MTH30 Chapter 1 Review

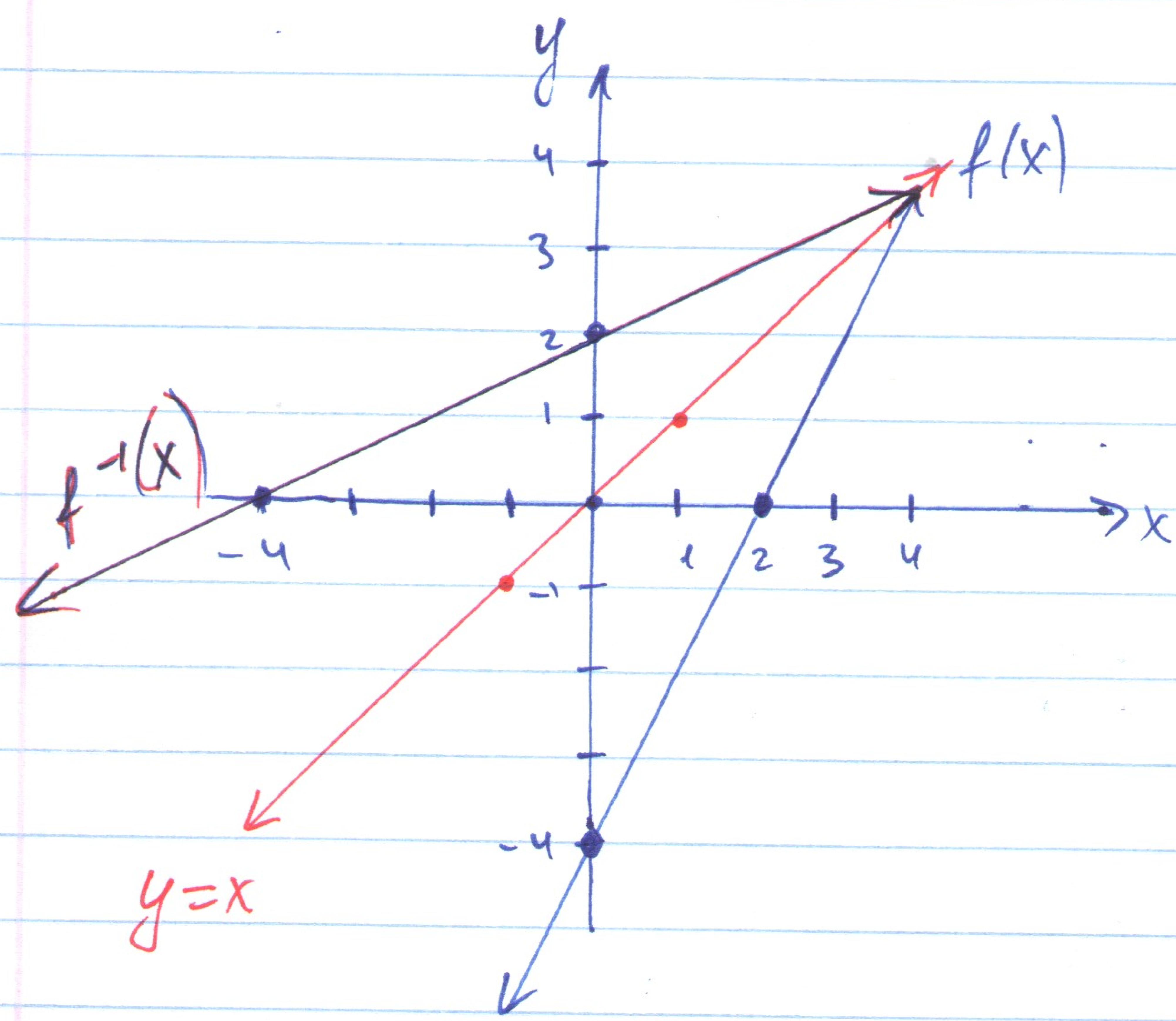
#13 $f(x) = 2x - 4$, graph $f(x)$ and $f^{-1}(x)$

x	$2x-4$
0	-4
2	0

x	f^{-1}
-4	0
0	2

graph this first

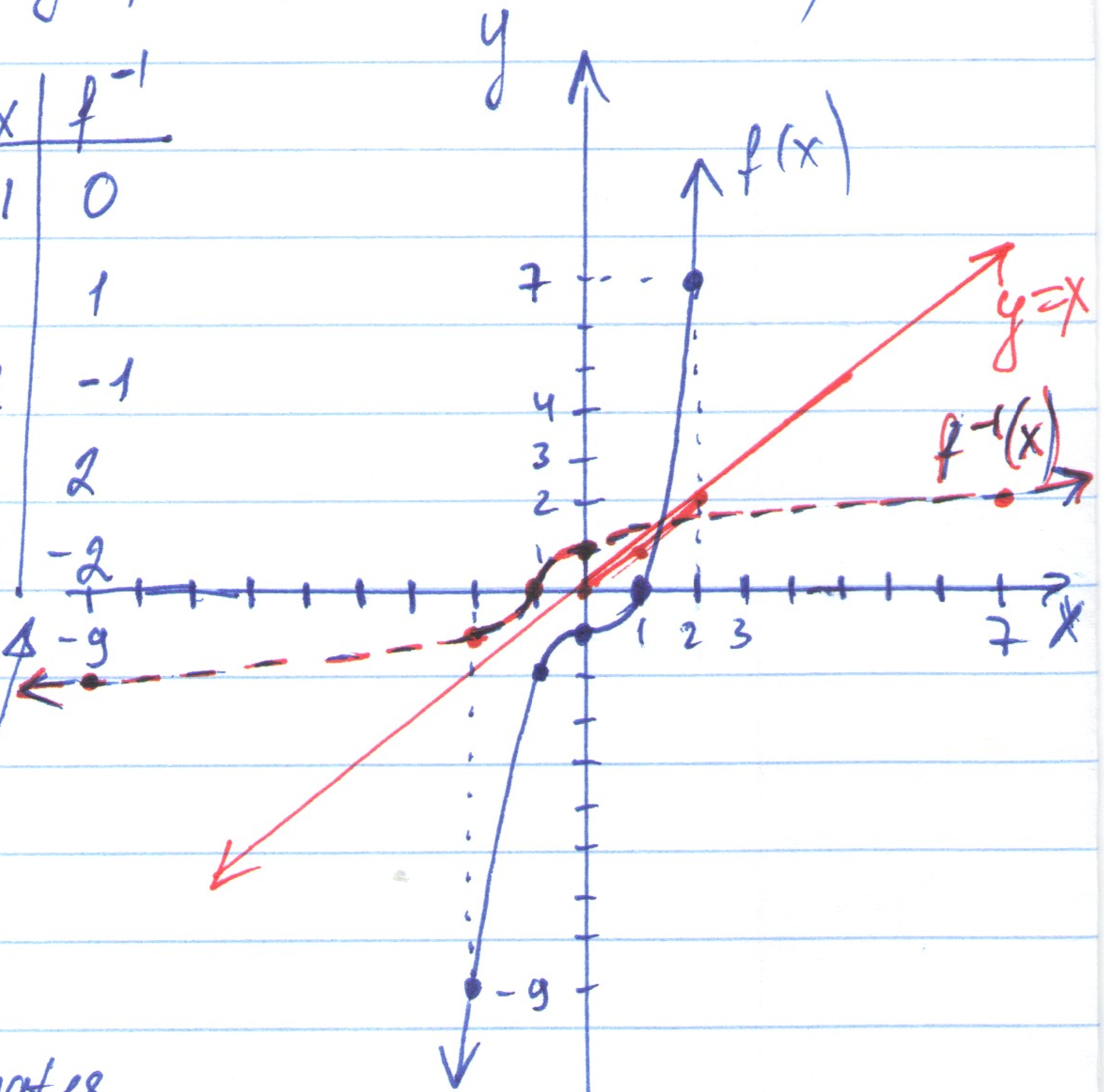
switch coordinates



#14 $f(x) = x^3 - 1$, graph $f(x)$ and $f^{-1}(x)$

x	$x^3 - 1$
0	-1
1	0
-1	-2
2	7
-2	-9

x	f^{-1}
-1	0
0	1
-2	-1
7	2
-9	-2



graph it first

then draw $y=x$

then switch the coordinates of the graph of $f(x)$

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#15 $f(x) = x^2 - 1, \ x \geq 0$ graph $f(x)$ and f^{-1}

x	x^2-1
0	-1
1	0
2	3
3	8

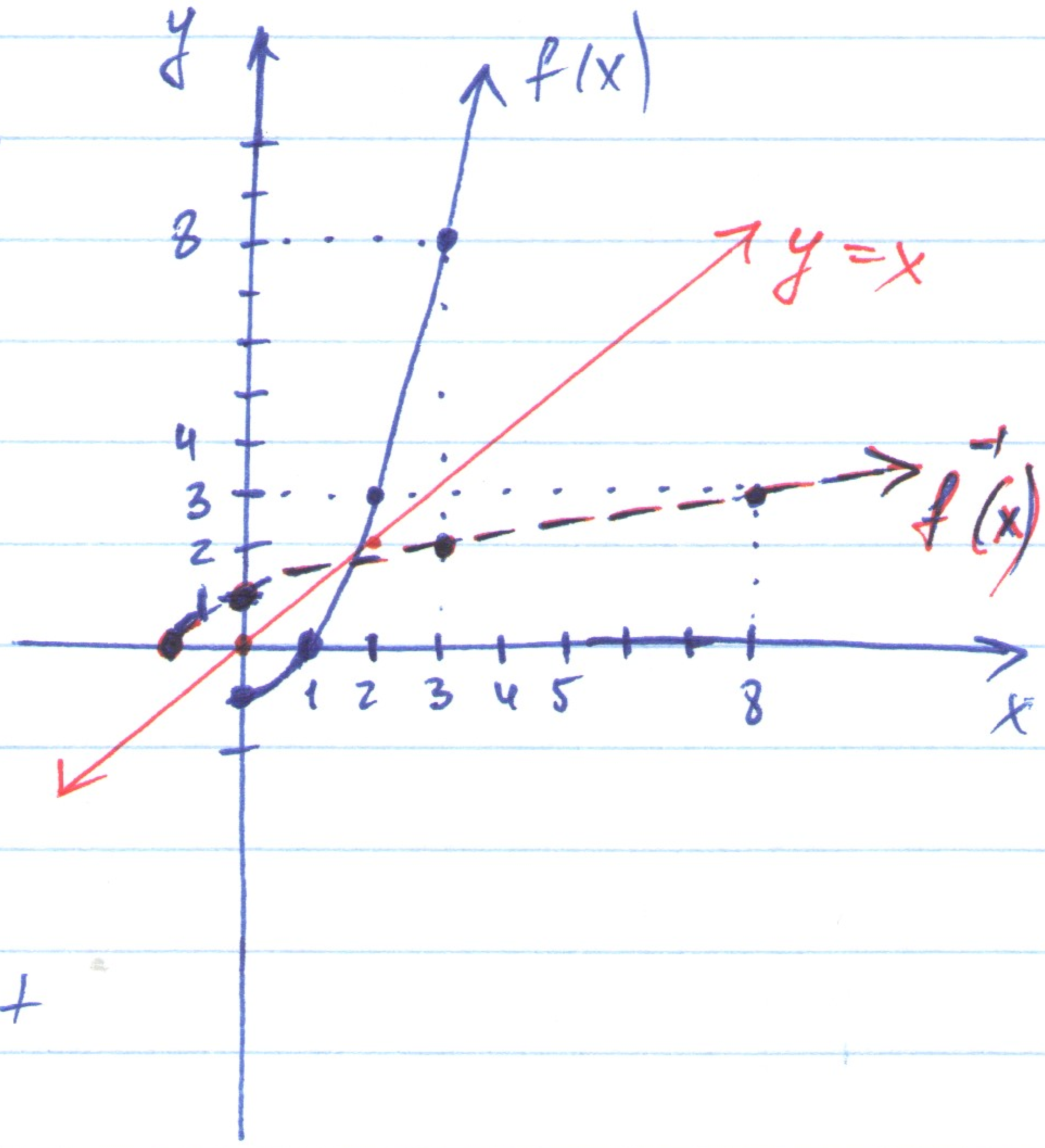
parabola

graph it first

switch the coordinates

x	f^{-1}
-1	0
0	1
3	2
8	3

graph it next



$\boxed{f(x) = x^2 - x - 4}$ $\boxed{g(x) = 2x - 6}$

#16 $f(x-1) = \underbrace{(x-1)^2}_{} - \underbrace{(x-1)}_{} - 4 =$

$= \boxed{x^2 - 2x + 1} \; \boxed{-x + 1} - 4 = x^2 - 3x - 2$

Answer: $\boxed{f(x-1) = x^2 - 3x - 2}$

#17 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{(x^2 + 2hx + h^2 - x - h - 4) - (x^2 - x - 4)}{h} =$

$h \neq 0$

$f(x+h) = (x+h)^2 - (x+h) - 4 = x^2 + 2xh + h^2 - x - h - 4$

$= \dfrac{\cancel{x^2} + 2xh + h^2 \cancel{-x} - h \cancel{-4} \cancel{-x^2} \cancel{+x} \cancel{+4}}{h} = \dfrac{2xh + h^2 - h}{h} = 2x + h - 1$

Answer: $\boxed{\dfrac{f(x+h) - f(x)}{h} = 2x + h - 1, \quad h \neq 0}$

#18 $(g - f)(x) = g(x) - f(x) = (2x - 6) - (x^2 - x - 4) = -x^2 + 3x - 2$

Answer: $\boxed{(g - f)(x) = -x^2 + 3x - 2}$

#19 $\boxed{\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 - x - 4}{2x - 6}}$ & nothing can be cancelled

$2x - 6 \neq 0$

$x \neq 3$

Answer: $\boxed{\text{domain: } (-\infty, 3) \cup (3, \infty) \text{ or } \{x \mid x \neq 3\}}$

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$\boxed{f(x) = x^2 - x - 4}$ $\boxed{g(x) = 2x - 6}$

#20 $(f \circ g)(x) = f(g(x)) = f(2x-6) = (2x-6)^2 - (2x-6) - 4 =$

$= 4x^2 - 24x + 36 - 2x + 6 - 4 = 4x^2 - 26x + 38$

Answer: $\boxed{(f \circ g)(x) = 4x^2 - 26x + 38}$

#21 $(g \circ f)(x) = g(f(x)) = g(x^2 - x - 4) = 2(x^2 - x - 4) - 6 =$

$= 2x^2 - 2x - 8 - 6 = 2x^2 - 2x - 14$

Answer: $\boxed{(g \circ f)(x) = 2x^2 - 2x - 14}$

#22 $g(f(-1)) = 2\cdot(-1)^2 - 2(-1) - 14 = 2 + 2 - 14 = \underline{-10}$

$2x^2 - 2x - 14$

or

$f(-1) = (-1)^2 - (-1) - 4 = 1 + 1 - 4 = -2$, then $g(-2) = 2\cdot(-2) - 6 = \underline{-10}$

Answer: $\boxed{g(f(-1)) = -10}$

#23 $f(-x) = (-x)^2 - (-x) - 4 = x^2 + x - 4$

$f(-x) \neq f(x)$ hence not even

$f(-x) \neq -f(x)$ hence not odd.

Answer: $f(x)$ is neither odd nor even

$f(-x) = x^2 + x - 4$

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#30 $\boxed{f(x) = \frac{3}{x+5} + \frac{7}{x-1}}$ find domain.

the problems we can encounter: division by zero.
- exclude them from $\mathbb{R}$.

$x+5 \neq 0$ $x-1 \neq 0$

$x \neq -5$ $x \neq 1$

domain: $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$ interval notation

or

$\{x \mid x \neq -5 \text{ and } x \neq 1\}$ set-builder notation

#31 $\boxed{f(x) = 3\sqrt{x+5} + 7\sqrt{x-1}}$ find domain

the problems we can encounter: negative number under the radical, i.e. $\sqrt{-2}$ is undefined in $\mathbb{R}$.
- exclude them from $\mathbb{R}$

$x+5 \geq 0$ and $x-1 \geq 0$

$x \geq -5$ and $x \geq 1$

- their intersection is $x \geq 1$

domain: $[1, \infty)$ or $\{x \mid x \geq 1\}$

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#32 $f(x) = \frac{7}{x-4}$ $x \neq 4$, $g(x) = \frac{2}{x}$ $x \neq 0$

a) $(f\circ g)(x) = f(g(x)) = f\left(\frac{2}{x}\right) = \frac{7}{\frac{2}{x} - 4} = \frac{7}{\frac{2}{x} - \frac{4x}{x}} =$

$= \frac{7}{\frac{2-4x}{x}} = \frac{7}{1} \cdot \frac{x}{2-4x} = \frac{7x}{2-4x}$

$2-4x \neq 0$ $x \neq \frac{1}{2}$

$$\boxed{(f\circ g)(x) = \frac{7x}{2-4x}}$$

b) domain of $(f\circ g)(x)$ is

- all values from the domain of $g(x)$ such that

$x \neq 0$

$g(x)$ is from the domain of f.

$x \neq \frac{1}{2}$

$$\boxed{\text{domain: } (-\infty, 0) \cup \left(0, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, \infty\right) \text{ or } \{x \mid x \neq 0 \text{ and } x \neq \tfrac{1}{2}\}}$$

P12 $h(x) = (2x+3)^7$ express as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$

$$h(x) = (2x+3)^{7} \quad x^7 \quad f(x)$$

$$g(x)$$

$$\boxed{\underline{\text{Answer}} \quad \begin{aligned} g(x) &= 2x+3 \\ f(x) &= x^7 \end{aligned}}$$

<u>Check</u> $(f \circ g)(x) = f(g(x)) = f(2x+3) = (2x+3)^7$

P19 $g(x) = \begin{cases} \sqrt{x-3} & x \geq 3 \\ 3-x & x < 3 \end{cases}$

$g(-1) = 3 - (-1) = 4$

$g(7) = \sqrt{7-3} = \sqrt{4} = 2$

$$\boxed{\underline{\text{Answer}} \quad \begin{aligned} g(-1) &= 4 \\ g(7) &= 2 \end{aligned}}$$