MTH30 Homework Section 1.8

#8 $f(x) = \frac{2}{x-5}$ and $g(x) = \frac{2}{x} + 5$

$$f(g(x)) = f\left(\frac{2}{x}+5\right) = \frac{2}{\left(\frac{2}{x}+5\right)-5} = \frac{2}{\frac{2}{x}} = \frac{2}{1}\cdot\frac{x}{2} = x$$

$$g(f(x)) = g\left(\frac{2}{x-5}\right) = \frac{2}{\left(\frac{2}{x-5}\right)} + 5 = \frac{2}{1}\cdot\frac{x-5}{2} + 5 =$$

$$= (x-5) + 5 = x$$

Therefore, $f(g(x)) = x$ and $g(f(x)) = x$.

f and g are inverses of each other.

#26 $f(x) = \frac{4}{x} + 9$

$y = \frac{4}{x} + 9$

$x = \frac{4}{y} + 9$ - solve for y:

$x = \frac{4}{y} + 9$ ← -9

$y\cdot(x-9) = \frac{4}{y}\cdot y$ ($y \neq 0$)

$\frac{y(x-9)}{x-9} = \frac{4}{x-9}$

$y = \frac{4}{x-9}$ ← defines a function of x

a) $f^{-1}(x) = \frac{4}{x-9}$

note that $x \neq 9$ i.e.

b) verify: $f(f^{-1}(x)) = f\left(\frac{4}{x-9}\right) = \frac{4}{\left(\frac{4}{x-9}\right)} + 9 = \frac{4}{1}\cdot\frac{(x-9)}{4} + 9 = x$ ✓

$f^{-1}(f(x)) = f^{-1}\left(\frac{4}{x}+9\right) = \frac{4}{\left(\frac{4}{x}+9\right)-9} = \frac{4}{\frac{4}{x}} = \frac{4}{1}\cdot\frac{x}{4} = x$ ✓