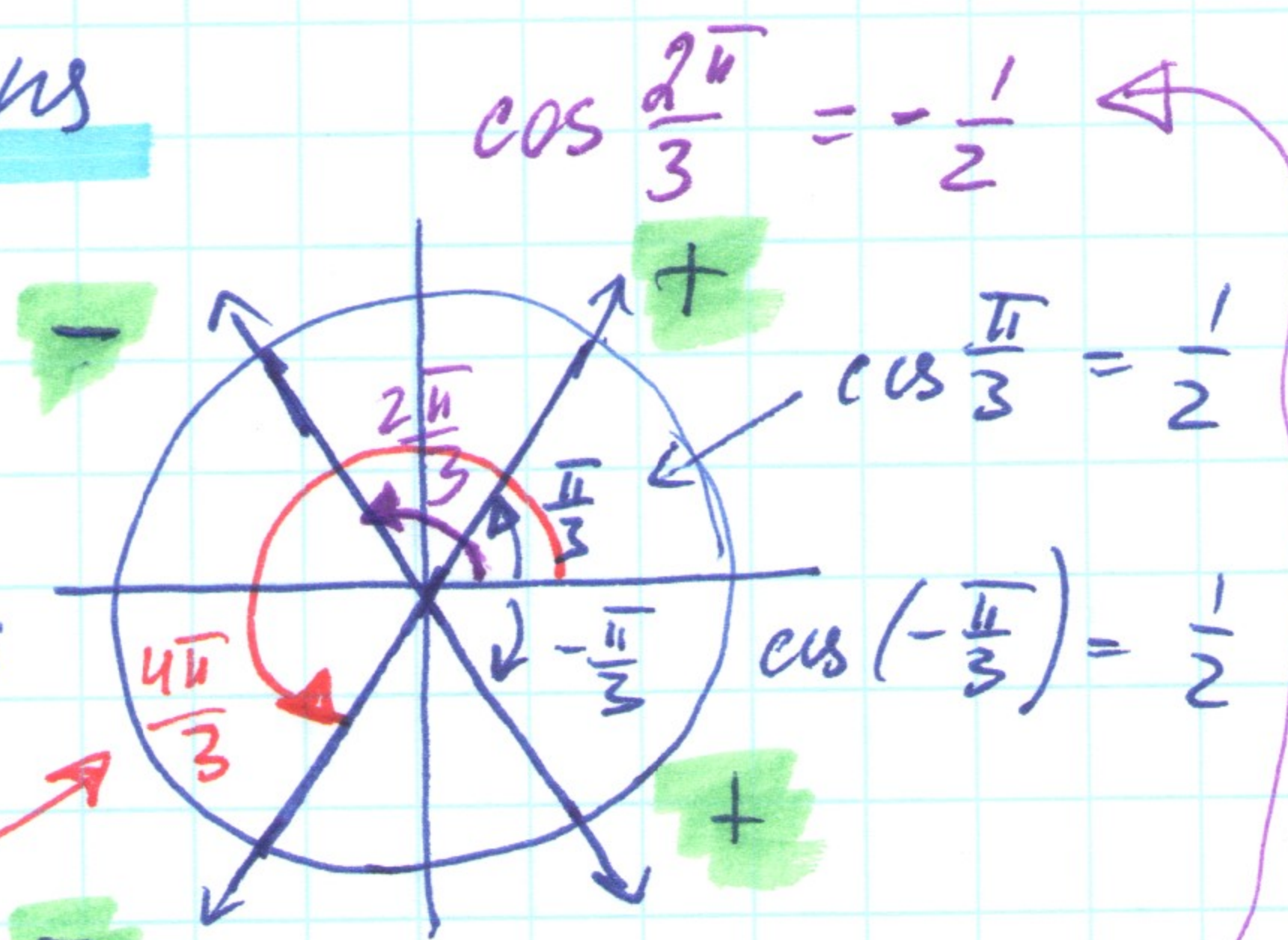
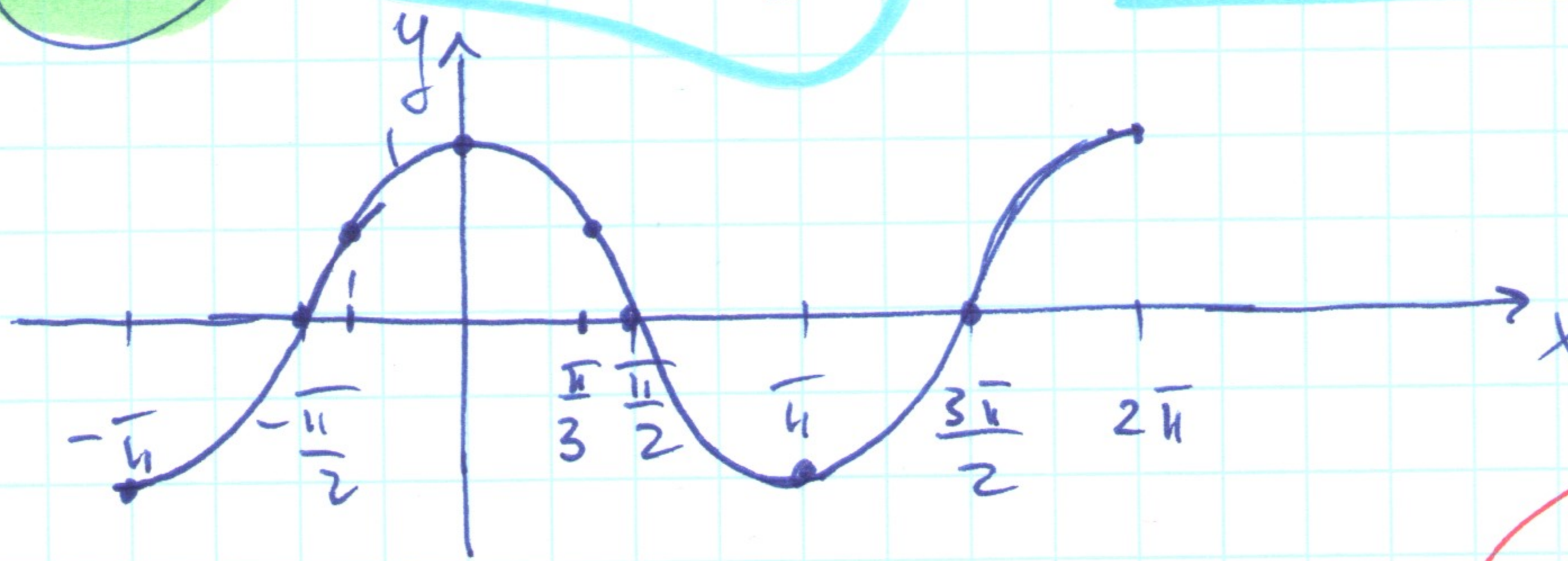


15

$\cos x = -\frac{1}{2}$

all solutions



$x = \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$

or

$x = \frac{4\pi}{3} + 2\pi n, n \in \mathbb{Z}$

$\frac{\pi}{2} \times 3 + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

$\frac{\pi}{3} + \pi = \frac{4\pi}{3}$

$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

19

$2 \cos x + \sqrt{3} = 0$

$\frac{2 \cos x}{2} = \frac{-\sqrt{3}}{2}$

$\cos x = -\frac{\sqrt{3}}{2}$

using reference angles and signs of cosine function

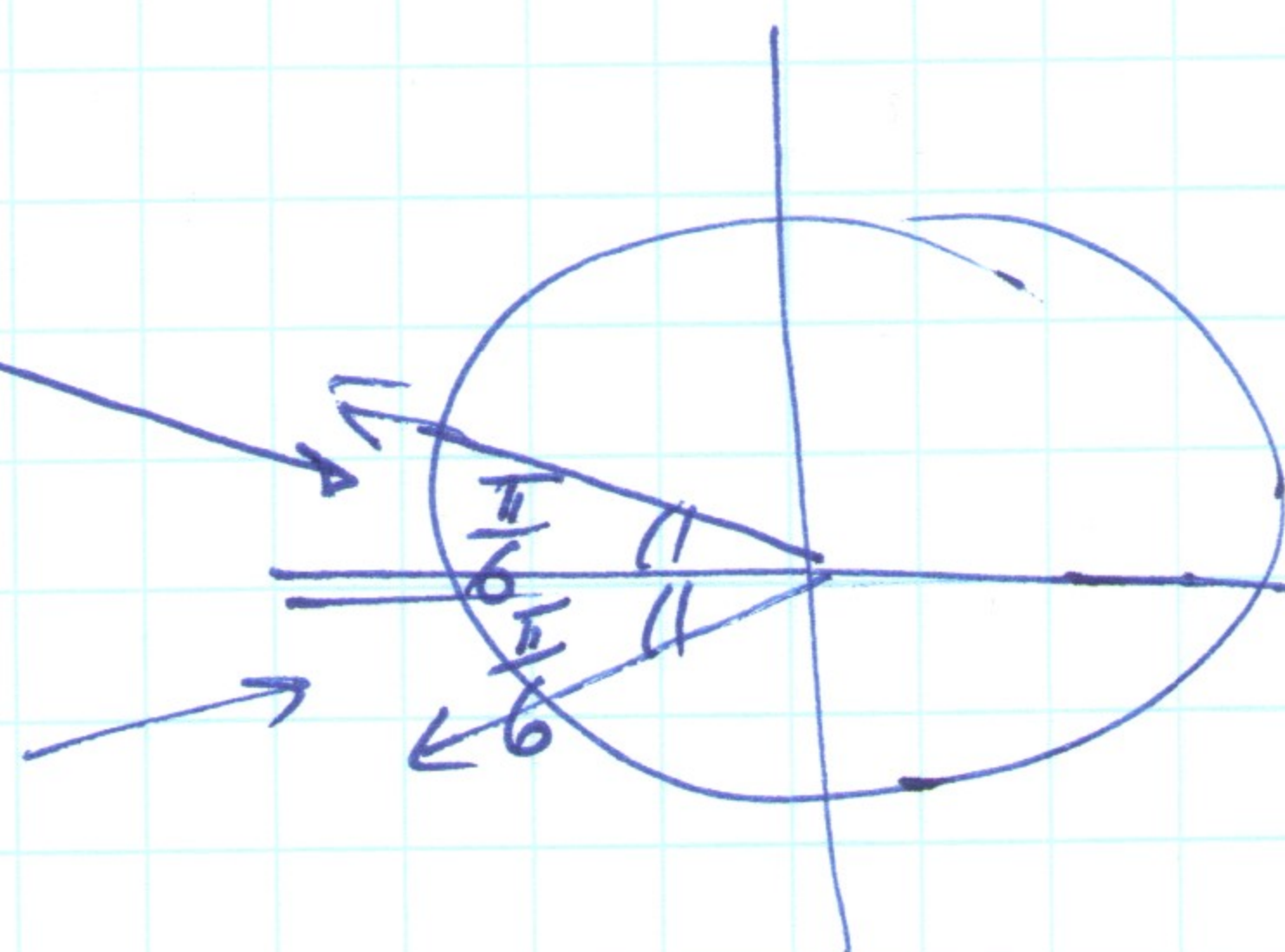
$\cos 30^\circ = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ need sign change

$x = \frac{5\pi}{6} + 2\pi n$

$\leftarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \frac{7\pi}{6} + 2\pi n$

$\leftarrow \pi + \frac{\pi}{6} = \frac{7\pi}{6}$



$n \in \mathbb{Z}$

#23

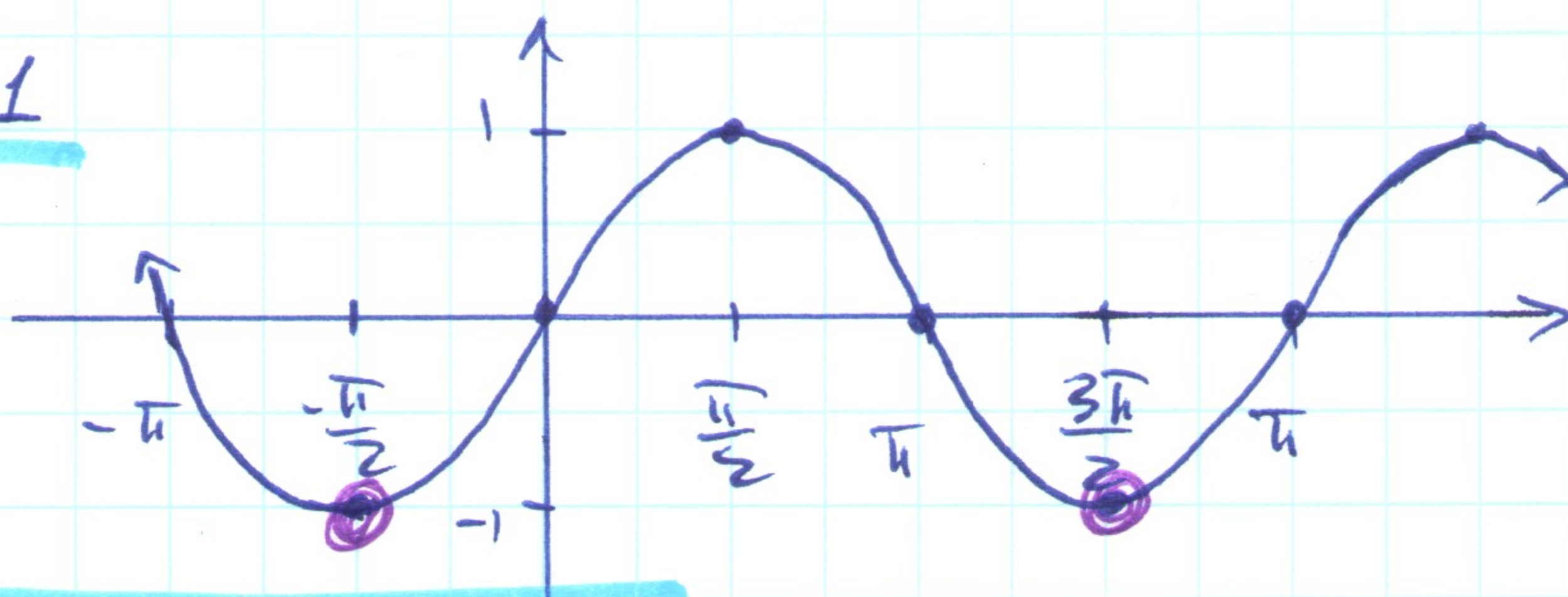
$$3 \sin \theta + 5 = -2 \sin \theta$$

$$5 \sin \theta + 5 = 0$$

$$\frac{5 \sin \theta}{5} = \frac{-5}{5}$$

$$\sin \theta = -1$$

period of
sine: 2π



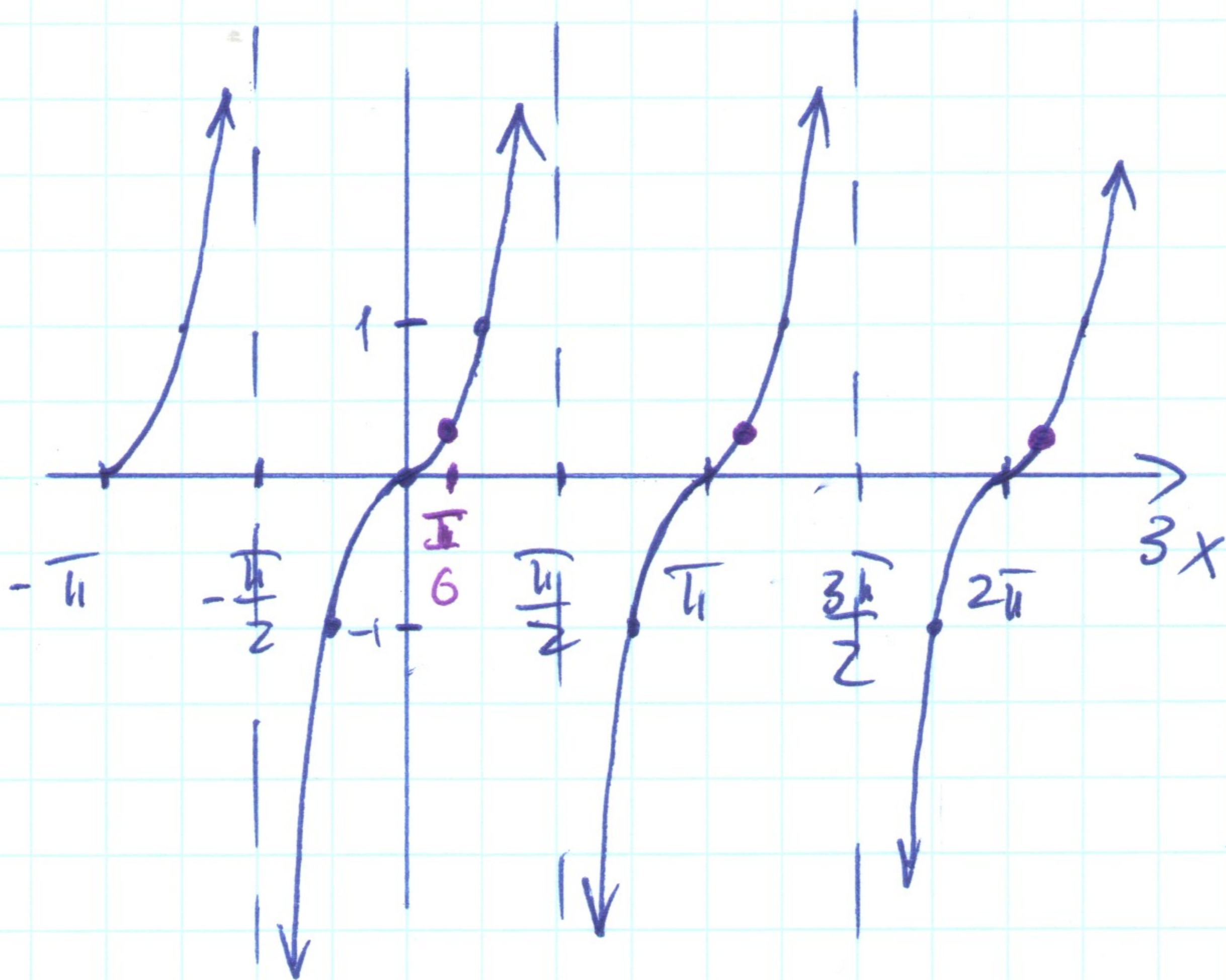
$$\theta = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

#29

interval: $[0, 2\pi)$

$$\tan 3x = \frac{\sqrt{3}}{3}$$

period of tan
function: π



$$\frac{3x}{3} = \frac{\frac{\pi}{6} + \pi n}{3}, n \in \mathbb{Z}$$

$$x = \frac{\pi}{18} + \frac{\pi n}{3}, n \in \mathbb{Z}$$

$n=0$

$$x = \frac{\pi}{18}$$

; $n=1$

$$x = \frac{\pi}{18} + \frac{\pi \times 6}{3 \times 6} = \frac{7\pi}{18}$$

$n=2$

$$x = \frac{\pi}{18} + \frac{2\pi \times 6}{3 \times 6} = \frac{13\pi}{18}$$

; $n=3$

$$x = \frac{\pi}{18} + \frac{3\pi \times 6}{3 \times 6} = \frac{19\pi}{18}$$

$n=4$

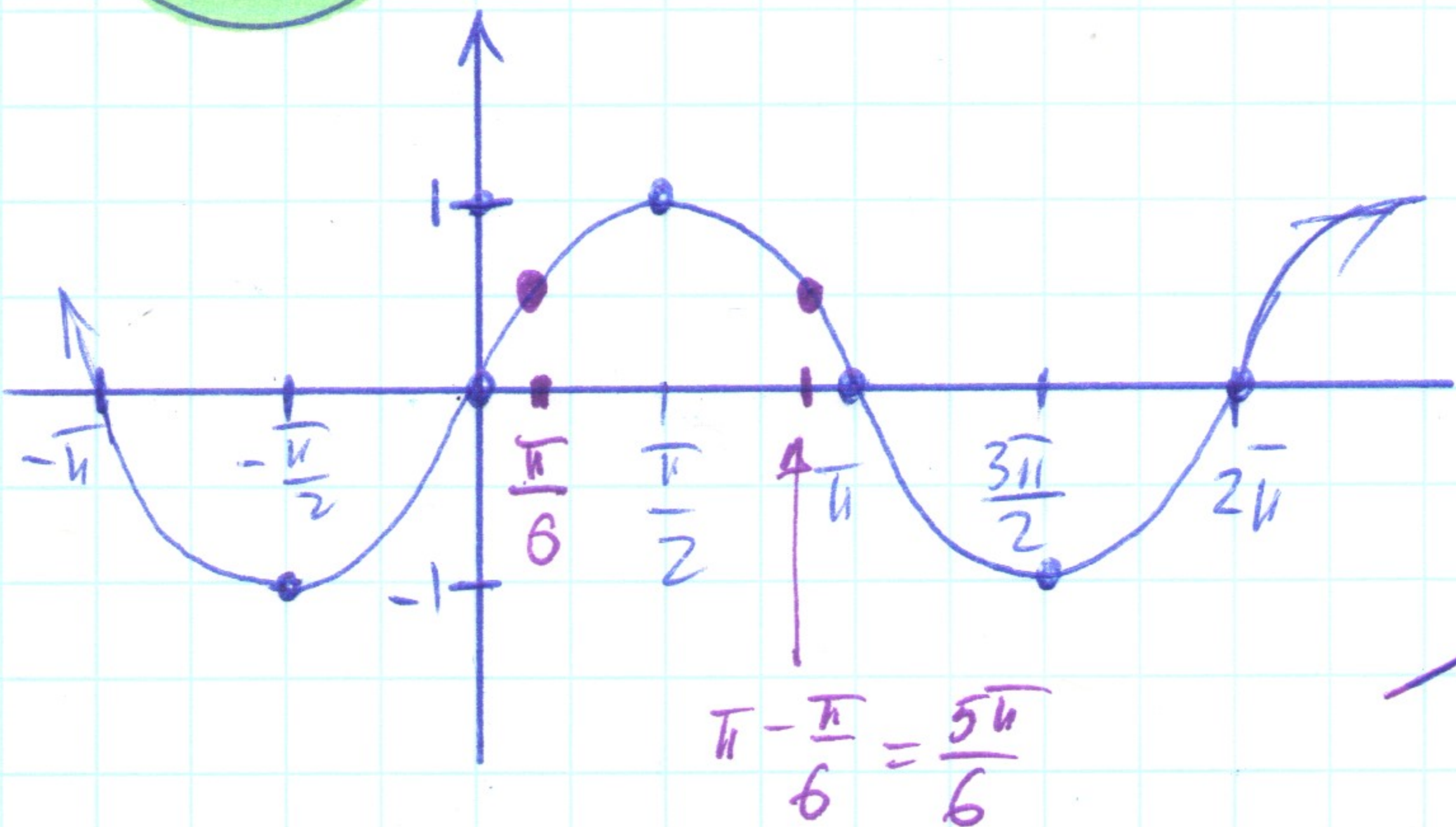
$$x = \frac{\pi}{18} + \frac{4\pi \times 6}{3 \times 6} = \frac{25\pi}{18}$$

; $n=5$

$$x = \frac{\pi}{18} + \frac{5\pi \times 6}{3 \times 6} = \frac{31\pi}{18}$$

#37

$$\sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$$



period of sine: 2π

$$2x + \frac{\pi}{6} = \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$$

or

$$2x + \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi n, n \in \mathbb{Z}$$

$$\frac{2x}{2} = \frac{2\pi n}{2}, n \in \mathbb{Z}$$

or

$$\frac{2x}{2} = \frac{2\pi n}{2} + \frac{2\pi n}{2}, n \in \mathbb{Z}$$

$$x = \pi n, n \in \mathbb{Z}$$

or

$$x = \frac{\pi}{3} + \pi n, n \in \mathbb{Z}$$

on interval $[0, 2\pi)$:

$$n=0: x=0$$

$$n=0: x = \frac{\pi}{3}$$

$$n=1: x = \pi$$

$$n=1: x = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

$$n=2: x = 2\pi$$

$$n=2: x = \frac{\pi}{3} + 2\pi > 2\pi$$

$$x = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$$

#43

$$2 \sin^2 x = \sin x + 3$$

$$2 \sin^2 x - \sin x - 3 = 0$$

Let $t = \sin x$, then we get $2t^2 - t - 3 = 0$

1st way: guessing: $(2t \quad)(t \quad)$
 $\begin{matrix} +1 & -3 \\ -3 & +1 \end{matrix} \rightarrow 2t^2 - 6t + t - 3$
 $\rightarrow 2t^2 + 2t - 3t - 3 = 2t^2 - t - 3 \checkmark$

$$(2t-3)(t+1) = 0 \quad t = -1, \frac{3}{2}$$

2nd way: or using quadratic formula: $D = b^2 - 4ac = (-1)^2 - 4 \cdot 2 \cdot (-3) =$

$$t_{1,2} = \frac{-(-1) \pm \sqrt{25}}{2 \cdot 2} = \frac{1 \pm 5}{4} = -1, + \frac{6}{4} = \frac{3}{2}$$

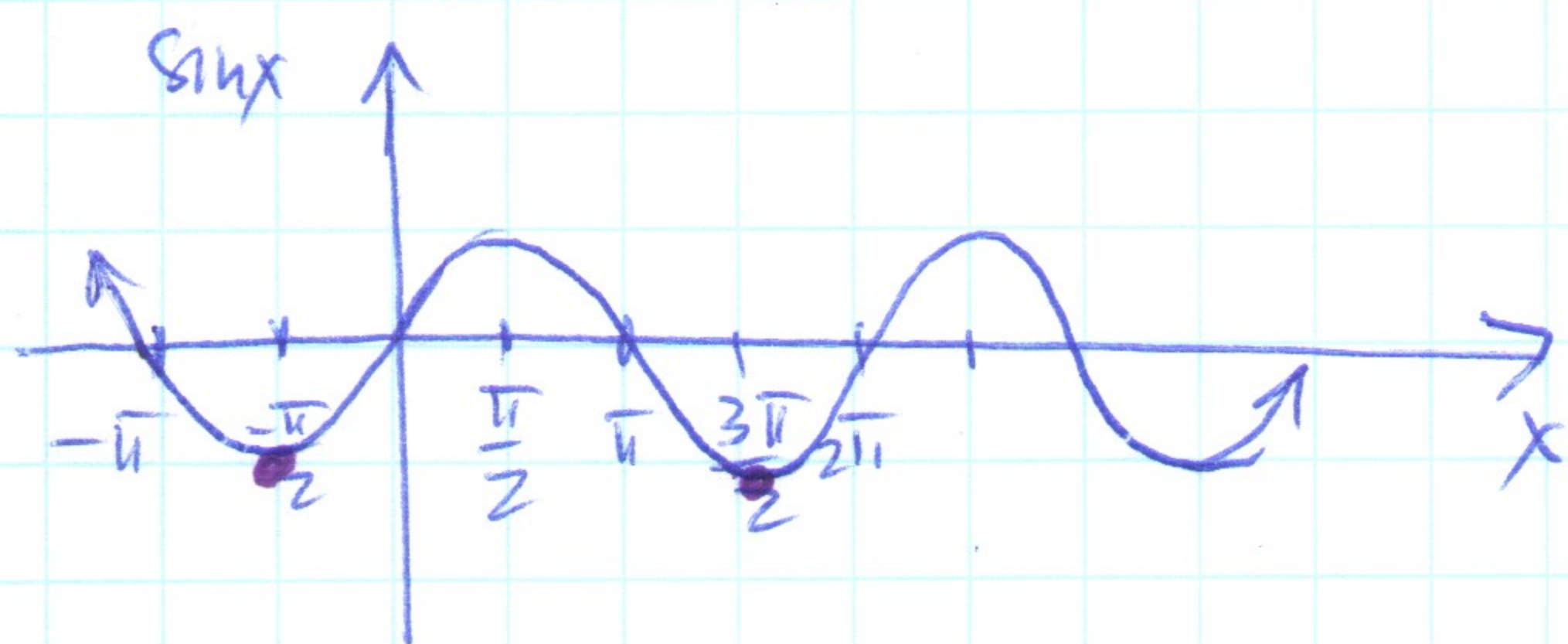
$$t_{1,2} = -1, \frac{3}{2}$$

$$\left(x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$\sin x = -1$$

or

~~$\sin x = \frac{3}{2} > 1$ cannot happen~~



$$x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

$$n=0:$$

$$x = \frac{3\pi}{2}$$

$$n=1:$$

~~$$x = \frac{3\pi}{2} + 2\pi > 2\pi$$~~

#47

$$4 \cos^2 x - 1 = 0$$

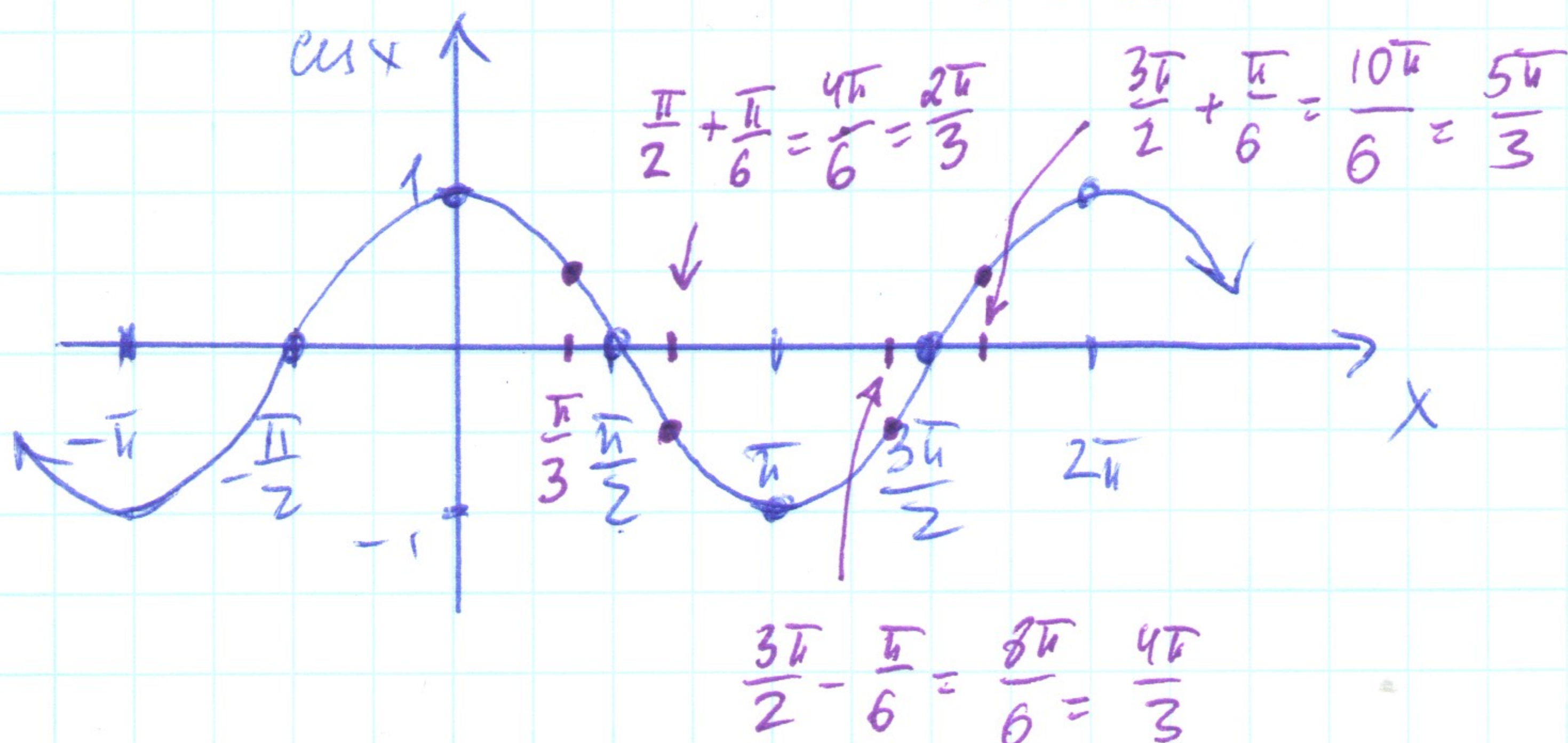
interval: $[0, 2\pi)$

$$\frac{4 \cos^2 x}{4} = \frac{1}{4}$$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

$\cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{2}$



$x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

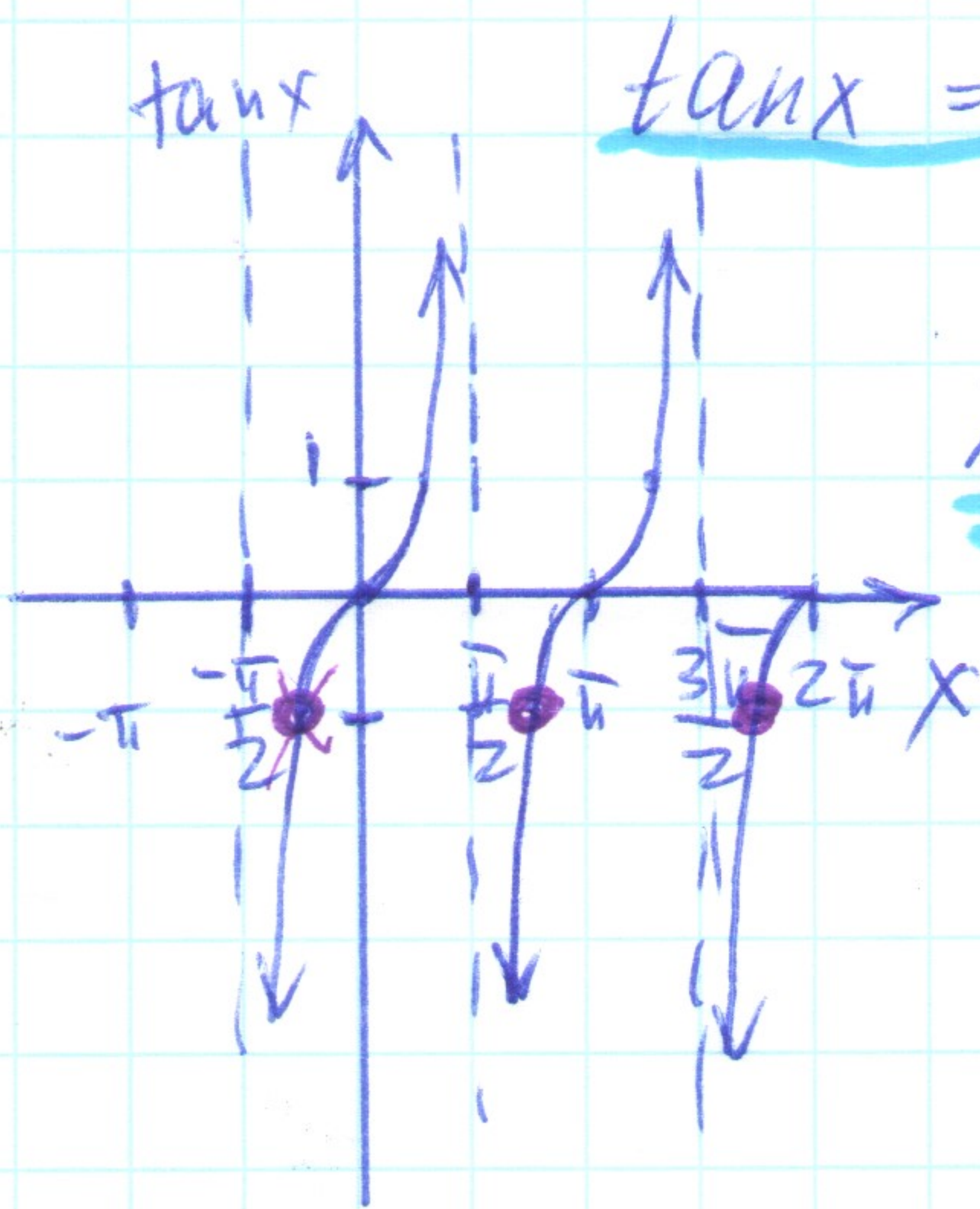
#54

$$(\tan x + 1)(\sin x - 1) = 0$$

interval: $[0, 2\pi)$

$$\tan x + 1 = 0$$

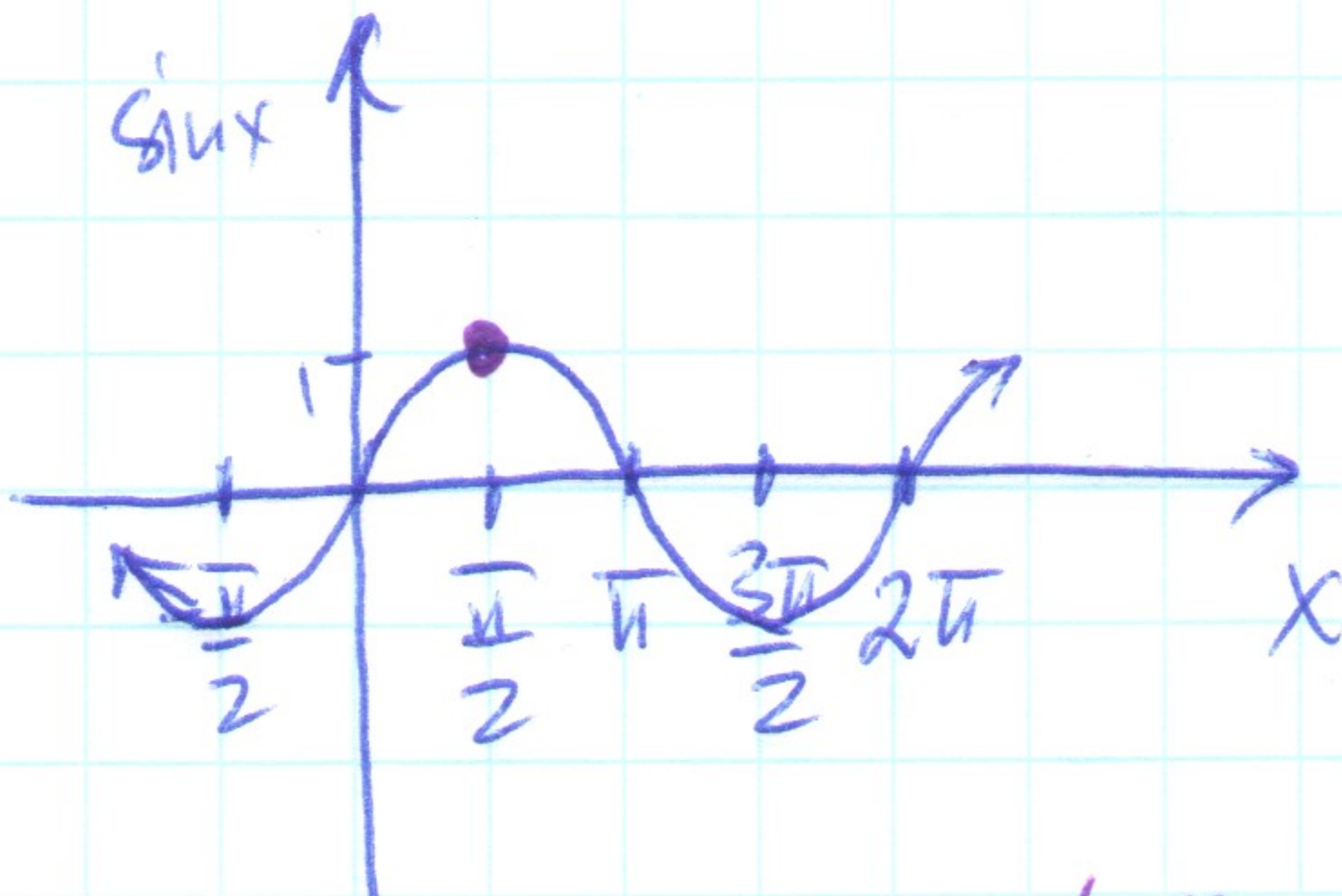
$$\text{or } \sin x - 1 = 0$$



$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\sin x = 1$



$x = \frac{\pi}{2}$

problem: $\tan x$ is undefined on $\frac{\pi}{2}$! exclude!

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

#60

$$\cos x - 2 \sin x \cos x = 0$$

interval: $[0, 2\pi)$

$$\cos x (1 - 2 \sin x) = 0$$

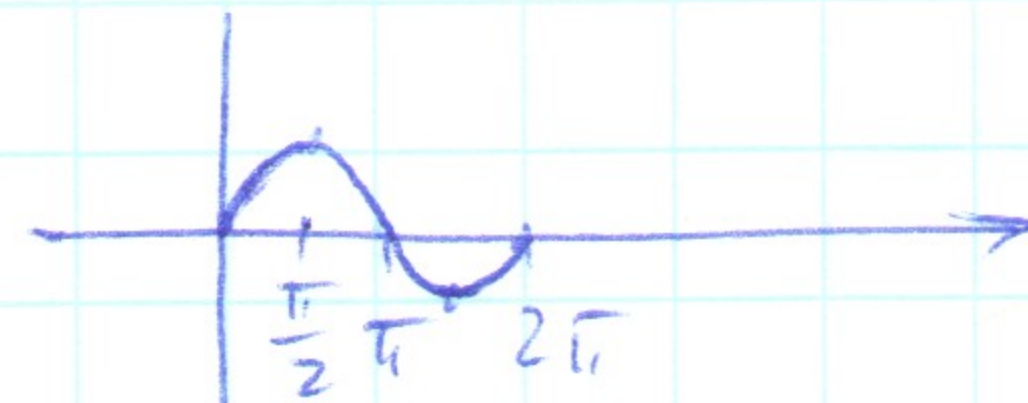
$$\cos x = 0$$

or

$$1 - 2 \sin x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = \frac{1}{2}$$



$$x = \frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$