

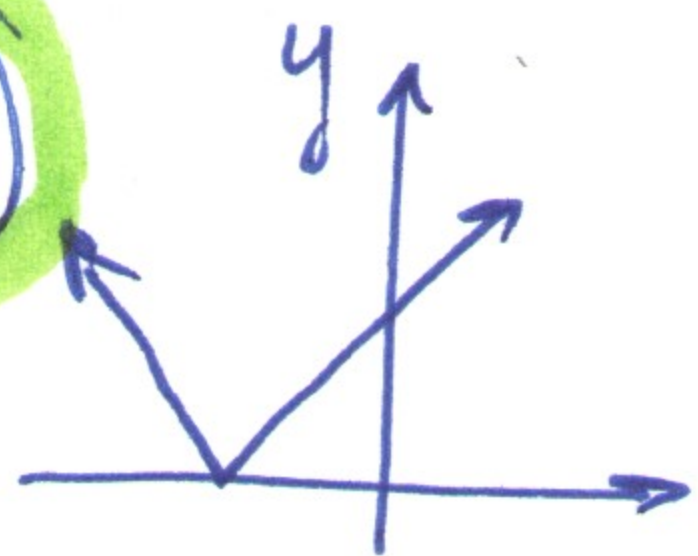
#4 $g(x) = 6x^7 + \pi x^5 + \frac{2}{3}x$

is a polynomial function of degree 7.

leading term: $6x^7$

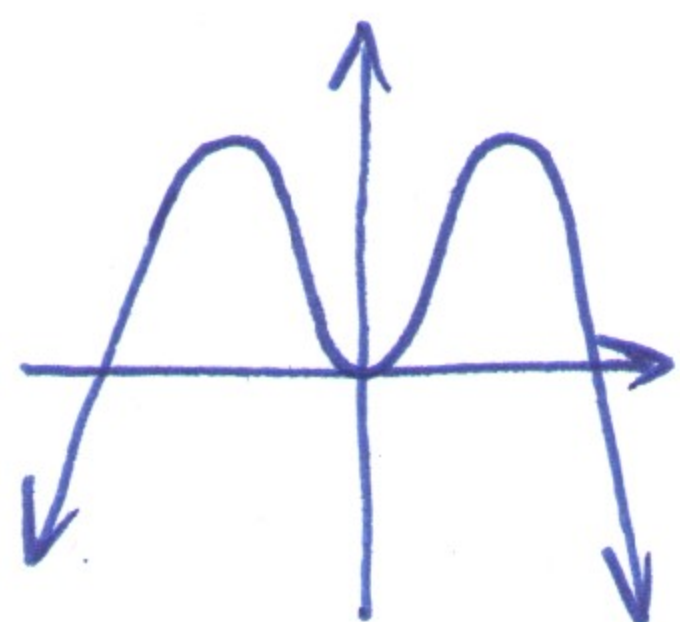
leading coefficient: 6

#12



it is not a graph of a polynomial function (not smooth)

#14



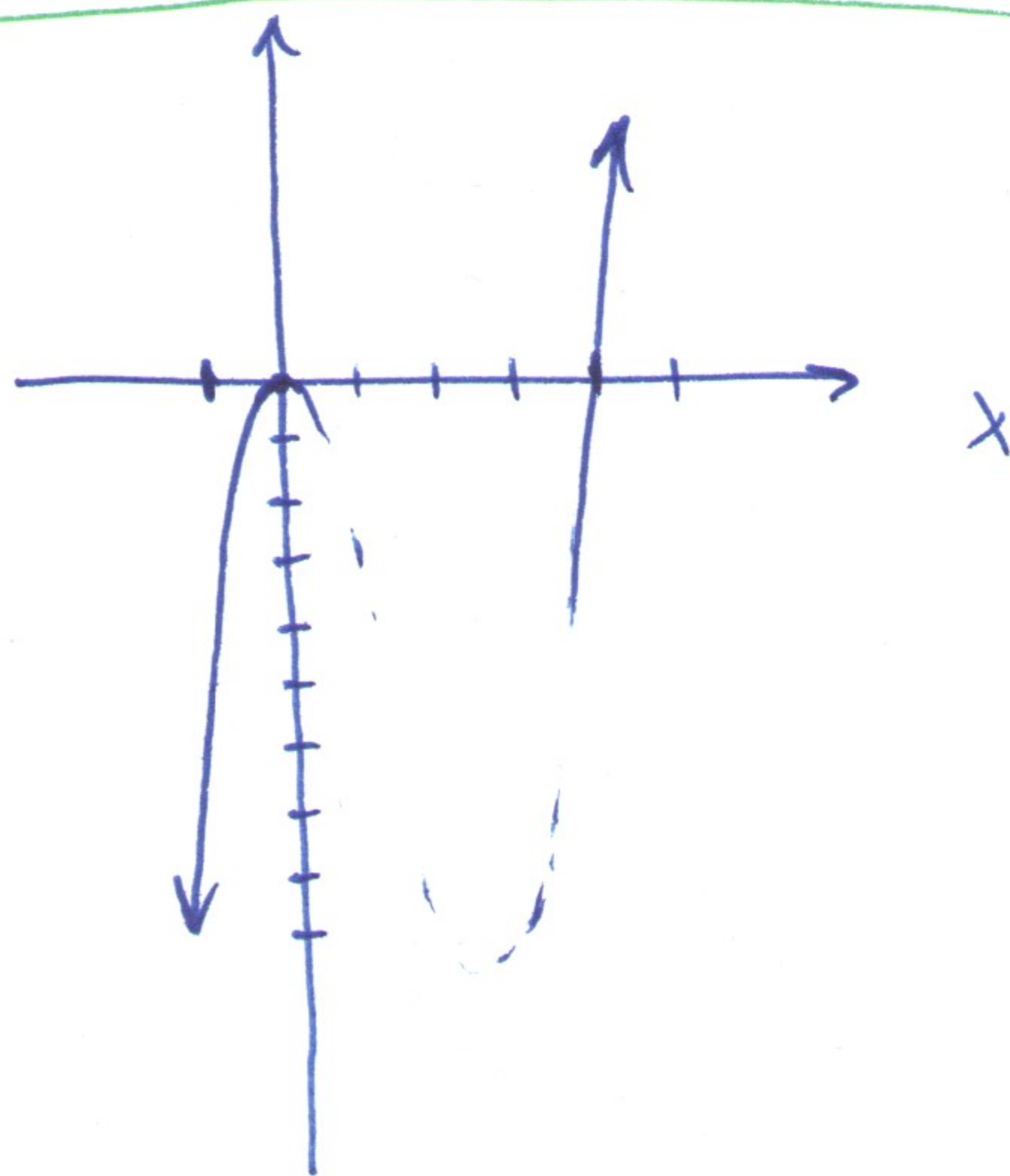
it is a graph of a polynomial function (smooth and continuous)

#16

$$f(x) = x^3 - 4x^2$$

leading coefficient = 1 > 0 } hence ↙ ↗
degree = 3

graph with label c:

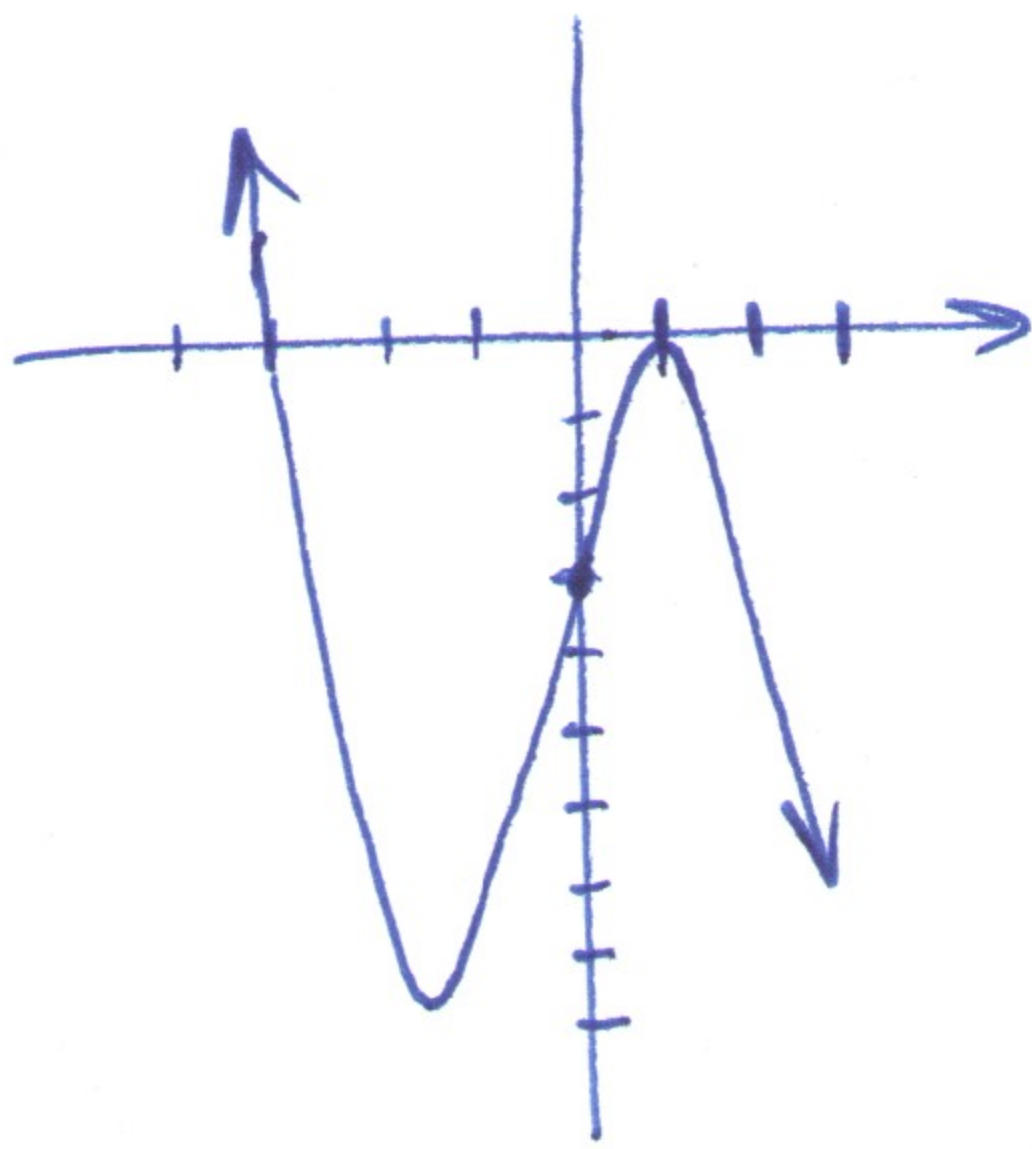


18

$$f(x) = -x^3 - x^2 + 5x - 3$$

leading coefficient $= -1 < 0$ $\uparrow \downarrow$
 degree $= 3$

graph with label d:



26

$$f(x) = 3(x+5)(x+2)^2$$

zeros:

$x = -5$ multiplicity of 1; crosses the x-axis

$x = -2$ multiplicity of 2; touches x-axis and turns around.

30

$$f(x) = x^3 + 4x^2 + 4x$$

let's factor!

$$f(x) = x(x^2 + 4x + 4) = x(x+2)(x+2) = x(x+2)^2$$

zeros:

$x = 0$ multiplicity of 1; crosses the x-axis

$x = -2$ multiplicity of 2; touches x-axis and turns around.

#42

$$f(x) = x^3 + x^2 - 4x - 4$$

a. leading coefficient: $1 > 0$, degree = 3, hence $\downarrow \uparrow$

b. y-intercept: $f(0) = -4$

$(0, -4)$

c.

x-intercepts: $f(x) = 0$ find x.

$$x^3 + x^2 - 4x - 4 = 0$$

$$x^2(x+1) - 4(x+1) = 0$$

$$(x^2 - 4)(x+1) = 0$$

$$x = \pm 2, x = -1$$

$(2, 0), (-2, 0), (-1, 0)$

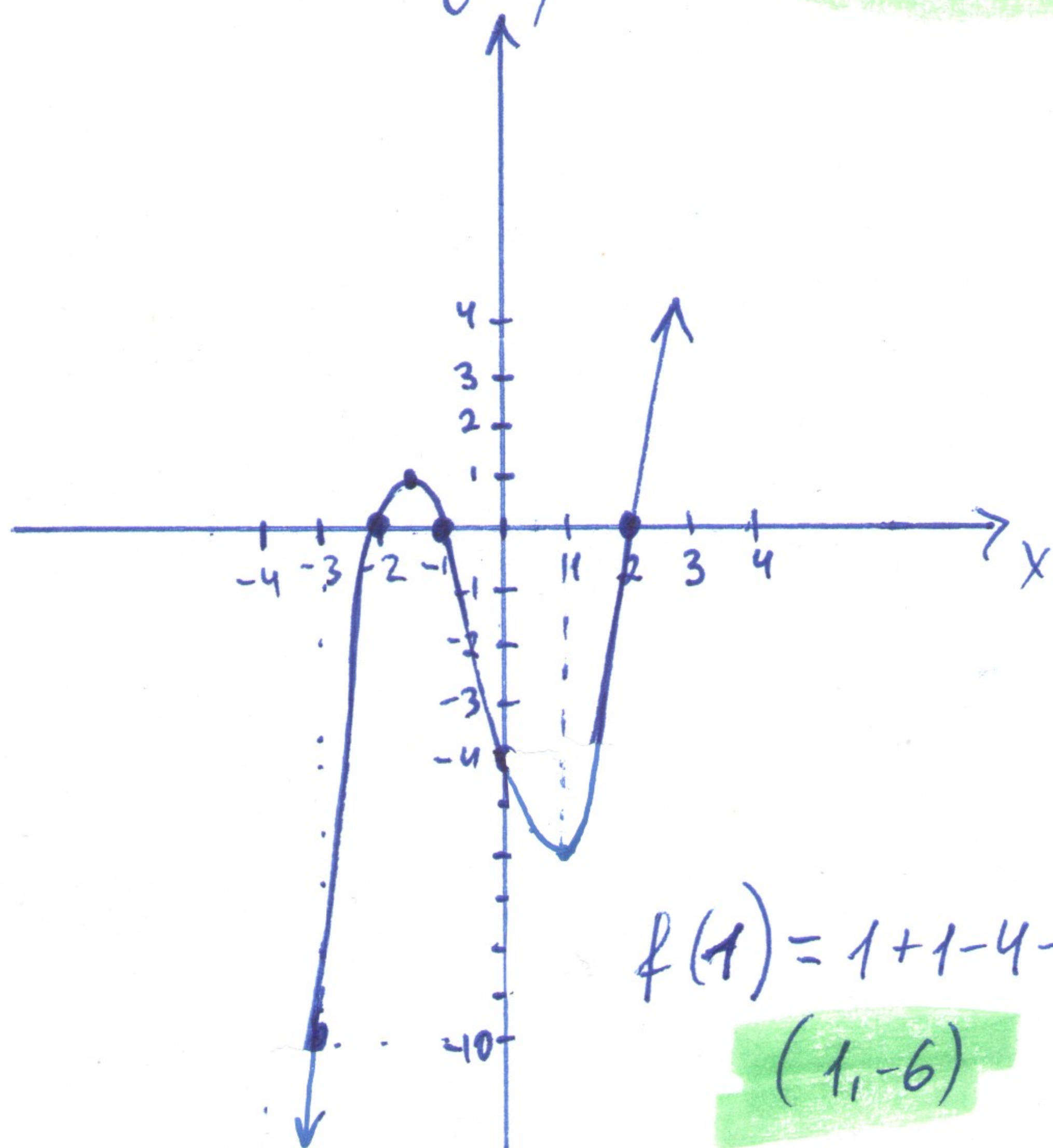
multiplicity of 1
crosses

d. even? odd?

$$f(-x) = (-x)^3 + (-x)^2 - 4(-x) - 4 = -x^3 + x^2 + 4x - 4 \neq -f(x) \neq f(x)$$

neither

e. turning points: at most 2 $(3-1)$



$$f(1) = 1 + 1 - 4 - 4 = -6$$

$(1, -6)$

Getting additional information:

$$f(-3) = -27 + 9 + 12 - 4 = -10 \quad (-3, -10)$$

$$f\left(-\frac{3}{2}\right) = -\frac{27}{8} + \frac{9}{4} + \frac{12}{2} - 4 = \frac{-27 + 18 + 48 - 32}{8}$$

$$= \frac{7}{8}$$

$\left(-\frac{3}{2}, \frac{7}{8}\right)$

$$f\left(\frac{1}{2}\right) = \frac{1}{8} + \frac{1}{4} - \frac{4}{2} - 4 = -\frac{13}{8} - 4 = -5\frac{5}{8}$$

$\uparrow \left(\frac{1}{2}, -5\frac{5}{8}\right)$

I chose $\frac{1}{2}$ because it is the midpoint of $[-1, 2]$.

#44

$$f(x) = x^4 - x^2$$

a) leading coefficient: $1 > 0$, degree = 4; hence $\uparrow \uparrow$

b) y-intercept: $f(0) = 0$ $(0,0)$

c) x-intercepts: $f(x) = 0$ find x

$$x^4 - x^2 = 0$$

zeros: $x = 0$, multiplicity of 2.

$$x^2(x^2 - 1) = 0$$

touches and turns around

$$x = 0, x = \pm 1$$

$x = 1$, multiplicity of 1, crosses

$$(0,0), (1,0), (-1,0)$$

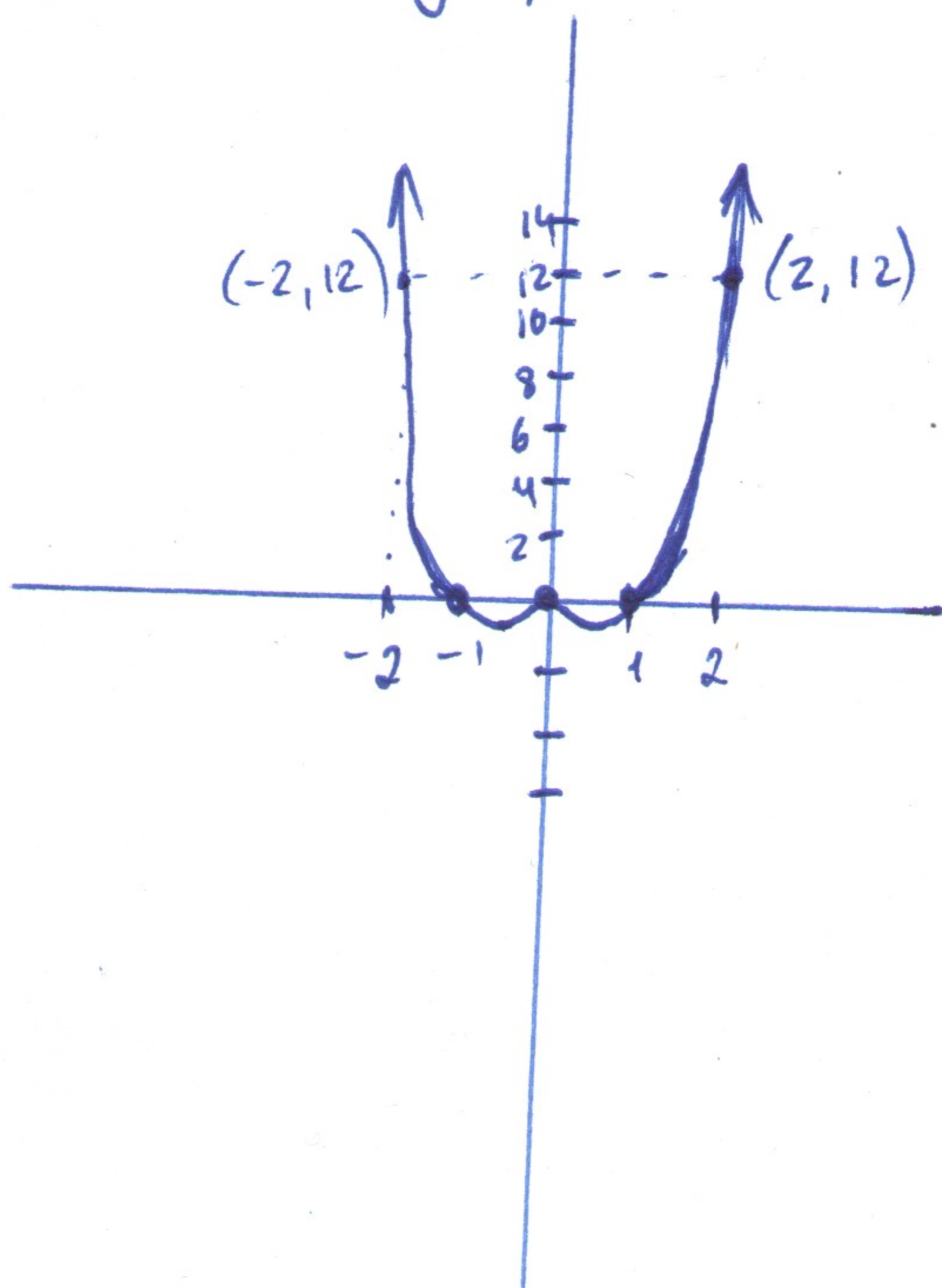
$x = -1$, multiplicity of 1, crosses.

d) even? odd?

$$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x) \text{ even!}$$

- symmetry around the y-axis

e) turning points: at least 3 $(4-1=3)$



more information:

$$f(-2) = 16 - 4 = 12 = f(2)$$

$$f\left(\frac{1}{2}\right) = \frac{1}{16} - \frac{1}{4} = \frac{1-4}{16} = -\frac{3}{16} = f\left(-\frac{1}{2}\right)$$

$$(-2, 12), (2, 12)$$

$$\left(\frac{1}{2}, -\frac{3}{16}\right), \left(-\frac{1}{2}, -\frac{3}{16}\right)$$