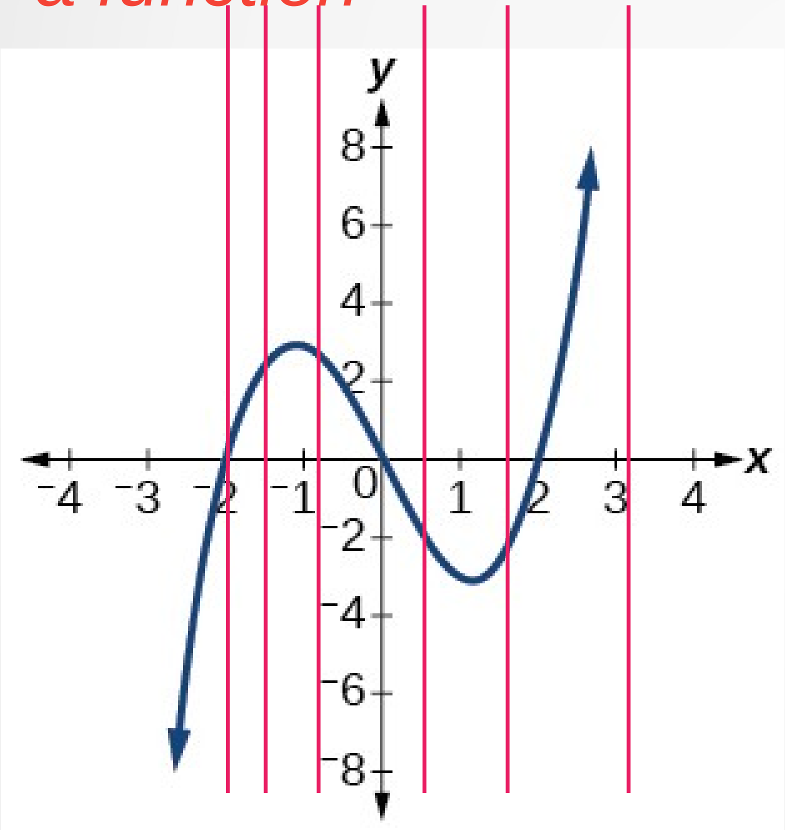
(b)



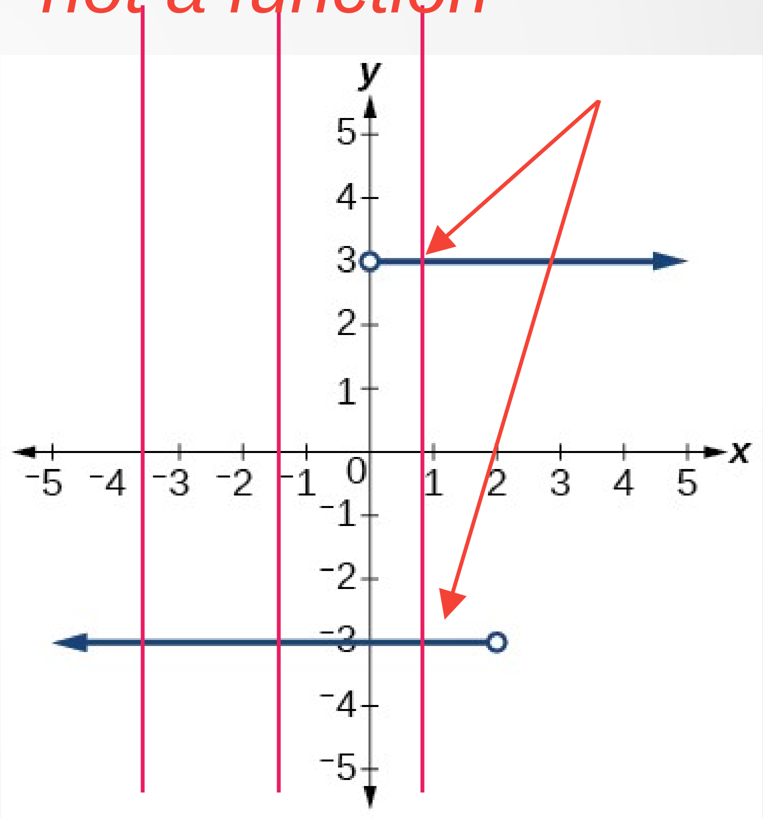
In-class practice

Exercise 2: use vertical line test to determine which graphs show relations that are *functions.*

(a) *a function*



(b) *vertical line test failed*
not a function



In-class practice

Exercise 3: For the given equations determine which ones define functions. Explain why.

(a) $y=\sqrt{npq}$

(b) $7x+y^2=100$

(c) $10x+7y=20$

In-class practice

Exercise 4: For the function $f(x)=x^2-x+10$. Find

(a) $f(3)$

(b) $f(x-2)$

(c) $f(-x)$

1.1 Functions and Function Notation

One-to-one functions

[Def] A *one-to-one function* is a function in which each range/output value corresponds to exactly one domain/input value.

[Def] A *one-to-one function* is a function in which no two elements in the domain/input correspond to the same element in the range/output.

y

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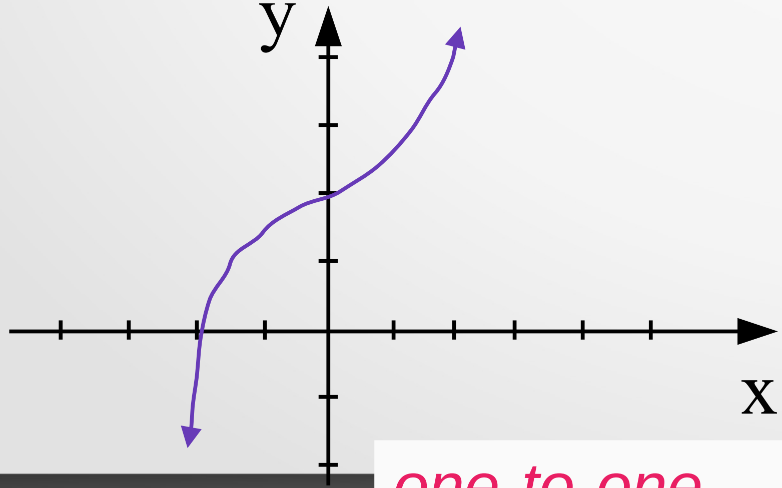
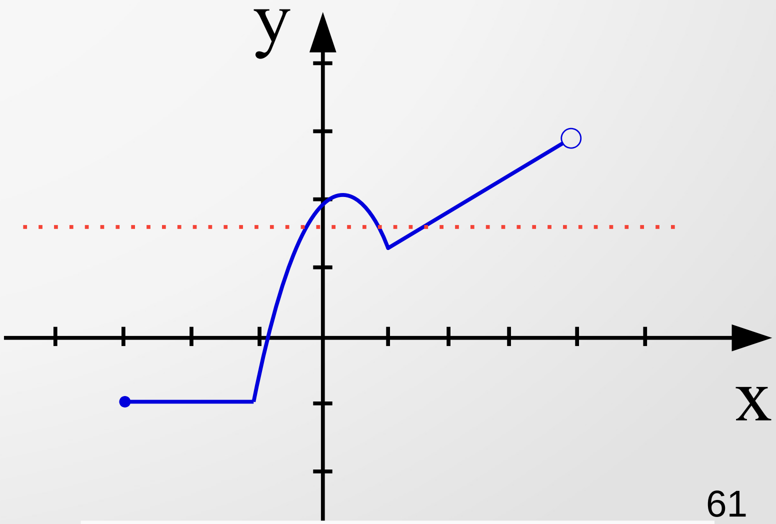


one-to-one



not one-to-one

1.1 Functions and Function Notation

Vertical Line Test

If any *vertical line* intersects a graph *in more than one point*, then the graph does not define a function

Horizontal Line Test

If any *horizontal line* intersects a graph *in more than one point*, then the graph does not define a one-to-one function

1.1 Functions and Function Notation

Basic Functions

See Section 1.1 of the book (Table 13) for the list of the basic toolkit functions.

1.1 Functions and Function Notation

Objectives:

- *Determine whether a relation represents a function.*
- *Find the value of a function.*
- *Determine whether a function is one-to-one.*
- *Use the vertical line test to identify functions.*
- *Graph the functions listed in the library of functions.*

Homework assignment

1) Precalculus textbook: read Section 1.1

2) WeBWorK:

- login into the webwork.
  If you tried several times, followed all the instructions and it still doesn't let you in, send me an email to natna20@gmail.com
- start working on HW 1 (due date is in one week)

3) Visit out website: https://natna.info/MTH30/