#2 $(x^2+3x-10) \div (x-2)$

divide using long division; state the quotient $q(x)$, and the remainder, $r(x)$

$$\begin{array}{r|l} & x+5 \\ x-2 & x^2+3x-10 \\ & -(x^2-2x) \\ & \quad 5x-10 \\ & -(5x-10) \\ & \quad 0 \end{array}$$

$\frac{x^2}{x} = x$

$\frac{5x}{x} = 5$

$$\boxed{\frac{x^2+3x-10}{x-2} = x+5}$$

$q(x) = x+5$

$r(x) = 0.$

check: $(x-2)(x+5) = x^2+5x-2x-10 = x^2+3x-10$

#8 $(4x^2-8x+6) \div (2x-1)$

$$\begin{array}{r|l} & 2x-3 \\ 2x-1 & 4x^2-8x+6 \\ & -(4x^2-2x) \\ & \quad -6x+6 \\ & -(-6x+3) \\ & \quad 3 \end{array}$$

$\frac{4x^2}{2x} = 2x$

$\frac{-6x}{2x} = -3$

$$\boxed{\frac{4x^2-8x+6}{2x-1} = 2x-3+\frac{3}{2x-1}}$$

$q(x) = 2x-3$

$r(x) = 3$

check: $\underbrace{(2x-1)(2x-3)}_{q(x)} + \underbrace{3}_{r(x)} = 4x^2-6x-2x+3+3 = 4x^2-8x+6$

#16 $\dfrac{2x^5-8x^4+2x^3+x^2}{2x^3+1}$

$$
\begin{array}{r|l}
 & x^2 - 4x + 1 \\
\hline
2x^3+1 & 2x^5 - 8x^4 + 2x^3 + x^2 \\
 & -\ (2x^5 + x^2) \\
\hline
 & -8x^4 + 2x^3 \\
 & -\ (-8x^4 - 4x) \\
\hline
 & 2x^3 + 4x \\
 & -\ (2x^3 + 1) \\
\hline
 & +4x - 1
\end{array}
$$

$\dfrac{2x^5}{2x^3} = x^2$

$\dfrac{-8x^4}{2x^3} = -4x$

$\dfrac{2x^3}{2x^3} = 1$

$$\boxed{\frac{2x^5-8x^4+2x^3+x^2}{2x^3+1} = x^2-4x+1 + \frac{4x-1}{2x^3+1}.}$$

$q(x) = x^2-4x+1$

$r(x) = (4x-1)$

check: $(x^2-4x+1)(2x^3+1) - (4x+1) = 4x^5 + x^2 - 8x^4 - 4x + 2x^3 + 1 + 4x - 1 =$

$= 4x^5 - 8x^4 + 2x^3 + x^2.$

#22 $(5x^3-6x^2+3x+11) \div (x-2)$ use synthetic division.

$x - C$ i.e. $C = 2.$

2	5	-6	3	11
*		10	8	22
	5	4	11	33
	coeff. of x^2	coeff. of x	const.	remainder

$$\boxed{\frac{5x^3-6x^2+3x+11}{x-2} = 5x^2+4x+11 + \frac{33}{x-2}}$$

check: $(5x^2+4x+11)(x-2)+33 = 5x^3 - 10x^2 + 4x^2 - 8x + 11x - 22 + 33 =$

$= 5x^3 - 6x^2 + 3x + 11.$

#26

$(x^2-6x-6x^3+x^4)\div(6+x)$ reorder by decreasing power of x!

$(x^4-6x^3+x^2-6x)\div(x+6)$

$x-c$ $\quad x+6=x-(-6)$, i.e. $c=-6$

-6	1	-6	1	-6	0 ← for constant
*	↓	-6	72	-438	$+2664$
	1	-12	73	-444	$+2664$
	x^3	x^2	x	const	remainder

$$\boxed{\frac{x^4-6x^3+x-6x}{x+6}=x^3-12x^2+73x-444+\frac{2664}{x+6}}$$

check: $(x+6)(x^3-12x^2+73x-444)-2664=x^4-12x^3+73x^2-444x+$
$+6x^3-72x^2+438x+2664-2664=x^4-6x^3+x^2-6x$

#38

use synthetic division and Remainder theorem to find the indicated function value.

$f(x)=x^4-5x^3+5x^2+5x-6$ $\quad f(2)$

according to R. theorem $f(2)$ is the remainder of the division of $f(x)$ by $x-2$., therefore let's divide $f(x)$ by $x-2$.

2	1	-5	5	5	-6
*	↓	2	-6	-2	6
	1	-3	-1	3	0
					↑ remainder.

$$\boxed{\text{Answer: } f(2)=0}$$

check: $f(2)=2^4-5\cdot2^3+5\cdot2^2+5\cdot2-6=16-40+20+10-6=0$

Section 2.4 Homework

#42 use synthetic division to divide $f(x) = x^3 - 2x^2 - x + 2$ by $x+1$.

Use the result to find all zeros of f.

Solution: 1) $c = -1$ $\quad (x+1 = x-(\overset{c}{-1}))$

$$\begin{array}{r|rrrr}
-1 & 1 & -2 & -1 & 2 \\
 & \downarrow & -1 & 3 & -2 \\
\hline
 & 1 & -3 & 2 & 0 \\
 & x^2 & x & \text{const.} & \text{remainder}
\end{array}$$

*

divisor (factor)

$$(x^3 - 2x^2 - x + 2) = (x^2 - 3x + 2)(x+1)$$

2) let's solve $x^2 - 3x + 2 = 0$

$$x^2 - 3x + 2 = (x-2)(x-1) = 0$$

$(x-2) = 0$ $\qquad$ $x - 1 = 0$

$x = 2$ $\qquad$ $x = 1$

Zeros of $f(x)$: $\boxed{x = -1, 1, 2}$

Answer

MTH30 Homework Section 2.4

#46 Solve the equation $3x^3+7x^2-22x-8=0$ given that $-\frac{1}{3}$ is a root.

Solution: using Factor Theorem:

1) if $f(-\frac{1}{3})=0$, then $x-(-\frac{1}{3})$ is a factor of f.

(! recall solution, zero, root - are the same things)

2) use synthetic division:

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 7 & -22 & -8 \\ * & \downarrow & -1 & -2 & 8 \\ \hline & 3 & 6 & -24 & 0 \\ & x^2 & x & c. & r \end{array}$$

$$3x^3+7x^2-22x-8 = (3x^2+6x-24)(x+\tfrac{1}{3})$$

3) let's solve $3x^2+6x-24=0$

$3(x^2+2x-8)=0$ $\quad x=-4$

$(x+4)(x-2)=0$ $\quad x=2$

Answer: $\boxed{x=-\frac{1}{3}, -4, 2}$