#16 $f(x) = x^3 - 4x^2 + 8x - 5$.

a) possible rational zeros $= \dfrac{\pm 1, \pm 5}{\pm 1} = \pm 1, \quad \pm 5.$

b) $f(1) = 1 - 4 + 8 - 5 = 0$ ← zero

1	1	-4	8	-5
*	↓	1	-3	5
	1	-3	5	0
	x^2	x	c	r

$$x^3 - 4x^2 + 8x - 5 = (x^2 - 3x + 5)(x - 1)$$

c) Let's solve $x^2 - 3x + 5 = 0$

$$D = b^2 - 4ac = 9 - 4 \cdot 1 \cdot 5 = 9 - 20 = -11.$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm \sqrt{-11}}{2 \cdot 1} = \frac{3 \pm \sqrt{11}\, i}{2}.$$

Answer: $\boxed{x = 1, \ \dfrac{3 - \sqrt{11}\, i}{2}, \ \dfrac{3 + \sqrt{11}\, i}{2}}$

HW #11. MTH30

p. 336/ 26, 30.

read the description in the book.

#26 $n = 3$; zeros: 4 and $2i$; $f(-1) = -50$.

$\downarrow$

$-2i$ – another zero.

$$f(x) = a_n (x-4)(x-2i)(x-(-2i)) =$$
$$= a_n (x-4)(x-2i)(x+2i) =$$
$$= a_n (x-4)(x^2 - 4i^2) = a_n (x-4)(x^2+4) =$$
$$= a_n (x^3 + 4x - 4x^2 - 16)$$

$f(-1) = -50$: let's use it:

~~[illegible]~~

$$a_n(-1-4-4-16) = -50$$
$$a_n(-25) = -50 \quad \text{i.e.} \quad \boxed{a_n = 2}$$

$$f(x) = 2(x^3 - 4x^2 + 4x - 16) = 2x^3 - 8x^2 + 8x - 32.$$

Answer: $\boxed{f(x) = 2x^3 - 8x^2 + 8x - 32}$

#30 $n=4$; zeros: $-2, -\frac{1}{2}, i$; $f(1)=18$

$$i \downarrow -i$$ ← one more zero.

$$f(x) = a_4\big(x-(-2)\big)\big(x-(-\tfrac{1}{2})\big)(x-i)\big(x-(-i)\big) =$$

$$= a_4 \underbrace{(x+2)(x+\tfrac{1}{2})}\,\underbrace{(x-i)(x+i)} =$$

$$= a_4\left(x^2+\tfrac{1}{2}x+2x+1\right)\left(x^2-i^2\right) =$$

$$= a_4\left(x^2+\tfrac{5}{2}x+1\right)\left(x^2+1\right) =$$

$$= a_4\left(x^4+x^2+\tfrac{5}{2}x^3+\tfrac{5}{2}x+x^2+1\right) =$$

$$= a_4\left(x^4+\tfrac{5}{2}x^3+2x^2+\tfrac{5}{2}x+1\right)$$

$f(1)=18$ - let's use it:

$$a_4\left(1^4+\tfrac{5}{2}+2+\tfrac{5}{2}+1\right)=18$$

$$a_4 \cdot 9 = 18 \qquad \boxed{a_4=2}$$

$$f(x) = 2\left(x^4+\tfrac{5}{2}x^3+2x^2+\tfrac{5}{2}x+1\right) = 2x^4+5x^3+4x^2+5x+2.$$

Answer: $\boxed{f(x)=2x^4+5x^3+4x^2+5x+2.}$

MTH30 Homework Section 2.5

#34 $f(x) = x^3 + 7x^2 + x + 7$ no sign changes!

- positive real zeros: either $\boxed{0}$ or $\cancel{1/2}$
- negative real zeros:

$f(-x) = -x^3 + 7x^2 - x + 7$ either $\boxed{3}$ $\boxed{\text{or}}$ $3-2=\boxed{1}$

1st 2nd 3rd

#38 $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

1st 2nd 3rd

$f(-x) = 4x^4 + x^3 + 5x^2 + 2x - 6$

1st

- positive real zeros: $\boxed{3}$ or $3-2=\boxed{1}$
- negative real zeros: $\boxed{1}$

MTH 30 Homework Section 2.5

#48 $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

1) leading coefficient $= 2 > 0$, degree $= 4$ even

Therefore end-behavior : $\uparrow\uparrow$ or $\nwarrow\nearrow$

2) $a_0 = 15$ $a_n = a_4 = 2$

possible real zeros $\frac{a_0}{a_n}$ $\frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2} =$

$$= \boxed{\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm 5, \pm 15, \pm \frac{15}{2}}$$

$f(1) = 2 + 3 - 11 - 9 + 15 = 20 - 20 = 0$ STOP!

We found one real zero: 1

3) use synthetic or long division to divide $f(x) \div x - 1$

	x^4	x^3	x^2	x	const
1	2	3	-11	-9	15
		+ 2	+ 5	+ -6	-15
	2	5	-6	-15	0
	x^3	x^2	x	const	remainder (as expected, zero)

$$\boxed{2x^3 + 5x^2 - 6x - 15}$$

or

$$\begin{array}{r} 2x^3 + 5x^2 - 6x - 15 \\ x - 1 \overline{\smash{)}\, 2x^4 + 3x^3 - 11x^2 - 9x + 15} \\ -\ (2x^4 - 2x^3) \qquad\qquad\qquad\qquad \\ 5x^3 - 11x^2 \qquad\qquad\quad \\ -\ (5x^3 - 5x^2) \qquad\qquad\quad \\ -6x^2 - 9x \qquad\quad \\ -\ (-6x^2 + 6x) \qquad\quad \\ -15x + 15 \\ -\ (-15x + 15) \\ 0 \end{array}$$

$\frac{2x^4}{x} = 2x^3$

$\frac{5x^3}{x} = 5x^2$

$\frac{-6x^2}{x} = -6x$

$\frac{-15x}{x} = -15$

4) let's solve

$$\underbrace{2x^3 + 5x^2}_{GCF = x^2} \underbrace{- 6x - 15}_{GCF = -3} = 0$$

$$x^2\underbrace{(2x+5)} - 3\underbrace{(2x+5)} = 0$$

Same factor - expressions

$$(2x+5)(x^2-3) = 0$$

$2x+5=0$ $\qquad$ $x^2-3=0$

$x = -\dfrac{5}{2}$ $\qquad$ $x = \pm\sqrt{3}$

Answer: $-\dfrac{5}{2}$, $-\sqrt{3}$, 1, $\sqrt{3}$

check: degree $= 4$

\# of zeros $= 4$ ✓

p. 336/12, 16
read assignment in the book

#12 $f(x) = 2x^3 - 5x^2 + x + 2$.

a) possible rational zeros $= \dfrac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \pm \frac{1}{2}, \pm 2$.

b) $f(1) = 2 - 5 + 1 + 2 = 0$ ← zero

$$
\begin{array}{c|cccc}
1 & 2 & -5 & 1 & 2 \\
* & \downarrow & 2 & -3 & -2 \\
\hline
 & 2 & -3 & -2 & 0 \\
 & x^2 & x & c & r
\end{array}
$$

$2x^3 - 5x^2 + x + 2 = (2x^2 - 3x - 2)(x - 1)$

c) solve $2x^2 - 3x - 2 = 0$

$MP = -4 \; : \; -4, 1$

$2x^2 - 4x + x - 2 = 0$

$2x(x-2) + 1(x-2) = 0$

$(2x+1)(x-2) = 0$

$x = 2, -\frac{1}{2}$

Answer: $\boxed{x = -\frac{1}{2}, 1, 2}$