5

Finance

5.2 Compound Interest

Objectives

- Understand the difference between simple interest and compound interest
- Use the Compound Interest Formula
- Understand and compute the annual yield

Compound Interest

Many forms of investment, including savings accounts, earn **compound interest**, in which interest is periodically paid on both the original principal and previous interest payments.

This results in earnings that are significantly higher over a longer period of time. It is important that you understand this difference in order to make wise financial decisions.

Compound Interest as Simple Interest, Repeated

Example 1 – *Understanding Compound Interest*

Tom and Betty deposit $1,000 into their new bank account. The account pays 8% interest compounded quarterly.

This means that interest is computed and deposited every quarter of a year.

Find the account balance after six months, using the *Simple Interest Future Value* formula to compute the balance at the end of each quarter.

Example 1 – *Solution*

At the end of the first quarter, P = \$1,000, r = 8% = 0.08, and t = one quarter or $\frac{1}{4}$ year.

$FV = P(1 + r\,t)$ the Simple Interest Future Value Formula

$= 1{,}000(1 + 0.08 \cdot \frac{1}{4})$ substituting

$= 1{,}000(1 + 0.02)$

$= \$1{,}020$

Example 1 – *Solution*

At the end of the first quarter, P = \$1,000, r = 8% = 0.08, and t = one quarter or $\frac{1}{4}$ year.

$$FV = P(1 + rt)$$ the Simple Interest Future Value Formula

$$= 1{,}000(1 + 0.08 \cdot \tfrac{1}{4})$$ substituting

$$= 1{,}000(1 + 0.02)$$

$$= \$1{,}020$$

This means that there is \$1,020 in Tom and Betty's account at the end of the first quarter. It also means that the second quarter's interest will be paid on this new principal.

Example 1 – *Solution*

cont'd

So at the end of the second quarter, P = \$1,020 and r = 0.08. Note that $t = \frac{1}{4}$, not $\frac{2}{4}$, because we are computing interest for one quarter.

$FV = P(1 + rt)$ the Simple Interest Future Value Formula

$= 1{,}020(1 + 0.08 \cdot \frac{1}{4})$ substituting

$= 1{,}020(1 + 0.02)$

$= \$1{,}040.40$

Example 1 – *Solution*

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So at the end of the second quarter, P = \$1,020 and $r = 0.08$. Note that $t = \frac{1}{4}$, not $\frac{2}{4}$, because we are computing interest for one quarter.

$$FV = P(1 + rt)$$ the Simple Interest Future Value Formula

$$= 1{,}020(1 + 0.08 \cdot \tfrac{1}{4})$$ substituting

$$= 1{,}020(1 + 0.02)$$

$$= \$1{,}040.40$$

At the end of six months, the account balance is \$1,040.40.

The Compound Interest Formula

The Compound Interest Formula

For each quarter's calculation in Example 1, we multiplied the annual rate of 8% by the time, one-quarter of a year, and got $\frac{1}{4} \cdot 8\% = 2\% = 0.02$.

This 2% is the *quarterly rate* (or more generally, the *periodic rate*). A *periodic rate* is any rate that is prorated in this manner from an annual rate.

The Compound Interest Formula

For each quarter's calculation in Example 1, we multiplied the annual rate of 8% by the time, one-quarter of a year, and got $\frac{1}{4} \cdot 8\% = 2\% = 0.02$.

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Compound Interest Terms

The **compounding period** is the time period over which any one interest payment is calculated. In Example 1, the compounding period was a quarter of a year.

The **periodic rate** i is the interest rate charged for each period. It is an appropriate fraction of the annual rate. In Example 1, the compound rate was $i = 2\%$.

The Compound Interest Formula

If i is the *periodic interest rate*, then the future value at the end of the first period is

$$FV = P(1 + i)$$

$FV = 1{,}000(1 + 0.02)$ in Example 1

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Because this is the account balance at the beginning of the second period, it becomes the new principal. The account balance at the end of the second period is

$$FV = P(1 + i) \cdot (1 + i)$$

substituting $P(1 + i)$ for P

$$= P(1 + i)^2$$

The Compound Interest Formula

This means that $P(1 + i)^2$ is the account balance at the beginning of the third period, and the future value at the end of the third period is

$$FV = [P(1 + i)^2] \cdot (1 + i) \qquad \text{substituting } P(1 + i)^2 \text{ for } P$$

$$= P(1 + i)^3$$

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If we generalize these results, we get the *Compound Interest Formula*.

Compound Interest Formula

If initial principal P earns compound interest at a periodic interest rate i for n periods, the future value is

$$FV = P(1 + i)^n$$

Example 2 – *Using the Compound Interest Formula*

$$FV = P\left(1+i\right)^n$$

Let's recall our Example 1: Tom and Betty deposit $1,000 into their new bank account. The account pays 8% interest compounded quarterly.

a. Let's find *i, periodic interest rate*, and *n (periods)*.
b. Use the compound interest formula to recompute Tom and Betty's account balance.

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Solution:

a. $i = \frac{1}{4} \cdot 8\%$

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b. $FV = P(1 + i)^n = 1{,}000(1 + 0.02)^2 = \$1{,}040.40$

Example 3 – *Compound Interest Over a Long Period of Time*

In 1777, it looked as though the Revolutionary War was about to be lost. George Washington's troops were camped at Valley Forge.

They had minimal supplies, and the winter was brutal. According to a 1990 class action suit, Jacob DeHaven, a wealthy Pennsylvania merchant, saved Washington's troops and the revolutionary cause by loaning Washington $450,000.

The suit, filed by DeHaven's descendants, asked the government to repay the still-outstanding loan plus compound interest at the then prevailing rate of 6%.

a. Find i and n.

b. How much did the government owe on the 1990 anniversary of the loan if the interest is compounded monthly?

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cont'd

In 1777, Jacob DeHaven loaned Washington $450,000.

In 1990 there was class action suit, with compound interest rate of 6%.

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$i = \frac{1}{12} \cdot 6\%$

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$i = \frac{1}{12} \cdot 6\% = 0.06/12 = 0.005$

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$n = 1990 - 1777 = 213$ years

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Compound Interest Compared with Simple Interest

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As we learned earlier compound interest is just simple interest, repeated.

However, there can be profound differences in their results.

Example 4 – *Comparing Simple Interest with Compound Interest Over a Long Period of Time*

How much would the government have owed the DeHavens if the interest was simple interest?

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Solution:

With simple interest, we use r and t, which are *annual* figures, rather than i and n, which are *periodic* figures.

So $r = 6\% = 0.06$, and $t = 213$ years.

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$= 450000(1 + 0.06 \cdot 213)$ substituting

$= \$6{,}201{,}000$ vs $\$154{,}762{,}723{,}400$

Example 4 – *Solution*

cont'd

Simple interest would have required a payment of only $6 million. This is a lot, but not in comparison with the $155 billion payment required by compound interest.

Finding the Interest and the Present Value

Example 5 – *Finding the Amount of Interest Earned*

Betty’s boss paid her an unexpected bonus of $2,500. Betty and her husband decided to save the money for their daughter’s education. They deposited it in account that pays 10.3% interest compounded daily. Find the amount of interest that they would earn in fifteen years by finding the future value and subtracting the principal.

Example 5 – *Finding the Amount of Interest Earned*

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$FV = P(1 + i)^n = 2{,}500\left(1 + \frac{0.103}{365}\right)^{5475} = 11{,}717.374\ldots \approx \$11{,}717.37$

Thus, the interest is \$11,717.37 – \$2,500 = \$9,217.37.

Example 6 – calculating the interest rate $FV = P(1 + i)^n$

Suppose $700 is invested for 2 years at a nominal yearly interest rate that is compounded monthly, further suppose it accumulates to $789.01 after 2 years. Find the annual nominal interest rate of the investment.

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using the formula: $700\left(1+\frac{r}{12}\right)^{24} = 789.01$

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using the formula: $$\frac{700\left(1+\frac{r}{12}\right)^{24}}{700} = \frac{789.01}{700}$$

dividing both sides by 700

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using the formula: $$\left(1 + \frac{r}{12}\right) = \sqrt[24]{\frac{789.01}{700}}$$

extracting 24th root of both sides

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using the formula:

$$1 + \frac{r}{12} = \sqrt[24]{\frac{789.01}{700}}$$

-1 -1

drop parentheses, subtract 1 from both sides

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using the formula: $$\frac{r}{12} = \sqrt[24]{\frac{789.01}{700}} - 1$$

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using the formula: $12 \times \frac{r}{12} = \left(\sqrt[24]{\frac{789.01}{700}} - 1\right) \times 12$ *multiply both sides by 12*

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hence $i = \frac{r}{12\,months}$

n – number of periods, hence n = 2 years $\times$12 months = 24 months

using the formula: $\cancel{12} \times \frac{r}{\cancel{12}} = \left(\sqrt[24]{\frac{789.01}{700}} - 1 \right) \times 12$ *cancellation of 12 on the left side*

Example 6 – calculating the interest rate $FV = P(1+i)^n$

Suppose \$700 is invested for 2 years at a nominal yearly interest rate that is compounded monthly, further suppose it accumulates to \$789.01 after 2 years. Find the annual nominal interest rate of the investment.

P = \$700, t = 2 years, FV = \$789.01
i – periodic rate (interest rate charged for each period),

hence $i = \frac{r}{12\,months}$

n – number of periods, hence n = 2 years $\times$12 months = 24 months

using the formula:

$$r = \left(\sqrt[24]{\frac{789.01}{700}} - 1\right) \times 12$$

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now use calculator to find r!

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using the formula: $$r = \left(\sqrt[24]{\frac{789.01}{700}} - 1\right) \times 12$$

now use calculator to find r!

$$r = 0.0599988260\ldots \approx 0.05999883 = 5.999883\,\%$$

Example 6 – calculating the interest rate $\quad FV = P(1 + i)^n$

Suppose \$700 is invested for 2 years at a nominal yearly interest rate that is compounded monthly, further suppose it accumulates to \$789.01 after 2 years. Find the annual nominal interest rate of the investment.

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i – periodic rate (interest rate charged for each period),

hence $i = \dfrac{r}{12\,months}$

n – number of periods, hence n = 2 years $\times$12 months = 24 months

using the formula: $$700\left(1+\frac{r}{12}\right)^{24} = 789.01$$

Annual Yield

Annual Yield

Bank One offers you a savings account with an interest rate of 5.80% compounded daily. Bank Two offers an account with an interest rate of 5.97% compounded annually.

You can't compare the rates *because the loans are compounded differently*.

Annual Yield

See Figure 5.4.

	Bank One	Bank Two	Which is better?
Compound rate	5.80% **compounded daily**	5.97% **compounded annually**	CAN'T TELL—you can't compare the interest rates because **the compounding periods are different**
Annual yield	5.97% **simple interest**	5.97% **simple interest**	THEY'RE THE SAME—you can compare the annual yields because **they're both simple interest rates**

Can't compare compound rates, can compare yields.

Figure 5.4

Annual Yield

Because of this, banks and other financial institutions are required to disclose their accounts' *annual yields*, as shown in Figure 5.5.

Banks disclose their yields as well as their rates.

Figure 5.5

Annual Yield

The **annual yield** corresponding to a compound interest rate is the *simple interest rate* that has the same future value as the compound rate has, in one year.

In Example 7, we'll see that the annual yield for each account is 5.97%.

Annual Yield

Annual Yield

The **annual yield** (also called the **yield** or the **annual percentage yield** or **APY**) corresponding to a compound interest rate is the *simple interest rate* that has the same future value as the compound rate has, in one year.

To calculate the annual yield:

$$FV(\text{simple interest}) = FV(\text{compound interest})$$ **because annual yield means the two future values are equal**

$$P(1 + rt) = P(1 + i)^n$$ **using the Future Value Formulas**

The values of t and n come from the fact that *annual* yield means *in one year*. The annual yield is r (round to the nearest hundredth of one percent).

Annual Yield

The annual yield should always be slightly higher than the compound rate, because compound interest is slightly more profitable than simple interest over a short period of time.

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

b. 5.97% compounded annually

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

b. 5.97% compounded annually

Solution:

a. *Finding the annual yield of 5.80% compounded daily:*

FV(simple interest) = FV(compound interest)

because annual yield means the two future values are equal

$$P(1 + rt) = P(1 + i)^n$$

using the future value formulas

Example 7 – *Solution*

cont'd

Annual yield means *in one year* so $t = 1$ year and
$n = 365$ days (we're compounding daily); 5.80% = 0.058

$$P(1 + r \cdot 1) = P\left(1 + \frac{0.058}{365}\right)^{365}$$ substituting for t, n, and i

$$1 + r \cdot 1 = \left(1 + \frac{0.058}{365}\right)^{365}$$ canceling the Ps

$$r = \left(1 + \frac{0.058}{365}\right)^{365} - 1$$ solving for r

$r = 0.0597101128\ldots \approx \mathbf{0.0597} =$ 5.97%

The annual yield corresponding to 5.80% compounded daily is 5.97%.

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

The annual yield corresponding to 5.80% compounded daily is 5.97%.

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$$P(1 + rt) = P(1 + i)^n$$

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

The annual yield corresponding to 5.80% compounded daily is 5.97%.

b. 5.97% compounded annually

$$P(1 + rt) = P(1 + i)^n$$

$$1 + r \cdot 1 = \left(1 + \frac{0.0597}{1}\right)^1$$

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

The annual yield corresponding to 5.80% compounded daily is 5.97%.

b. 5.97% compounded annually

$$P(1 + rt) = P(1 + i)^n$$

$$1 + r \cdot 1 = \left(1 + \frac{0.0597}{1}\right)^1$$

$$r = \left(1 + \frac{0.0597}{1}\right)^1 - 1$$

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

The annual yield corresponding to 5.80% compounded daily is 5.97%.

b. 5.97% compounded annually

$$P(1 + rt) = P(1 + i)^n$$

$$1 + r \cdot 1 = \left(1 + \frac{0.0597}{1}\right)^1$$

$$r = \left(1 + \frac{0.0597}{1}\right)^1 - 1 \qquad r = 0.0597 = 5.97\%$$

Example 7 – *Finding the Annual Yield*

Find the annual yield corresponding to an interest rate of:

a. 5.80% compounded daily

The annual yield corresponding to 5.80% compounded daily is 5.97%.

b. 5.97% compounded annually

$$P(1 + rt) = P(1 + i)^n$$

$$1 + r \cdot 1 = \left(1 + \frac{0.0597}{1}\right)^1$$

$$r = \left(1 + \frac{0.0597}{1}\right)^1 - 1 \qquad r = 0.0597 = 5.97\%$$

The annual yield corresponding to 5.97% compounded annually is 5.97%.

Annual Yield

In Example 7(b), the interest rate is 5.97% compounded annually, and the annual yield is also 5.97%. This always happens when we're compounding annually—the annual yield equals the compound rate.