

#28

$$f(x) = \frac{2x-3}{x+1}$$

$$y = \frac{2x-3}{x+1}$$

$$x = \frac{2y-3}{y+1}$$

- solve for y:

$$(y+1) \cdot x = \frac{2y-3}{y+1} \cdot (y+1)^1$$

$$x(y+1) = 2y-3$$

$$xy + x = 2y - 3$$

$$xy - 2y = -x - 3$$

$$y(x-2) = \frac{-x-3}{x-2}$$

$$y = \frac{-x-3}{x-2} = \frac{x+3}{2-x}$$

- defines a function of x

$$a) f^{-1}(x) = \frac{x+3}{2-x}$$

note that
 $x \neq 2$

b) verify:

$$f(f^{-1}(x)) = f\left(\frac{x+3}{2-x}\right) =$$

$$= \frac{2 \cdot \left(\frac{x+3}{2-x}\right) - 3}{\left(\frac{x+3}{2-x}\right) + 1} = \frac{\frac{2(x+3)}{2-x} - 3 \cdot \frac{2-x}{2-x}}{\frac{x+3}{2-x} + 1 \cdot \frac{2-x}{2-x}} =$$

$$= \frac{2(x+3) - 3(2-x)}{2-x} = \frac{2x+6-6+3x}{2-x} = \frac{5x}{2-x}$$

$$= \frac{(x+3) + (2-x)}{2-x} = \frac{x+3+2-x}{2-x} = \frac{5}{2-x}$$

$$= \frac{5x}{2-x} \cdot \frac{2-x}{5} = \frac{5x}{5} = x$$

we can do this cancellation
because $2-x \neq 0$

$f^{-1}(f(x))$ check is similar