Definitions, Rules, and Formulas

THE REAL NUMBERS

Natural Numbers: $\{1, 2, 3, \ldots\}$
Whole Numbers: $\{0, 1, 2, 3, \ldots\}$
Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers: $\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers}, b \neq 0\right\}$
Irrational Numbers: $\{x \mid x \text{ is real and not rational}\}$

PROPERTIES OF ADDITION AND MULTIPLICATION

Commutative: $a + b = b + a; ab = ba$
Associative: $(a + b) + c = a + (b + c);$
$(ab)c = a(bc)$
Distributive: $a(b + c) = ab + ac; a(b - c) = ab - ac$
Identity: $a + 0 = a; a \cdot 1 = a$
Inverse: $a + (-a) = 0; a \cdot \frac{1}{a} = 1 \ (a \neq 0)$
Multiplication Properties: $(-1)a = -a;$
$(-1)(-a) = a; a \cdot 0 = 0; (-a)(b) = (a)(-b) = -ab;$
$(-a)(-b) = ab$

EXPONENTS

Definitions of Rational Exponents

1. $a^{\frac{1}{n}} = \sqrt[n]{a}$

2. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

3. $a^{-\frac{m}{n}} = \dfrac{1}{a^{\frac{m}{n}}}$

Properties of Rational Exponents

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m \cdot b^n = b^{m+n}$

2. $\dfrac{b^m}{b^n} = b^{m-n}$

3. $\left(b^m\right)^n = b^{mn}$

4. $(ab)^n = a^n b^n$

5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

RADICALS

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

1. If n is even, then $\sqrt[n]{a^n} = |a|$.

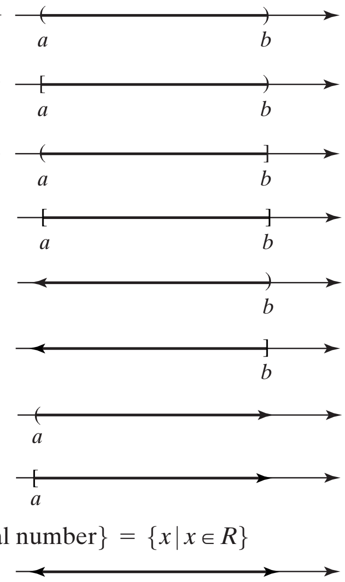
2. If n is odd, then $\sqrt[n]{a^n} = a$.

3. The product rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

4. The quotient rule: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

INTERVAL NOTATION, SET-BUILDER NOTATION, AND GRAPHS

$(a, b) = \{x \mid a < x < b\}$
$[a, b) = \{x \mid a \le x < b\}$
$(a, b] = \{x \mid a < x \le b\}$
$[a, b] = \{x \mid a \le x \le b\}$
$(-\infty, b) = \{x \mid x < b\}$
$(-\infty, b] = \{x \mid x \le b\}$
$(a, \infty) = \{x \mid x > a\}$
$[a, \infty) = \{x \mid x \ge a\}$
$(-\infty, \infty) = \{x \mid x \text{ is a real number}\} = \{x \mid x \in R\}$



SLOPE FORMULA

$$\text{slope } (m) = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2)$$

EQUATIONS OF LINES

1. *Slope-intercept form:* $y = mx + b$
m is the line's slope and b is its y-intercept.

2. *General form:* $Ax + By + C = 0$

3. *Point-slope form:* $y - y_1 = m(x - x_1)$
m is the line's slope and (x_1, y_1) is a fixed point on the line.

4. *Horizontal line parallel to the x-axis:* $y = b$

5. *Vertical line parallel to the y-axis:* $x = a$

ABSOLUTE VALUE

1. $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

2. If $|x| = c$, then $x = c$ or $x = -c$. $(c > 0)$

3. If $|x| < c$, then $-c < x < c$. $(c > 0)$

4. If $|x| > c$, then $x < -c$ or $x > c$. $(c > 0)$

SPECIAL FACTORIZATIONS

1. *Difference of two squares:*

$$A^2 - B^2 = (A + B)(A - B)$$

2. *Perfect square trinomials:*

$$A^2 + 2AB + B^2 = (A + B)^2$$
$$A^2 - 2AB + B^2 = (A - B)^2$$

3. *Sum of two cubes:*

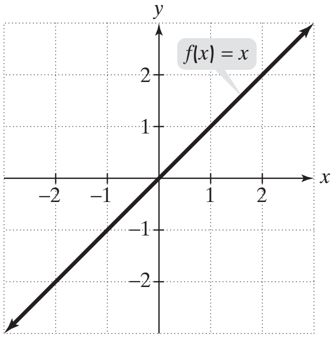
$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

4. *Difference of two cubes:*

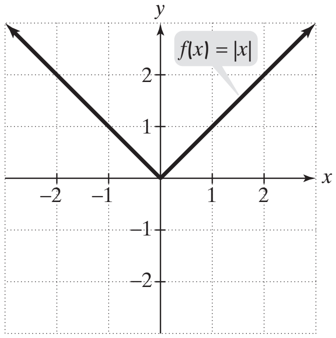
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

ALGEBRA'S COMMON GRAPHS

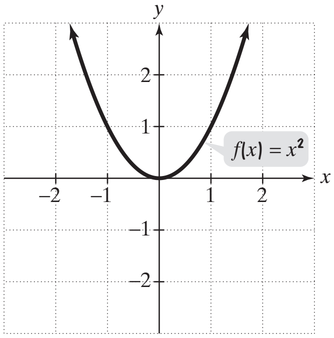
Identity Function



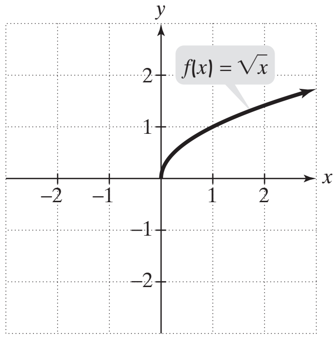
Absolute Value Function



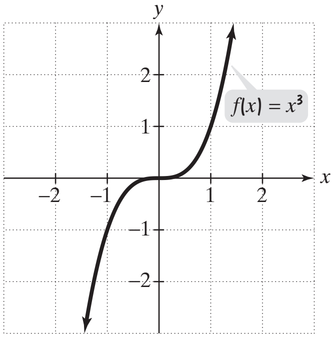
Standard Quadratic Function



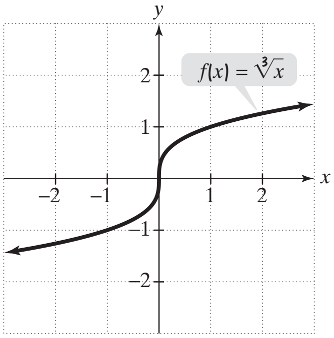
Square Root Function



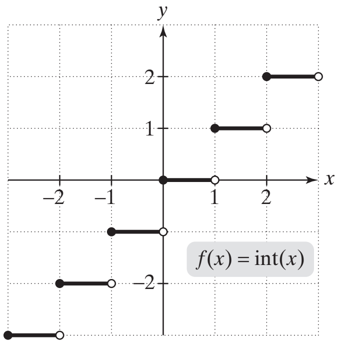
Standard Cubic Function



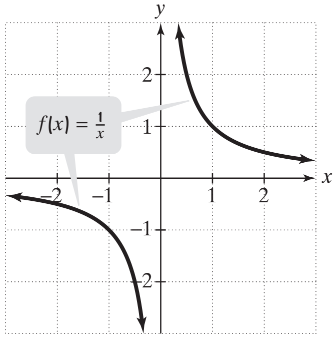
Cube Root Function



Greatest Integer Function



Reciprocal Function



TRANSFORMATIONS

In each case, c represents a positive real number.

	Function	Draw the graph of f and:
Vertical translations	$y = f(x) + c$	Shift f upward c units.
	$y = f(x) - c$	Shift f downward c units.
Horizontal translations	$y = f(x - c)$	Shift f to the right c units.
	$y = f(x + c)$	Shift f to the left c units.
Reflections	$y = -f(x)$	Reflect f about the x-axis.
	$y = f(-x)$	Reflect f about the y-axis.
Vertical Stretching or Shrinking	$y = cf(x); c > 1$	Vertically stretch f, multiplying each of its y-coordinates by c.
	$y = cf(x); 0 < c < 1$	Vertically shrink f, multiplying each of its y-coordinates by c.
Horizontal Stretching or Shrinking	$y = f(cx); c > 1$	Horizontally shrink f, dividing each of its x-coordinates by c.
	$y = f(cx); 0 < c < 1$	Horizontally stretch f, dividing each of its x-coordinates by c.

DISTANCE AND MIDPOINT FORMULAS

1. The distance from (x_1, y_1) to (x_2, y_2) is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

2. The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

QUADRATIC FORMULA

The solutions of $ax^2 + bx + c = 0$ with $a \neq 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

FUNCTIONS

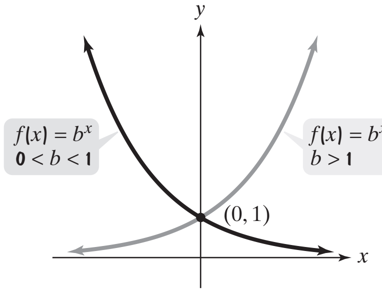
1. Linear Function: $f(x) = mx + b$
Graph is a line with slope m and y-intercept b.

2. Quadratic Function: $f(x) = ax^2 + bx + c, a \neq 0$
Graph is a parabola with vertex at $x = -\dfrac{b}{2a}$.
Quadratic Function: $f(x) = a(x - h)^2 + k$
In this form, the parabola's vertex is (h, k).

3. nth-Degree Polynomial Function:
$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0, a_n \neq 0$
For n odd and $a_n > 0$, graph falls to the left and rises to the right.
For n odd and $a_n < 0$, graph rises to the left and falls to the right.
For n even and $a_n > 0$, graph rises to the left and rises to the right.
For n even and $a_n < 0$, graph falls to the left and falls to the right.

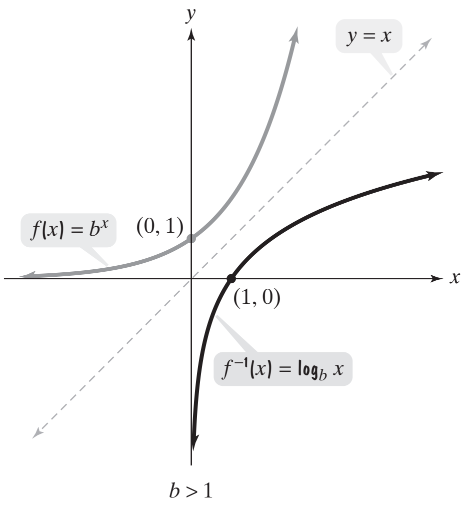
4. Rational Function: $f(x) = \dfrac{p(x)}{q(x)}$, $p(x)$ and $q(x)$ are polynomials, $q(x) \neq 0$

5. Exponential Function: $f(x) = b^x, b > 0, b \neq 1$
Graphs:



6. Logarithmic Function: $f(x) = \log_b x, b > 0, b \neq 1$
$y = \log_b x$ is equivalent to $x = b^y$.

Graph:



PROPERTIES OF LOGARITHMS

1. $\log_b(MN) = \log_b M + \log_b N$

2. $\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$

3. $\log_b M^p = p \log_b M$

4. $\log_b M = \dfrac{\log_a M}{\log_a b} = \dfrac{\ln M}{\ln b} = \dfrac{\log M}{\log b}$

5. $\log_b b^x = x; \quad \ln e^x = x$

6. $b^{\log_b x} = x; \quad e^{\ln x} = x$

INVERSE OF A 2 × 2 MATRIX

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

CRAMER'S RULE

If

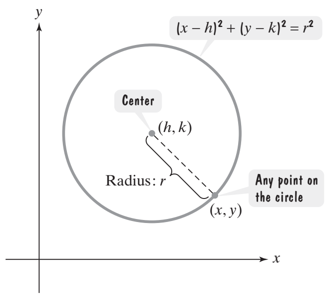
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

then $x_i = \dfrac{D_i}{D}$, $D \neq 0$.

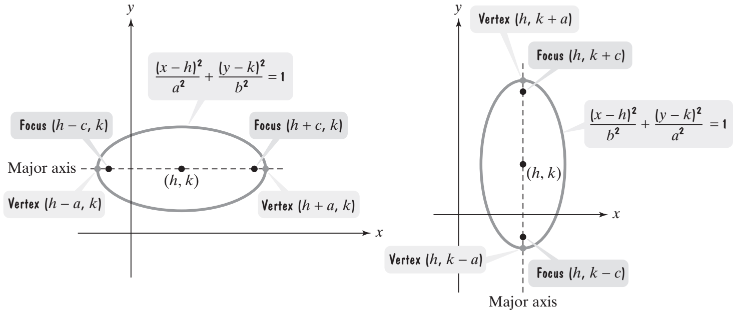
D: determinant of the system's coefficients
D_i: determinant in which coefficients of x_i are replaced by $b_1, b_2, b_3, \ldots, b_n$.

CONIC SECTIONS

Circle

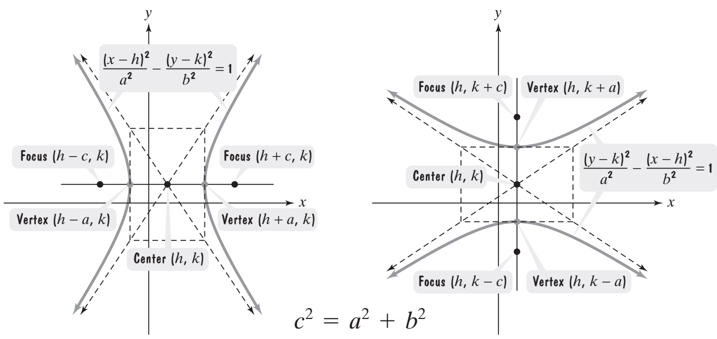


Ellipse



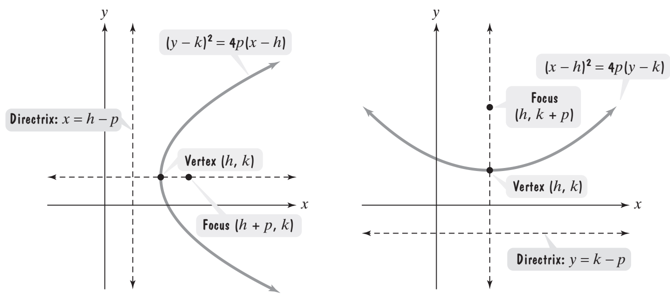
$a^2 > b^2$ and $c^2 = a^2 - b^2$

Hyperbola



$c^2 = a^2 + b^2$

Parabola



SEQUENCES

1. Infinite Sequence:

$$\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$$

2. Summation Notation:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

3. nth Term of an Arithmetic Sequence:

$$a_n = a_1 + (n-1)d$$

4. Sum of First n Terms of an Arithmetic Sequence:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

5. nth Term of a Geometric Sequence: $a_n = a_1 r^{n-1}$

6. Sum of First n Terms of a Geometric Sequence:

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad (r \neq 1)$$

7. Sum of an Infinite Geometric Series with $|r| < 1$:

$$S = \frac{a_1}{1-r}$$

THE BINOMIAL THEOREM

1. $n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1; 0! = 1$

2. $\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$

3. Binomial Theorem:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$$

PERMUTATIONS, COMBINATIONS, AND PROBABILITY

1. ${}_nP_r$, the number of permutations of n elements taken r at a time, is given by

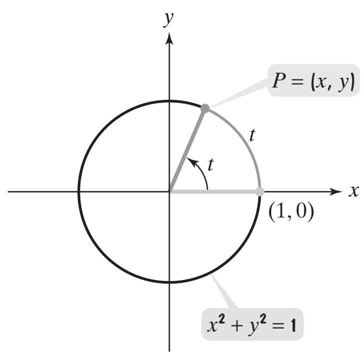
$$ {}_nP_r = \frac{n!}{(n-r)!}.$$

2. ${}_nC_r$, the number of combinations of n elements taken r at a time, is given by

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!}.$$

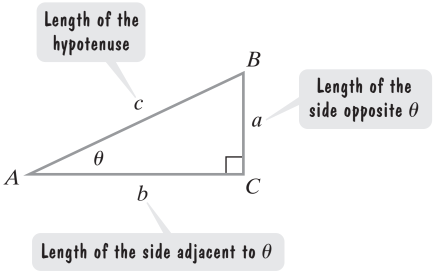
3. *Probability of an Event:* $P(E) = \frac{n(E)}{n(S)}$, where $n(E)$ = the number of outcomes in event E and $n(S)$ = the number of outcomes in the sample space.

UNIT CIRCLE DEFINITIONS OF TRIGONOMETRIC FUNCTIONS



$\sin t = y$	$\csc t = \frac{1}{y}, \quad y \neq 0$
$\cos t = x$	$\sec t = \frac{1}{x}, \quad x \neq 0$
$\tan t = \frac{y}{x}, \quad x \neq 0$	$\cot t = \frac{x}{y}, \quad y \neq 0$

RIGHT TRIANGLE DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

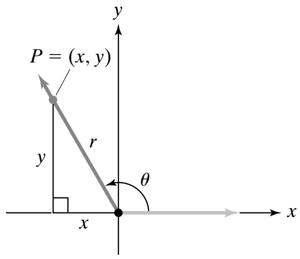


$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{hyp.}}{\text{opp.}} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{hyp.}}{\text{adj.}} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{a}{b} \qquad \cot\theta = \frac{\text{adj.}}{\text{opp.}} = \frac{b}{a}$$

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

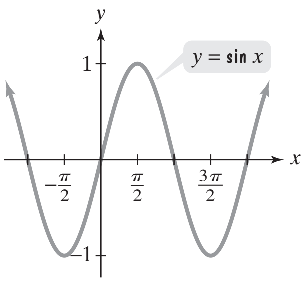


$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$$

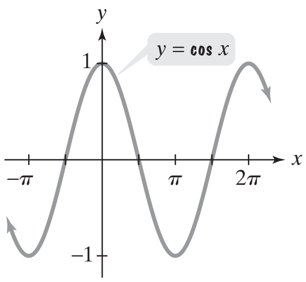
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}, \quad x \neq 0$$

$$\tan\theta = \frac{y}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

GRAPHS OF TRIGONOMETRIC FUNCTIONS



Domain: all real numbers: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π



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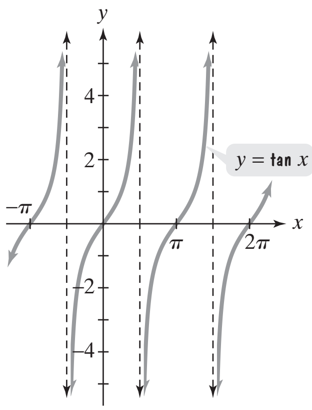
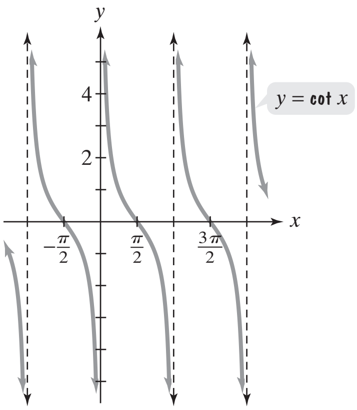
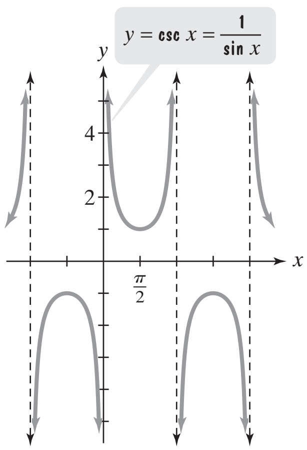


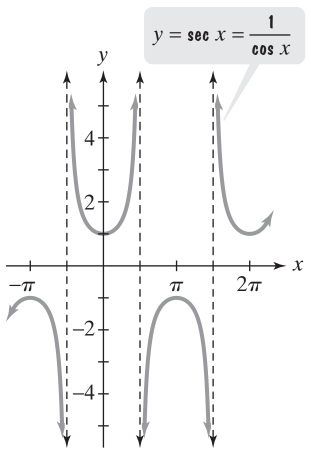
Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: all real numbers
Period: π



Domain: all real numbers except integral multiples of π
Range: all real numbers
Period: π



Domain: all real numbers except integral multiples of π
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π



Domain: all real numbers except odd multiples of $\frac{\pi}{2}$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \csc x = \frac{1}{\sin x}$$
$$\cos x = \frac{1}{\sec x} \quad \sec x = \frac{1}{\cos x}$$
$$\tan x = \frac{1}{\cot x} \quad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$
$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$

OTHER TRIGONOMETRIC IDENTITIES

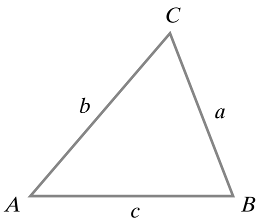
Sum and Difference Formulas

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

OBLIQUE TRIANGLES



Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$