5

Finance

5.1 Simple Interest

Objectives

- Make simple interest calculations
- Determine a credit card finance charge
- Find the payment required by an add-on interest loan

Simple Interest

Loans and investments are very similar financial transactions. Each involves

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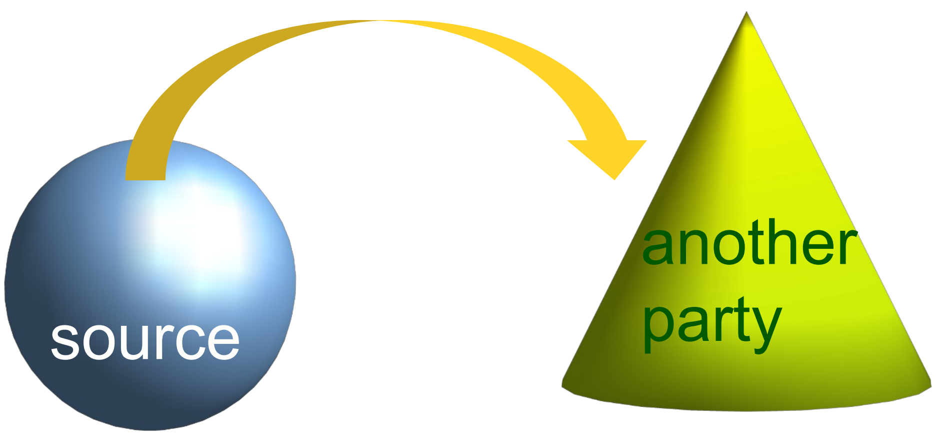
- the flow of money from a source to another party



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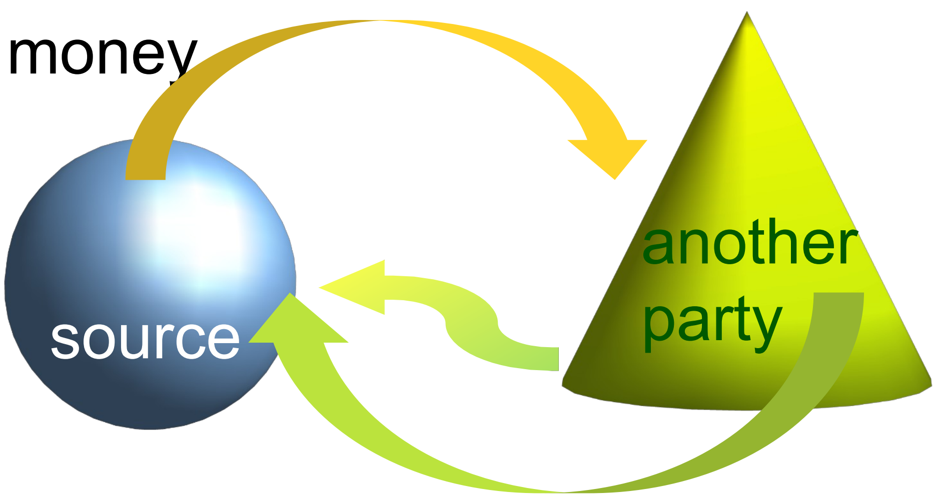
- the flow of money from a source to another party
- the return of the money to its source
- the payment of a fee to the source for the use of the money



Simple Interest

You make a deposit in your savings account:

- you are making an investment, but
- the bank views it as a loan; you are lending the bank your money, which they will lend to another customer, perhaps to buy a house.

Simple Interest

You borrow money to buy a car:

- you view the transaction as a loan, but
- the bank views it as an investment; the bank is investing its money in you in order to make a profit.

Simple Interest

Basic Finance Terms

Principal: the amount of money that is invested

Interest: the investor's profit

Interest rate: the percent of the principal that becomes interest

Time (or term): the amount of time that the money is invested

Simple Interest

In this section, we'll explore simple interest in investments and short-term loans.

Simple Interest

Simple interest means that the amount of interest is calculated as a percent per year of the principal. The simple interest I on a principal P at an annual simple interest rate of r for a timespan of t years is

$$I = Prt$$

Example 1 – *Using the Simple Interest Formula*

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They invest $150,000, so the principal is *P* = $150,000. The interest rate is *r* = 5.1% = 0.051, and the term is *t* = 2 years.

a. Find the interest that the investment earns.

b. Find the value of the CD at the end of its term.

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150,000 + 15,300 = 165,300 (\$)

- future value of the CD (i.e. what it will be worth in 2 years)

Using the Simple Interest Formula

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Exercise: Jeff bought a three-year CD that pays 7.3% simple interest from his bank for $70,000. Find the value of the CD at the end of its term.

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Exercise: Jeff bought a three-year CD that pays 7.3% simple interest from his bank for $70,000. Find the value of the CD at the end of its term.

Answer: $I = Prt = 70{,}000 \times 0.073 \times 3 = 15{,}330\,(\$)$

70,000 + 15,330 = 85,330 ($)

Future Value

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$$= P(1 + rt) \qquad \text{factoring}$$

Simple Interest Future Value

The **future value** of an account is the amount of money in that account at some time in the future.

The *FV* of a principal *P* at an annual simple interest rate *r* for *t* years is

$FV = P + I$ **use if you know *I***

$FV = P(1 + rt)$ **use if you don't know *I***

Capital Letters for Money

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It's common to confuse the interest *I* with the interest rate *r*. The interest rate *r* is a percentage, and the interest *I* is an amount of money. We will always use capital letters for variables that measure amounts of money and lowercase letters for other variables.

- *I*, *P*, and *FV* — capital letters so they measure amounts of money
- *r* and *t* — lowercase letters so they don't measure amounts of money

Finding the Number of Days: “Through” versus “To”

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In finance, it’s important to accurately determine the number of days your money is deposited, because you earn interest for each day. If your money was deposited for 890 days, it would be unfair if you were paid interest for only 889 days.

An important factor in finding the number of days is the use of the words “to” and “through.”

Finding the Number of Days: “Through” versus “To”

- There are two days from January 3 *to* January 5; they are January 3 and January 4. “*to* January 5” means “up to but not including the fifth.”

- There are three days from January 3 *through* January 5; they are January 3, January 4, and January 5. “*through* January 5” means to include the fifth.

Finding the Number of Days: “Through” versus “To”

- There are two days from January 3 *to* January 5;

  we could find it by subtracting 4 – 2 = 2

  - we start the subtraction with 4, the last day that we want to count;

  - we subtract 2 because we don’t want to include the first two days of the month.

Finding the Number of Days: “Through” versus “To”

- There are <u>three days</u> from January 3 *through* January 5; we could calculate this by subtracting 5 – 2 = 3.

  - we start the subtraction with 5, the last day that we want to count

  - we subtract 2 because we don’t want to include the first two days of the month.

Finding the Number of Days: “Through” versus “To”

Exercise: from March 5 *to* March 30, there are ___ days.

Finding the Number of Days: “Through” versus “To”

Exercise: from March 5 *to* March 30, there are ___ days.

Answer: 29 – 4 = 25 days

- we start the subtraction with 29, the last day we want to count

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Exercise: from March 5 *through* March 30, there are ___ days.

Answer: 30 – 4 = 26 days

-we start the subtraction with 30, the last day we want to count.

- we subtract 4 because we don’t want to include the first four days of the month.

Short-Term Loans

Short-Term Loans

One of the more common uses of simple interest is a *short-term* (such as a year or less) *loan* that requires a single lump sum payment at the end of the term.

Businesses routinely obtain these loans to purchase equipment or inventory, to pay operating expenses, or to pay taxes.

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A **lump sum** payment is a *single payment that pays off an entire loan*. Some loans require smaller monthly payments rather than a single lump sum payment.

Example 2 – *Using the Simple Interest Future Value Formula*

$$FV = P + I = P\left(1 + rt\right)$$

Espree Clothing borrowed $185,000 at $7\frac{1}{4}\%$ from January 1 to February 28.

a. Find the future value of the loan.

b. Interpret the future value.

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Solution:

a. We are given: P = 185,000, $r = 7\frac{1}{4}\%$ = 0.0725

Let's find t: 31 days in Jan. + 27 days to Feb. 28Th = 58 days

Note that interest rate is per year, so let's convert 58 days to year: $58 \div 365 \approx 0.1589041$

Using $FV = P(1+rt) = 185{,}000(1 + 0.0725 \times 0.1589041) \approx 187{,}131.30$

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Solution:

b. $FV \approx 187{,}131.30$

This means that Espree has agreed to make a lump sum payment to their lender $187,168.05 on February 28.

Calculation Notes

Calculation Notes

In Example 2, we do not use t = 2 months $\cdot \frac{1 \text{ year}}{12 \text{ months}} = \frac{2}{12}$ years. If we did, we would get an inaccurate answer because some months are longer and some are shorter. Instead, we use the number of days, converted to years: $t = \frac{58}{365}$ years. This represents the time more accurately.

In Example 2, we naturally used 365 days per year, but some institutions traditionally count a year as 360 days and a month as 30 days (especially if that tradition works in their favor).

This is a holdover from the days before calculators and computers—the numbers were simply easier to work with.

Calculation Notes

Also, we used normal round-off rules to round \$187,131.301 . . . to \$187,131.30; some institutions round off some interest calculations in their favor. In this book, we will count a year as 365 days and use normal round-off rules (unless stated otherwise).

A written contract signed by the lender and the borrower is called a **loan agreement** or a **note**. The **maturity value** of the note (or just the **value** of the note) refers to the note's future value. Thus, the value of the note in Example 2 was \$187,131.30. This is what the note is worth to the lender in the future—that is, when the note matures.

National Debt

National Debt

In almost every year since 1931, the U.S. federal budget called for deficit spending, that is, spending more money than is received.

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$$r = 0.0243344669\ldots \approx 0.024 = 2.4\,\%$$

Present Value

Example 4 – *Finding how Much to Invest Now*

Let's find the amount of money that must be invested now at a $5\frac{3}{4}\%$ simple interest so that it will be worth $1,000 in two years.

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r = $5\frac{3}{4}\%$ = 0.0575. Let's use $FV = P\left(1+rt\right)$ to find P:

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$$1000 = P(1.115)$$

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$$1000 = P(1 + 0.0575 \times 2)$$

$$1000 = P(1.115)$$

$$P = \frac{1000}{1.115} = 896.86098\ldots \approx 896.86\,(\$)$$

Present Value

In Example 4, the investment is worth \$1,000 two years *in the future*; that is, \$1000 is the investment's *future value*. But the same investment is worth \$896.86 *in the present*.

For this reason, we say that \$896.86 is the investment's **present value**.

In this case, this is the same thing as the principal; it is just called the *present value* to emphasize that this is its value in the present.

Add-on Interest

Add-on Interest

An **add-on interest loan** is an older type of loan that was common before calculators and computers, because calculations of such loans can easily be done by hand.

With this type of loan, the interest for the entire term of the loan is calculated as if no payments will be made until the loan matures.

The total interest is added on to the principal, and the borrower must repay this amount with equal payments over the term of the loan.

Add-on Interest

Add-on Interest Loan

To find the monthly payment of an add-on interest loan:

1. Find P, the loan amount.
2. Use the Simple Interest Formula to find I, the interest for the entire term of the loan.
3. Find the amount to be repaid by adding on the interest I to the loan amount P.
4. Divide the result by the number of payments.

Example 5 – *An Add-on Interest Loan*

Chip Douglas's car died, and he must replace it right away. Centerville Auto Sales has a nine-year-old Ford that's "like new" for $5,988.

The sign in their window says, "Bad credit? No problem!" They offered Chip a 5% two-year add-on interest loan if he made a $600 down payment. Let's find the monthly payment.

Example 5 – *An Add-on Interest Loan*

Centerville Auto Sales has a nine-year-old Ford that's "like new" for \$5,988. They offered Chip a 5% two-year add-on interest loan if he made a \$600 down payment. Let's find the monthly payment.

Solution:

- The loan amount is P = 5,988 – 600 = \$5,388.
- The interest for the entire two years of the loan is
  $I = Prt = 5{,}388 \cdot 0.05 \cdot 2 = \$538.80$
- The amount to be repaid is $P + I$ = 5,388 + 538.80 = \$5,926.80
- This total is to be spread out over 24 monthly payments, so the monthly payment is \$5926.80 ÷ 24 = \$246.95

Chip has to pay \$600 at the time of purchase, and \$246.95 a month for 24 months.

Credit Card Finance Charge

Credit Card Finance Charge

One of the most common methods of calculating credit card interest is the average daily balance method. The **average daily balance** is the weighted average of each daily balance, with each balance weighted to reflect the number of days at that balance. The finance charge consists of simple interest, charged on the result.

Credit Card Finance Charge

To find the credit card finance charge with the average daily balance method, do the following:

1. Find the balance for each day in the billing period and the number of days at that balance.
2. The average daily balance is the weighted average of these daily balances, weighted to reflect the number of days at that balance.
3. The finance charge is simple interest applied to the average daily balance.

Example 6 – *Finding a Credit Card Finance Charge*

The activity on Tom and Betty's Visa account for one billing is shown below. The billing period is October 15 through November 14, the previous balance was $346.57, and the annual in rate is 21%.

October 21	payment	$50.00
October 23	restaurant	$42.14
November 7	clothing	$18.55

a. Find the average daily balance.

b. Find the finance charge.

Example 6(a) – *Solution*

(a) To find the average daily balance, we have to know the balance for each day in the billing period and the number of days at that balance, as shown in Figure 5.1.

Time Interval	Days	Daily Balance
October 15 through October 20	$20 - 14 = 6$	\$346.57
October 21 through October 22	$22 - 20 = 2$	\$346.57 − \$50 = \$296.57
October 23 through November 6 (October has 31 days)	$31 - 22 = 9$ $9 + 6 = 15$	\$296.57 + \$42.14 = \$338.71
November 7 through November 14	$14 - 6 = 8$	\$338.71 + \$18.55 = \$357.26

Preparing to find the average daily balance.

Figure 5.1

Example 6(a) – *Solution*

cont'd

The daily balance of \$346.57 occurred for 6 days, and the daily balance of \$296.57 occurred for 2 days, so we multiply \$346.57 by 6 and we multiply \$296.57 by 2, to reflect this.

Average daily balance = $\dfrac{6 \cdot 346.57 + 2 \cdot 296.57 + 15 \cdot 338.71 + 8 \cdot 357.26}{6 + 2 + 15 + 8}$

$= 342.29967 \ldots \approx \$342.30$

(b) $P = \$342.29967 \ldots$; $r = 21\% = 0.21$; $t = 31$ days $= \dfrac{31}{356}$ years

Finding *I*:

$I = Prt = 342.29967 \ldots \cdot 0.21 \cdot \dfrac{31}{356} = 6.1051258 \ldots \approx \$6.11$

Answer: the average daily balance is ≈ \$342.30 and the finance charge is ≈ \$6.11

How Many Days?

How Many Days?

Financial calculations usually involve computing the term t either in a whole number of years or in the number of days converted to years.

This requires that you know the number of days in each month.

How Many Days?

As you can see from Figure 5.2, the months alternate between thirty-one days and thirty days, with two exceptions:

- February has twenty-eight days (twenty-nine in leap years).
- The alternation does not happen from July to August.

January	31 days	February	28 days	March	31 days
April	30 days	May	31 days	June	30 days
July	31 days	August	31 days	September	30 days
October	31 days	November	30 days	December	31 days

The number of days in a month.

Figure 5.2